SL TYPE 1

Portfolio: Infinte Summation

Math SL Section 1

2011-06-18

Infinity has always been the mysterious dark realm of the human kind's knowledge. Not only is the idea of being able to find out a meaning for infinite sequences of numbers unbelievable, but the process also possesses a level of excitement. In this task, an attempt will be made to discover an exact value for an infinite sequence. Despite the limitations of human—or any other beings—being unable to see the end of the endlessly-continuing series of numbers, steps of mathematical process will be implemented to provide a generalization that is as accurate as possible.

This report will investigate the sum of infinite sequences t_n, where

$$t_0 = 1, t_1 = \frac{(x\ln a)}{1}, t_2 = \frac{(x\ln a)^2}{2\times 1}, t_3 = \frac{(x\ln a)^3}{3\times 2\times 1}\cdots, t_n = \frac{(x\ln a)^n}{n!}\cdots$$

During the process of obtaining the sum of t_n, a generalized statement will be reached, based on the pattern of the sequence attained by assessing the influences of n, x, and a on the sequence.

First of all, the assignment provides the values $x = 1$ and $a = 2$. Applying these values into the initial sequences t_n, a new sequences is achieved.

$$1, \frac{(\ln 2)}{1!}, \frac{(\ln 2)^2}{2!}, \frac{(\ln 2)^3}{3!}\cdots$$

In order to deduce a pattern from the sequences, the varying values for S_n, the sum of the sequences for different values of n will be analyzed. Calculations of the first 10 terms are implemented via GDC and are shown in the following equations. The data is also organized in **Table 1**. *(Note: All data in this assignment are correct to six decimal points.)*

$$S_0 = 1$$

$$S_1 = 1 + \frac{(\ln 2)}{1!} = 1.693147$$

$$S_2 = 1 + \frac{(\ln 2)}{1!} + \frac{(\ln 2)^2}{2!} = 1.933374$$

$$S_3 = 1 + \frac{(\ln 2)}{1!} + \frac{(\ln 2)^2}{2!} + \frac{(\ln 2)^3}{3!} = 1.988878$$

$$S_4 = 1 + \frac{(\ln 2)}{1!} + \frac{(\ln 2)^2}{2!} + \frac{(\ln 2)^3}{3!} + \frac{(\ln 2)^4}{4!} = 1.998496$$

$$S_5 = 1 + \frac{(\ln 2)}{1!} + \frac{(\ln 2)^2}{2!} + \frac{(\ln 2)^3}{3!} + \frac{(\ln 2)^4}{4!} + \frac{(\ln 2)^5}{5!} = 1.999829$$

$$S_6 = 1 + \frac{(\ln 2)}{1!} + \frac{(\ln 2)^2}{2!} + \frac{(\ln 2)^3}{3!} + \frac{(\ln 2)^4}{4!} + \frac{(\ln 2)^5}{5!} + \frac{(\ln 2)^6}{6!} = 1.999983$$

$$S_7 = 1 + \frac{(\ln 2)}{1!} + \frac{(\ln 2)^2}{2!} + \frac{(\ln 2)^3}{3!} + \frac{(\ln 2)^4}{4!} + \frac{(\ln 2)^5}{5!} + \frac{(\ln 2)^6}{6!} + \frac{(\ln 2)^7}{7!} = 1.999999$$

$$S_8 = 1 + \frac{(\ln 2)}{1!} + \frac{(\ln 2)^2}{2!} + \frac{(\ln 2)^3}{3!} + \frac{(\ln 2)^4}{4!} + \frac{(\ln 2)^5}{5!} + \frac{(\ln 2)^6}{6!} + \frac{(\ln 2)^7}{7!} + \frac{(\ln 2)^8}{8!} = 2$$

$$S_9 = 1 + \frac{(\ln 2)}{1!} + \frac{(\ln 2)^2}{2!} + \frac{(\ln 2)^3}{3!} + \frac{(\ln 2)^4}{4!} + \frac{(\ln 2)^5}{5!} + \frac{(\ln 2)^6}{6!} + \frac{(\ln 2)^7}{7!} + \frac{(\ln 2)^8}{8!} + \frac{(\ln 2)^9}{9!} = 2$$

$$S_{10} = 1 + \frac{(\ln 2)}{1!} + \frac{(\ln 2)^2}{2!} + \frac{(\ln 2)^3}{3!} + \frac{(\ln 2)^4}{4!} + \frac{(\ln 2)^5}{5!} + \frac{(\ln 2)^6}{6!} + \frac{(\ln 2)^7}{7!} + \frac{(\ln 2)^8}{8!} + \frac{(\ln 2)^9}{9!} + \frac{(\ln 2)^{10}}{10!} = 2$$

$n=0$	1.000000
$n=1$	1.693147
$n=2$	1.933374
$n=3$	1.988878
$n=4$	1.998496
$n=5$	1.999829
$n=6$	1.999983
$n=7$	1.999999
$n=8$	2.000000
$n=9$	2.000000
$n=10$	2.000000

Table 1: Values of S_n **when** $x=1$, $a=3$, $0 \le n \le 10$

In order to visualize the set of data, a graph can be plotted based on the values of S_n.

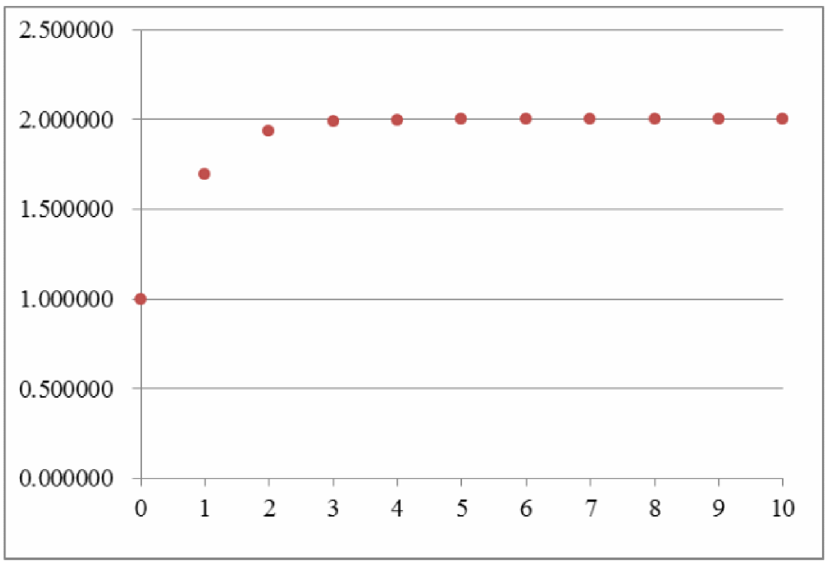

Figure 1: Value of S_n **when** $a=2$

Observation of both **Table 1** and **Figure 1** suggest that the value of S_n increases, approaching 2 as the value of n approaches ∞. With the numbers rounded up to six decimal places, the result of the last three terms round up to the number 2. Thus, a statement about the relationship between the values of n and S_n can be made using limitation. This is denoted as following:

$$\lim_{n\to\infty}\sum_{r=0}^{n}\frac{(\ln 2)^r}{r!}=2$$

Next, another set of values, $x=1$ and $a=3$ will be used to formulate another sequence:

$$1, \frac{(\ln 3)}{1!}, \frac{(\ln 3)^2}{2!}, \frac{(\ln 3)^3}{3!} \cdots$$

Using the same mechanism as done with values $x=1$ and $a=2$, the value of S_n can be calculated using GDC. **Table 2** shows the values of S_n as n increases, when $x=1$ and $a=3$. Using the table, a graph can be plotted in order to visualize the relationship between the values of S_n and n.

$n=0$	1.000000
$n=1$	2.098612
$n=2$	2.702087
$n=3$	2.923082
$n=4$	2.983779
$n=5$	2.997115
$n=6$	2.999557
$n=7$	2.999940
$n=8$	2.999993
$n=9$	2.999999
$n=10$	3.000000

Table 2: Values of S_n **when** $x=1$, $a=3$, $0 \le n \le 10$

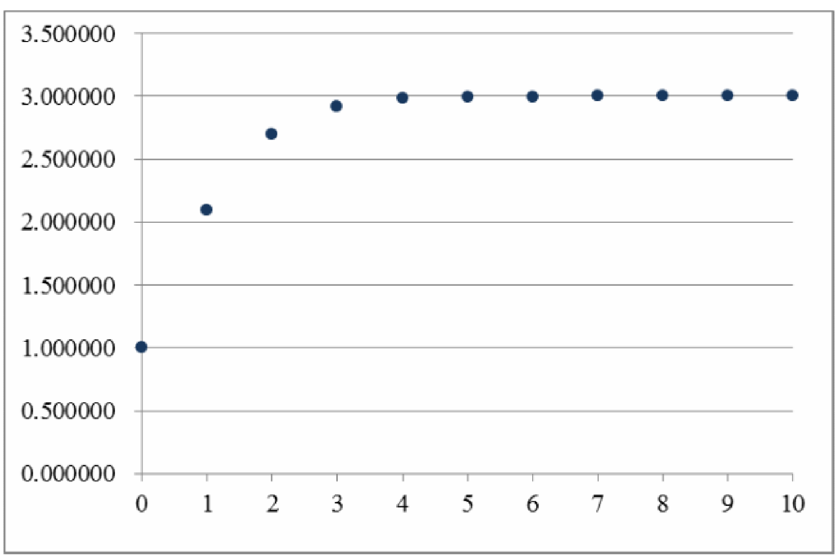

Figure 2: Value of S_n **when** $a=3$

Again, observing Table 2 and Figure 2, a noticeable pattern of the numbers is that the values of S_n approaches 3 as n approaches ∞. Another generalization using limitation can be made, as denoted following:

$$\lim_{n\to\infty}\sum_{r=0}^{n}\frac{(\ln 3)^r}{r!}=3$$

Now, a general sequence with the x value as 1 will be used to derive a generalized statement.

$$1,\frac{(\ln a)}{1!},\frac{(\ln a)^2}{2!},\frac{(\ln a)^3}{3!}\cdots$$

Since calculating an exact value of an undefined number is not possible, an attempt will be made to generalize a statement about a using multiple values for a. Below, in **Figure 3**, a graph was plotted using the S_n values when $a=2$, $a=3$, $a=4$, $a=5$, $a=6$, $a=7$, $a=8$, $a=9$, and $a=10$, using the method used previously.

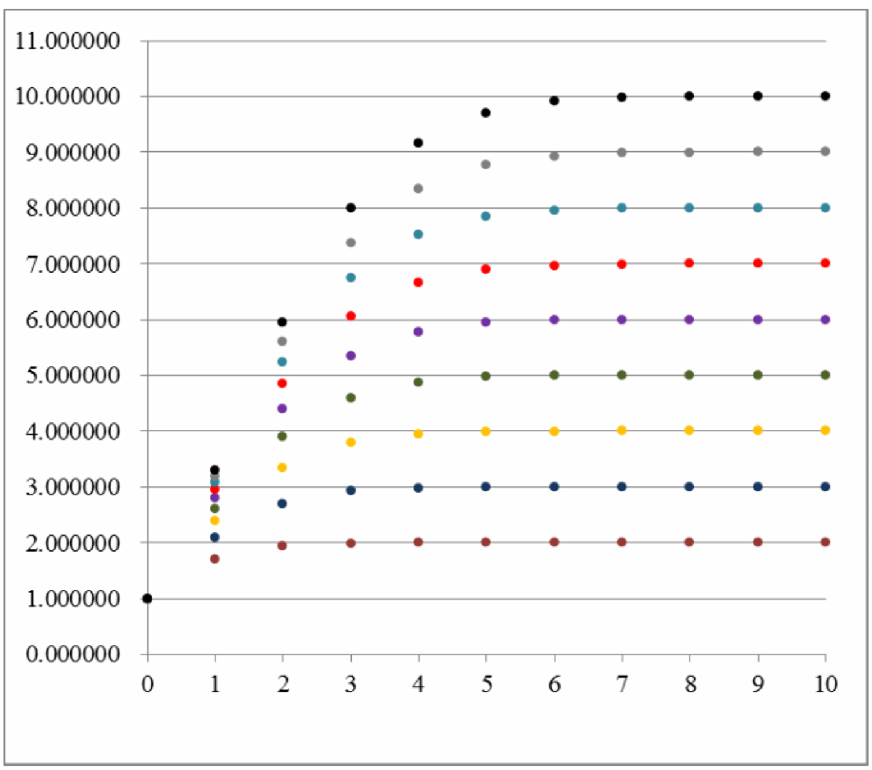

Figure 3: Values fo S_n with multiple a values

Here, depending on the value of a, the values of S_n approaches relative values of a. When $a = 4$, S_n approaches 4 as n approaches ∞, and when $a = 10$, S_n approaches 10 as n approaches ∞. Without loss of generality, a general statement can be drawn, denoted as following:

$$\lim_{n \to \infty} \sum_{r=0}^{n} \frac{(\ln a)^r}{r!} = a$$

Now that a general statement for an undefined a is established, the investigation will expand to encompass two undefined numbers, x and a. The sum of t_n will be determined, where

$$t_0 = 1, t_1 = \frac{(x \ln a)}{1!}, t_2 = \frac{(x \ln a)^2}{2!}, t_3 = \frac{(x \ln a)^3}{3!} \cdots$$

Here, $T_n(a, x)$, the sum of the first n terms for various values of a and x. In order to compare the sums and discover patterns from various x values, the n value will be set at 9 and the a value will be set at 2. Below, **Table 3** shows the calculated values for $T_9(2, x)$, using the same method for calculating values of S_n.

	T_9
$x = 1$	2.000000
$x = 2$	3.999992
$x = 3$	7.999488
$x = 4$	15.990193
$x = 5$	31.900922
$x = 6$	63.331066
$x = 7$	124.572949

Table 3: Values of T_9 for $a = 2$

A visible trait in **Table 3** is that as the values of T_9 increase in a nearly exponential pattern. For $x = 1$, the value of T_9 approaches 2, $x = 2$ approaches 4, $x = 3$ 8, and so on. This draws a hint about the general pattern of the sum of an infinite sequence when the a value is fixed. In order to visualize the values for an analysis of a pattern, **Figure 4** plots the values on a graph.

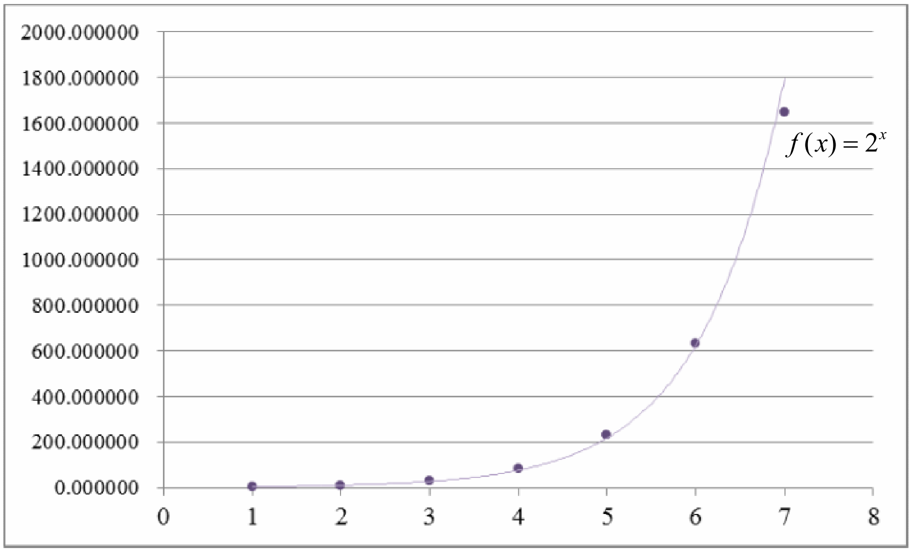

Figure 4: Values of T_n when $a=2$

Along with the data values of T_n, the exponential graph of $f(x)=2^x$ is plotted for comparative analysis. Observing the values suggests that the exponential graph of $f(x)=2^x$ matches the values of T_n, which allows a generalized statement:

$$\lim_{n\to\infty} T_n(2,x) = 2^x$$

In order to reach a conclusion, values of T_n for $a=3$, $a=4$, and $a=5$ will be analyzed. **Table 4** shows the data calculated via the same method used previously.

	$a=3$	$a=4$	$a=5$
$x=1$	2.999999	3.999991	4.999962
$x=2$	8.999101	15.990192	24.954056
$x=3$	26.941276	63.331066	121.747436
$x=4$	79.803290	241.624473	551.663426
$x=5$	230.019331	857.473473	2221.100395
$x=6$	633.777653	2770.933209	7836.811421
$x=7$	1649.507247	8117.577207	22402.123516

Table 4: Values of T_9 for different a values

In order to analyze the influence of a values on the graph of T_9 values, a graph is plotted below. (See **Figure 5**)

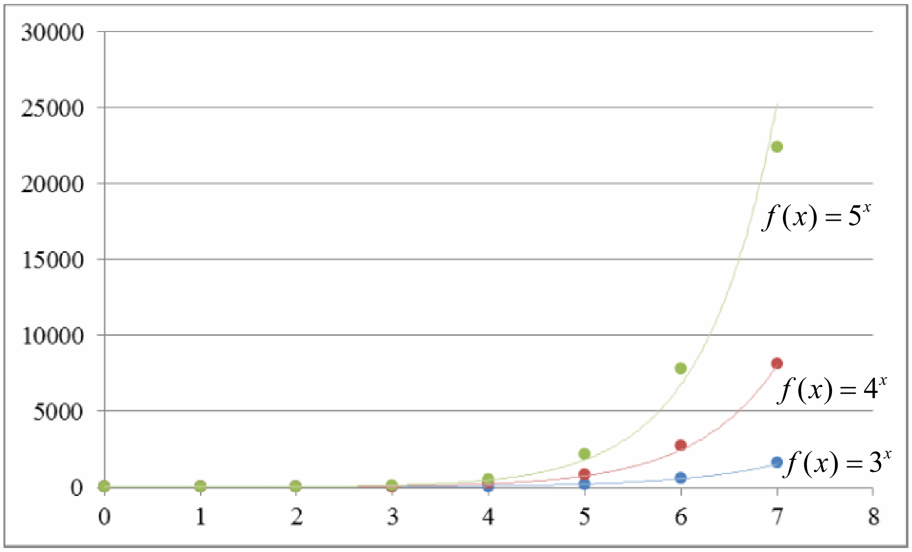

Figure 5: T_9 **with different** x **and** a **values**

Figure 5, along with exponential graphs $f(x)=3^x$, $f(x)=4^x$, and $f(x)=5^x$, show the tendency of of the T_9 to match the relative exponential graphs with each a values matching the base of the relative exponential functions. This leads to a general statement about the tendency of each equation.

$$\lim_{n\to\infty} T_n(3,x)=3^x$$
$$\lim_{n\to\infty} T_n(4,x)=4^x$$
$$\lim_{n\to\infty} T_n(5,x)=5^x$$

This leads to a general statement:

$$\lim_{n\to\infty} T_n(a,x)=a^x$$

In final statement of the general traits discovered in this assignment, the values of n in the sums of the infinite sequences t_n are accountable for the precision of the sums to the value of a. The x values are accountable for the exponent of the value of a that is reached by the sequence. Finally, the a values are accountable for the base of the equation, whereas also influencing the distance of the data values to the tendency of the values to match the function: $f(x)=a^x$.

Thus, encompassing all the data and statements derived from previous analyses, a general conclusion can be drawn:

$$\lim_{n \to \infty} \sum_{r=0}^{n} \frac{(x \ln a)^r}{r!} = a^x$$

There are limitations to this statement. Since the values of all data stretch infinitely when put in their actual form, the values were correct to six decimal places. Thus, at some point as the n values increased, the data showed no difference in the sum of the sequence despite the increase of the n value. For instance, in **Table 1**, the values of S_n for $n=8$, $n=9$, and $n=10$ showed no visible difference. The expression of using six decimal places also accounts for the decline of the inclination of the data to match the general statement as the a values increased.

Through this assignment, an attempt to discover a general statement about an infinite sequence was undertaken. A general statement has been reached, taking a step further into 'knowing the infinity.' Despite the limitations of the precision of number values, the general statement provides a glimpse into the unknown realm of infinity. Mankind has always strived to achieve the 'impossible.' This report, along with many great works of numerous mathematicians, shall be an additional proof of mankind's attempt to tame the infinity.

END