

The manager of a wine bottling plant has received a special order for two wine bottles. The two wine bottles require corks with diameters between 2.7 centimeters and 3.3 centimeters to properly fit the opening of the bottle. From past experience, the manager knows that corks used at his bottling plant have diameters that are normally distributed with a mean of 3 centimeters and a standard deviation of 1 centimeter.

1. Determine the probability of finding a cork of the appropriate size in plant cork bin selected at random. Round your answer to the nearest thousandth.
 - a. A Normal Probability Distribution is symmetric, single-peaked, and bell shaped. The points of inflection lie between plus and minus one standard deviations from the mean. The total area under the curve is zero, and the empirical rule values should fit relatively well.
 - b. To solve this problem, we are looking for the $P(2.7 \text{ cm} < x < 3.3 \text{ cm})$. To find this probability, we need to standardize the 2.7 cm and 3.3 cm. Do to so, we have to use the formula $z = (x_i - \bar{x}) / s$ which is your data value minus the mean, and all of that divided by the standard deviation.
 - i. Z-score for 2.7 is $(2.7 - 3) / 1 = -0.3$
 - ii. Z-score for 3.3 is $(3.3 - 3) / 1 = 0.3$
 - c. Once the z-score is determined, we use the z-score probability chart to determine the probability associated with the z-score.
 - i. $P(z = -0.3) = .3821$
 - ii. $P(z = 0.3) = .6179$
 - d. To find the probability with the associated interval, we subtract the probability from -0.3 from 0.3.
 - i. $.6179 - .3821 = .2358$
 - e. The $P(2.7 \text{ cm} < x < 3.3 \text{ cm})$ rounded to the nearest thousandth place is .236 or 23.6%.
2. Describe how you would use a table of random digits to carry out a simulation to determine the number of corks that must be examined to find two corks of the appropriate size. Include a complete description of what each of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 will represent in your simulation.
 - a. Problem:
 - i. To simulate 23.6% probability of selecting corks between the diameters of 2.7 centimeters and 3.3 centimeters.
 - b. Key Components:
 - i. Each cork is chosen independently; one cork has no effect on the next selection.
 - ii. We are modeling 23.6% of choosing corks between 2.7 cm and 3.3 cm.
 - iii. Each trial must be at least two tries because we are looking for two corks between the stated interval.
 - c. Underlying Assumptions
 - i. None of the corks are damaged.
 - d. Model:
 - i. Use the numbers 0 – 235 to stand for “yes”—meaning that these corks are between 2.7 centimeters and 3.3 centimeters.
 - ii. 236 – 999 to stand for “no”—meaning that these corks are not between 2.7 centimeters and 3.3 centimeters.
 - iii. We are going to use a random digit table.
 - e. Trial:
 - i. A trial must be at least two tries so that it ensures that at least two corks are selected of the right size.

- ii. For the trial to end, there must be two “yes”s, so that there is two corks with the diameters desired.
 - f. Observation of Interest:
 - i. The diameters of the corks. Specifically, the corks with the diameters between 2.7 centimeters and 3.3 centimeters.
3. Perform your simulation three times. (That is, run 3 trials of your simulation.) Start at the left most digit in the **first row** of the table and move across. Describe your simulation run in sufficient detail to make clear the reader. Demonstrate by marking your table.
 - a. Refer to previous question for trial in detail.
 - b. Table of Random Digits:

<u>Number from</u> <u>Table of Random</u> <u>Digits</u>	<u>Yes or No</u>	<u>Number from</u> <u>Table of Random</u> <u>Digits</u>	<u>Yes or No</u>
630	No	167	Yes
357	No	222	Yes/END TRIAL
684	No	024	Yes
093	Yes/END TRIAL	756	No
552	No	457	No
012	Yes	570	No
618	No	197	Yes/END TRIAL

The results of two 115-trial simulations, one searching for two corks between 2.7 cm and 3.3 cm in diameters, and the other searching for two corks between 1.8 cm and 2.5 cm are shown below.

4. Identify which distribution A or B, represents the search for the two corks between 2.7 cm and 3.3 cm in diameters. Explain your reasoning.
 - a. Distribution B represents the search for corks between 2.7 cm and 3.3 cm in diameter, because the probability to find a cork between 1.8 cm and 2.5 cm is smaller than the probability for finding a cork between 2.7 cm and 3.3 cm. This is because the smaller the probability value, the more corks one must examine. Since the interval $2.7 \text{ cm} < x < 3.3 \text{ cm}$ is greater than the $1.8 \text{ cm} < x < 2.5 \text{ cm}$, it should require a less amount of corks to be examined—only exemplified by the graph of Distribution B.

- b. To solve this problem, we are looking for the $P(1.8 \text{ cm} < x < 2.5 \text{ cm})$. To find this probability, we need to standardize the 2.7 cm and 3.3 cm. Do to so, we have to use the formula $z = (x_i - \bar{x}) / s$ which is your data value minus the mean, and all of that divided by the standard deviation.
 - i. Z-score for 1.8 = $(1.8 - 3) / 1 = -1.2$
 - ii. Z-score for 2.5 = $(2.5 - 3) / 1 = -0.5$
 - c. Once the z-score is determined, we use the z-score probability chart to determine the probability associated with the z-score.
 - i. $P(z = -1.2) = .1151$
 - ii. $P(z = -0.5) = .3085$
 - d. To find the probability with the associated interval, we subtract the probability from -1.2 from -0.5.
 - i. $.3085 - .1151 = .1934$
 - e. Rounded to the nearest thousandth place, $P(1.8 \text{ cm} < x < 2.5 \text{ cm})$ is approximately .193 or 19.3%.
 5. Compute an estimate of the expected number of corks that must be examined in the cork bin to find two corks between 2.7 cm and 3.3 cm in diameter. Use the appropriate distribution above.
 - a. The best estimate of the expected number of corks is the sample mean of approximately 6.7, for distribution B. This can be arrived by using the formula:
 - i. $x = (\sum x * f) / n$ where f stands for frequency (or number of trials for this situation), n is the number of trial simulations performed in total, and x is the value on the x-axis for that particular bar.
 - o $(3*6) + (4*10) + (5*16) + (6*22) = (7*24) + (8*13) + (9*12) + (10*5) + (11*2) + (12*4) = 770$.
 - o $770 / 15 = 6.69565217391$ which rounds to 6.7.