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Maths Internal Assessment: Shady Areas

Introduction:

In this investigation you will attempt to find a rule to approximate the area under a curve (i.e. between the curve and the x – axis) using trapeziums (trapezoids).

Let us consider using the function:$g(x) = x^2 + 3$. Before finding the area under the curve using trapeziums. I will use integration to help me find the exact area. This will enable me to compare the results I acquire using the trapezium method and to find out whether the use of more trapeziums will provide me with a more accurate estimation of the area under the graph.

$$g(x) = x^2 + 3$$

$$\int_0^1 x^2 + 3\, dx$$

$$\int_0^1 \frac{x^3}{3} + 3x$$

$$\left(\frac{1}{3} + 3\right) - (0) = \frac{10}{3}$$

Using the integration method, I have found the exact area for the function $g(x) = x^2 + 3$ is $\frac{10}{3}$. I will now use the trapezium method to find the approximation of the area under the curve. I have used Autograph to show the graph for the function of g.

0 0.5

It can clearly be seen from the graph that the area under the curve is roughly by the sum of the areas of two trapeziums.

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The area of a trapezium can be found using the formula: $\frac{1}{2}(a+b)h$

Let a = the length of one side of the trapezium
b = the length of the other side of the trapezium
h = the vertical height between the two lengths

In order,

Although there are two trapeziums, there are three different values and in order to determine the area of the trapeziums at each value I will work out the values on the graph using the trapezium method.

I will now input the values the graph displays into the function:

$$g(x_0)\ or\ g(x) = x^2+3$$
$$g(0) = (0)^2+3$$
$$x_0 = 3$$

$$g(x_1)\ or\ g(x) = x^2+3$$
$$g(0.5) = (0.5)^2+3$$
$$x_1 = 3.25$$

x	y
0	3
0.5	3.25
1	4

$$g(x_2)\ or\ g(x) = x^2+3$$
$$g(1) = (1)^2+3$$
$$x_2 = 4$$

Now, I have worked out the answers using the values from the graph. I have shown you that I can work out the area of each trapezium individually.

For the area (A) of the first trapezium, I will replace 'a' and 'b' with the 'x_0' and 'x_1' values that I had calculated earlier but I will show my working:

A= $\frac{1}{2}(a+b)h$

A= $\frac{1}{2}(3+3.25)\times 0.5$

A_1= 1.5625

For the area (A) of the second trapezium, I will replace 'a' and 'b' with the 'x_1' and 'x_2' values that I had calculated earlier but I will show my working:

A= $\frac{1}{2}(a+b)h$

A= $\frac{1}{2}(3.25+4)\times 0.5$

A_2= 1.8125

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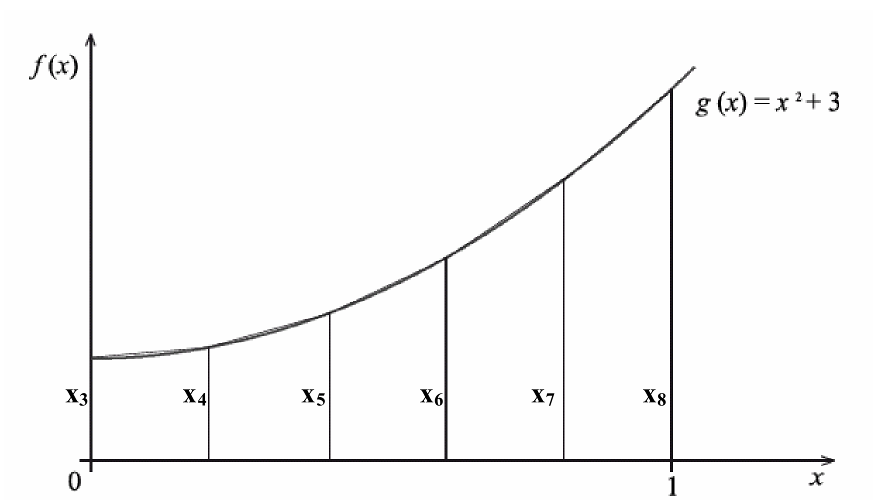
As you know that $A_1 + A_2$ is equal to the area under the curve. So the area under the curve is equal to 1.5625 + 1.8125 = 3.375

The difference between the exact value previously done using the integration method and the estimated area under the curve using the trapeziums = $-\frac{1}{24}$.

$$\left(\frac{10}{3} - 3.375 = -\frac{1}{24}\right)$$

This shows us that the estimates using trapeziums is an over-estimate of the actual area (the estimated area under the curve of the two trapeziums is greater than the actual area under the curve).

I am now going to increase the number of trapeziums to five and find second set of estimates for the area under the curve. This will tell me whether increasing the number of trapeziums will make the estimates more accurate. The graph below is now divided into five trapeziums.



Using the same method I used when there were only two trapeziums, I will calculate the approximated area under this curve. As there are five trapeziums, I will divide the horizontal height (horizontal height =1) divide by the five trapeziums displayed in the graph.

$h = \frac{1}{5}$

$h = 0.2$

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To find each of the lengths (labeled x_3-x_8), I will substitute the values on the X – axis into the function as I had done previously for x_0, x_1 and x_2. I know that when I substitute the value 0, I get the answer for $g(x_3)\ or\ g(0) = 3$. When I substitute the value 1, I get the answer for $g(x_8)\ or\ g(1) = 4$.

$$g(x_4)\ or\ g(x) = x^2 + 3$$
$$g(0.2) = (0.2)^2 + 3$$
$$x_4 = 3.04$$

$$g(x_5)\ or\ g(x) = x^2 + 3$$
$$g(0.4) = (0.4)^2 + 3$$
$$x_5 = 3$$

$$g(x_6)\ or\ g(x) = x^2 + 3$$
$$g(0.6) = (0.6)^2 + 3$$
$$x_6 = 3$$

$$g(x_7)\ or\ g(x) = x^2 + 3$$
$$g(0.8) = (0.8)^2 + 3$$
$$x_7 = 3$$

x	y
0	3
0.2	3.04
0.4	3.16
0.6	3.36
0.8	3.64
1	4

I have worked out the value for the lengths (x_4 - x_7), through my working I can work out the areas of the five trapeziums. If I add all the areas together, I get an estimated area under the curve.

Area under the curve	Area for each trapezium	a	b	Height (h)
	Area for first Trapezium			
		3	3.04	0.2
Area	0.604			
	Area for second Trapezium			
		3.04	3.16	0.2
Area	0.62			
	Area for Third Trapezium			
		3.16	3.36	0.2
Area	0.652			
	Area for Fourth Trapezium			
		3.36	3.64	0.2
Area	0.7			
	Area for Fifth Trapezium			
		3.64	4	0.2
Area	0.764			
Total Area	3.34			

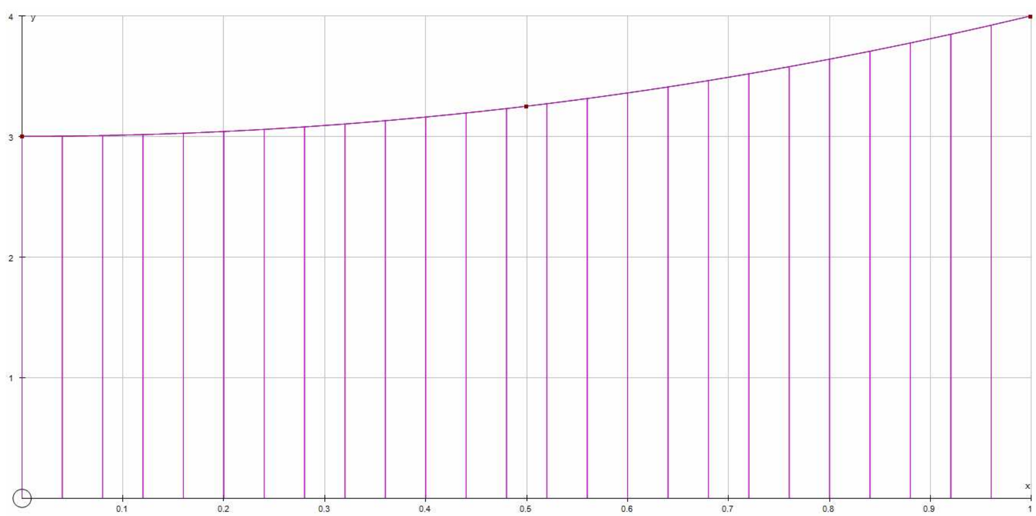
If the areas of the five trapeziums are added together, they give me the estimated area under the curve which is 3.34

0.604 + 0.62 + 0.652 + 0.7 + 0.764 = 3.34

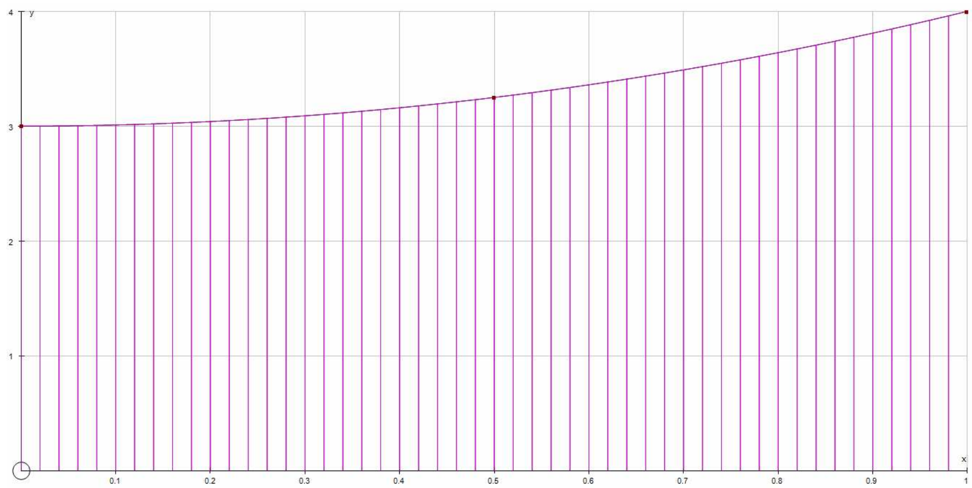
The area given using the five trapeziums is still larger than the actual area found using the integration method ($\frac{10}{3} - 3.34 = -\frac{1}{150}$). Therefore it is an over – estimate of the actual area.

However, the area obtained using five trapeziums are closer to the actual area than the area obtained using two trapeziums. The difference is not much. It can be said that using more trapeziums provides us with a lesser under/over estimate of the estimated area under the curve. To test this hypothesis, I will use 25,50,75 and 100 trapeziums to reach an estimate of the area under the curve.

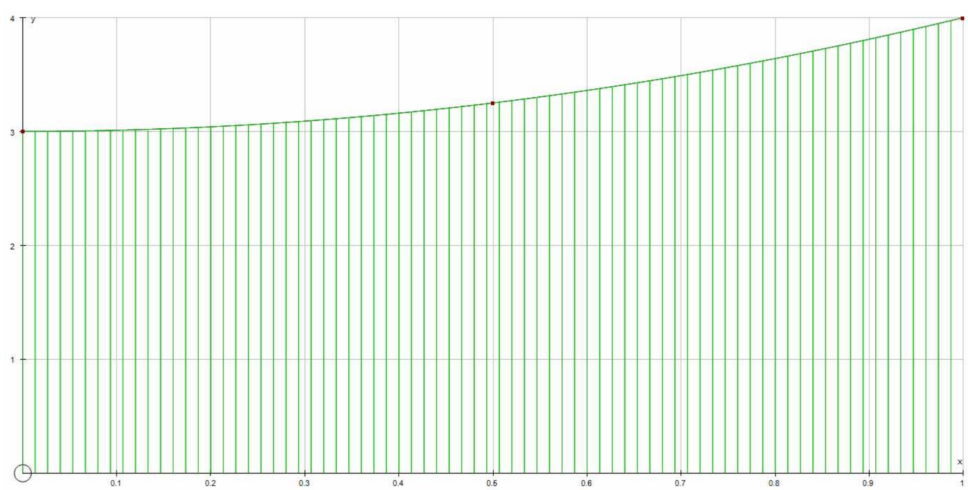
Using Autograph and Microsoft Excel 2007, I have created the following tables and graphs:



Graph of the function $g(x) = x^2 + 3$ .The area under the curve is divided into 25 trapeziums, and the area is given as 3.334 (4 s.f.).



Graph of the function $g(x) = x^2 + 3$ .The area under the curve is divided into 50 trapeziums, and the area is given as 3.334 (4 s.f.).



Graph of the function $g(x) = x^2 + 3$ .The area under the curve is divided into 75 trapeziums, and the area is given as 3.334 (4 s.f.).

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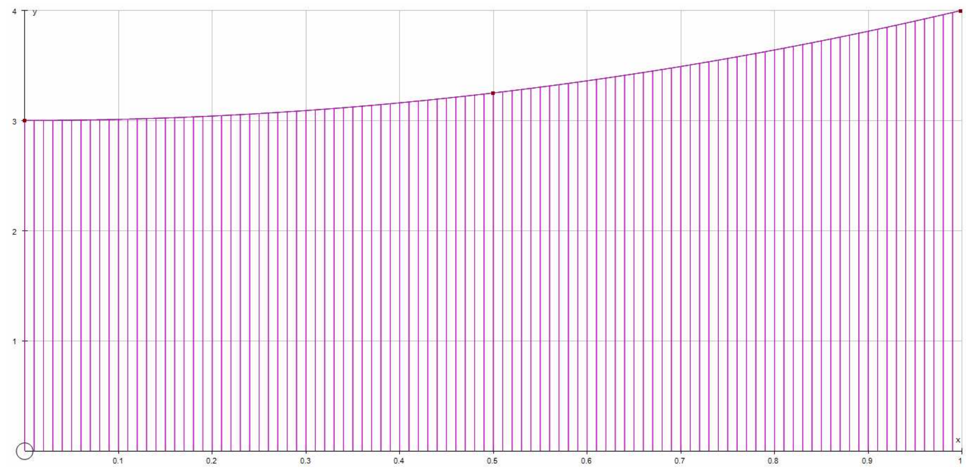

Graph of the function $g(x) = x^2 + 3$ .The area under the curve is divided into 100 trapeziums, and the area is given as 3.334 (4 s.f.).