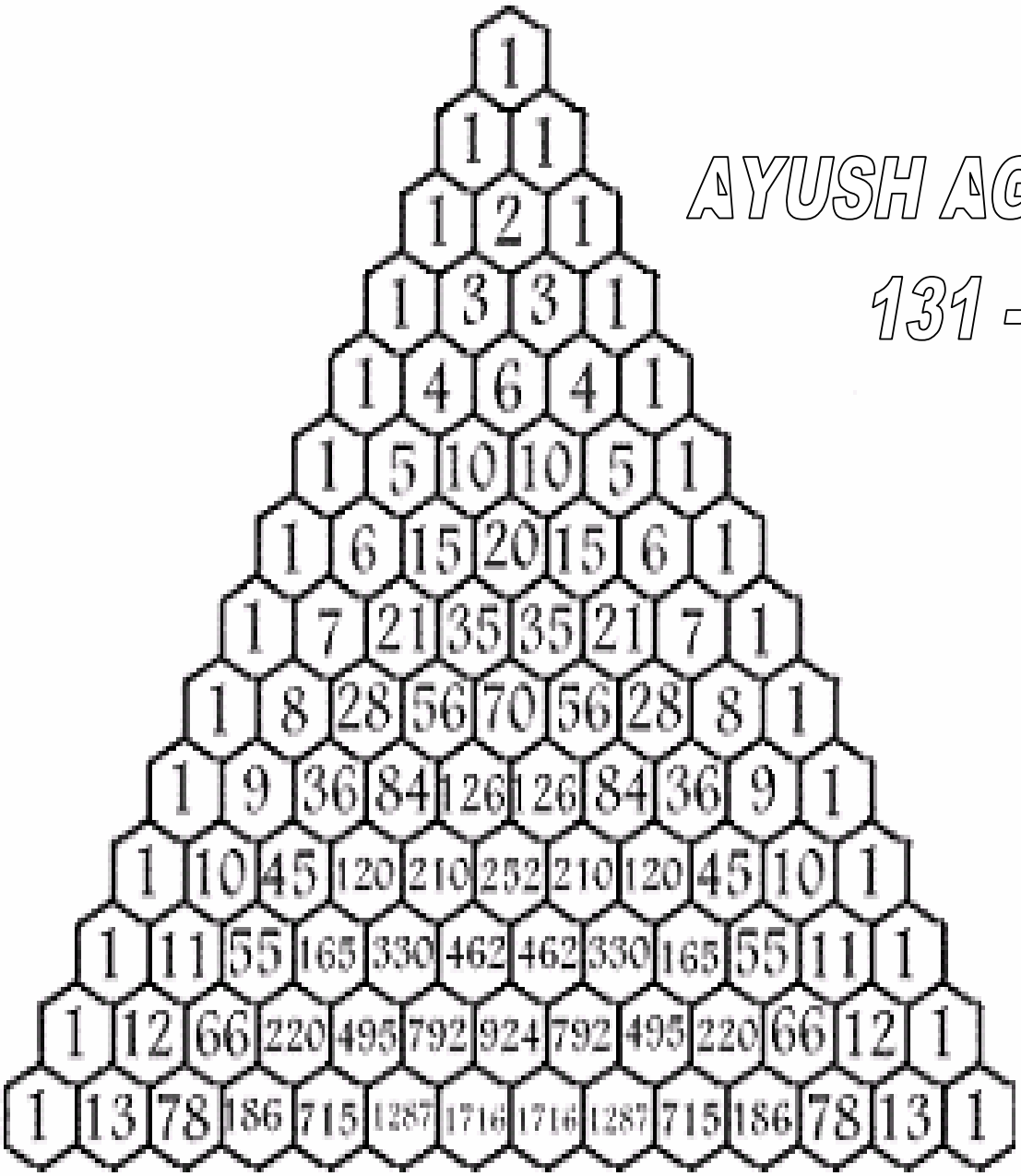
Marked by Teachers

This document was downloaded from www.markedbyteachers.com

SEQUENCES AND SERIES

AYUSH AGARWAL

131 - J



Marked by Teachers
This document was downloaded from www.markedbyteachers.com

INTRODUCTION

The definition of **sequence** is a list of numbers written in a particular order. These numbers are called the terms of the sequence. For example, 2,4,6,8 are the first four terms of positive even integers. When the sum of the terms in a sequence is taken, we will get something called a **series**. For example, 2+4+6+8+… is a series. The terms in a sequence is denoted by T_n where n is the number of the term in question.
In my portfolio there are certain conventions used such as:
'+' denotes addition
'–' denotes subtraction
'x' denotes multiplication
'/' denotes division
'∑' denotes summation
'⇒' denotes equal to

OBSERVATION & ANALYSIS

Q1. Consider the sequence $\{a_n\}^{\infty}_{n=1}$ where

$a_1 = 1 \times 2$
$a_2 = 2 \times 3$
$a_3 = 3 \times 4$
$a_4 = 4 \times 5$
.
.
.

Find an expression for a_n, the general term in the sequence.

A1.

Let p = 1
$a_1 = 1 \times 2 = p(p+1)$
$a_2 = 2 \times 3 = (p+1)(p+2)$
$a_3 = 3 \times 4 = (p+2)(p+3)$
$a_4 = 4 \times 5 = (p+3)(p+4)$
.
.
Thus,
$a_n = (p+n-1)(p+n)$
Since p = 1
Therefore,
$\boxed{\mathbf{a_n = n(n+1)}}$

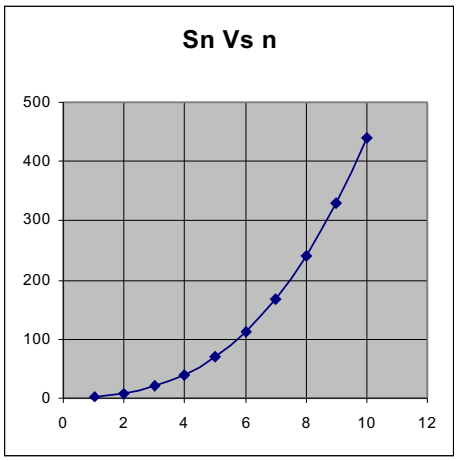
Marked by Teachers
This document was downloaded from www.markedbyteachers.com

Q2. Consider the series $S_n = a_1 + a_2 + a_3 + \ldots + a_n$ where a_k is defined as above.
(a). Determine several values of S_k, including S_1, S_2, S_3, …, S_6 and note observations.
(b). Formulate a conjecture for a general expression for S_n.
(c). Prove your conjecture by induction.
(d). Using the above result, calculate $1^2 + 2^2 + 3^2 + \ldots + n^2$.

A2.
(a).
$S_n = a_1 + a_2 + a_3 + \ldots\ldots\ldots\ldots + a_n$

$S_1 = a_1 = 1 \times 2$
$= 2$
$S_2 = a_1 + a_2 = (1 \times 2) + (2 \times 3)$
$= 2 + 6$
$= 8$
$S_3 = a_1 + a_2 + a_3 = (1 \times 2) + (2 \times 3) + (3 \times 4)$
$= 2 + 6 + 12$
$= 20$
$S_4 = a_1 + a_2 + a_3 + a_4 = (1 \times 2) + (2 \times 3) + (3 \times 4) + (4 \times 5)$
$= 2 + 6 + 12 + 20$
$= 40$
$S_5 = a_1 + a_2 + a_3 + a_4 + a_5 = (1 \times 2) + (2 \times 3) + (3 \times 4) + (4 \times 5) + (5 \times 6)$
$= 2 + 6 + 12 + 20 + 30$
$= 70$
$S_6 = a_1 + a_2 + a_3 + a_4 + a_5 + a_6 = (1 \times 2) + (2 \times 3) + (3 \times 4) + (4 \times 5) + (5 \times 6) + (6 \times 7)$
$= 2 + 6 + 12 + 20 + 30 + 42$
$= 112$

A graph between S_n and *n* showing how S_n changes with a change in *n*.



Marked by Teachers
This document was downloaded from www.markedbyteachers.com

(b).

$S_n = (1 \times 2) + (2 \times 3) + (3 \times 4) + \ldots\ldots + n(n+1)$

$S_n = \sum n(n+1) = \sum (n^2 + n)$
$= \sum n^2 + \sum n$

We know that $n^2 - (n-1)^2 = n^2 - n^2 + 2n - 1$
$= 2n - 1$

Thus,

$$
\begin{aligned}
1^2 - 0^2 &= 2 \times 1 - 1 \\
2^2 - 1^2 &= 2 \times 2 - 1 \\
3^2 - 2^2 &= 2 \times 3 - 1 \\
&\;\;\vdots \\
+\; n^2 - (n-1)^2 &= 2 \times n - 1 \\
\hline
n^2 - 0^2 &= 2 \times (1+2+3+\ldots+n) - n
\end{aligned}
$$

$\Rightarrow n^2 + n = 2 \times (\sum n)$
$\Rightarrow \sum n = (n^2 + n)/2$
$= n(n + 1)/2$

We know that $n^3 - (n-1)^3 = n^3 - n^3 + 3n^2 - 3n + 1$
$= 3n^2 - 3n + 1$

Thus,

$$
\begin{aligned}
1^3 - 0^3 &= 3(1)^2 - 3(1) + 1 \\
2^3 - 1^3 &= 3(2)^2 - 3(2) + 1 \\
3^3 - 2^3 &= 3(3)^2 - 3(3) + 1 \\
&\;\;\vdots \\
+\; n^3 - (n-1)^3 &= 3(n)^2 - 2(n) + 1 \\
\hline
n^3 - 0^3 &= 3(1^2+2^2+3^2+\ldots+n^2) - 3(1+2+3+\ldots+n) + n
\end{aligned}
$$

$\Rightarrow n^3 - n = 3[(\sum n^2) - (\sum n)]$
$\Rightarrow (n^3 - n)/3 = (\sum n^2) - [n(n + 1)/2]$
$\Rightarrow \sum n^2 = (n^3 - n)/3 + n(n + 1)/2$
$= (2n^3 - 2n + 3n^2 + 3n)/6$
$= (2n^3 + 3n^2 + n)/6$
$= n(2n^2 + 3n + 1)/6$ (taking *n* as common)
$= n(n + 1)(2n + 1)/6$

Marked by Teachers
This document was downloaded from www.markedbyteachers.com

Hence,
$S_n = \sum n^2 + \sum n$
$= [n(n + 1)(2n + 1)/6] + [n(n + 1)/2]$
$= [n(n + 1)(2n + 1) + 3n(n + 1)]/6$ (taking 6 as the LCM)
$= [n(n + 1)(2n + 1 + 3)]/6$
$= [2n(n + 1)(n + 2)]/6$
$= n(n + 1)(n + 2)/3$

$\boxed{\mathbf{S_n = n(n + 1)(n + 2)/3}}$

(c).
$1x2 + 2x3 + 3x4 + \ldots + n(n+1) = n(n + 1)(n + 2)/3$

When n = 1
LHS = 1x2 = 2; RHS = (1 x 2 x 3)/3 = 6/3 = 2
Thus, S_1 is true.
When n = 2
LHS = 1x2 + 2x3 = 8; RHS = (2 x 3 x 4)/3 = 24/3 = 8
Thus, S_2 is true.

Let it be true for n = k
Thus,
$1x2 + 2x3 + 3x4 + \ldots + k(k+1) = k(k + 1)(k + 2)/3$ ……[1]

To prove for n = k + 1
$\boxed{1x2 + 2x3 + 3x4 + \ldots + (k + 1)(k + 2) = (k + 1)(k + 2)(k + 3)/3}$

<u>LHS</u>
$= 1x2 + 2x3 + 3x4 + \ldots + (k + 1)(k + 2)$
$= 1x2 + 2x3 + 3x4 + \ldots + k(k + 1) + (k + 1)(k + 2)$
$= [k(k + 1)(k + 2)/3] + (k + 1)(k + 2)$ ……From [1]
$= [k(k + 1)(k + 2) + 3(k + 1)(k + 2)]/3$
$= (k + 1)(k + 2)(k + 3)/3 =$ <u>RHS</u>

Thus, as S_{k+1} is true, S_k is also true. Since S_1 and S_2 are also true, so S_n is true for all $n \in Z^+$.

(d).

As formulated in (b) above,
$\mathbf{1^2 + 2^2 + 3^2 + \ldots + n^2 = n(n + 1)(2n + 1)/6}$

Marked by Teachers
This document was downloaded from www.markedbyteachers.com

Q3. Consider $T_n = 1x2x3 + 2x3x4 + 3x4x5 + \ldots + n(n + 1)(n + 2)$

(a). Determine several values of T_k, including T_1, T_2, T_3, …, T_6 and note observations.

(b). Formulate a conjecture for a general expression for T_n.

(c). Prove your conjecture by induction.

(d). Using the above result, calculate $1^3 + 2^3 + 3^3 + \ldots + n^3$.

A3.

(a).

$T_n = 1x2x3 + 2x3x4 + 3x4x5 + \ldots + n(n + 1)(n + 2)$

$T_1 = 1 \text{ x } 2 \text{ x } 3$
$= 6$

$T_2 = (1 \text{ x } 2 \text{ x } 3) + (2 \text{ x } 3 \text{ x } 4)$
$= 6 + 24$
$= 30$

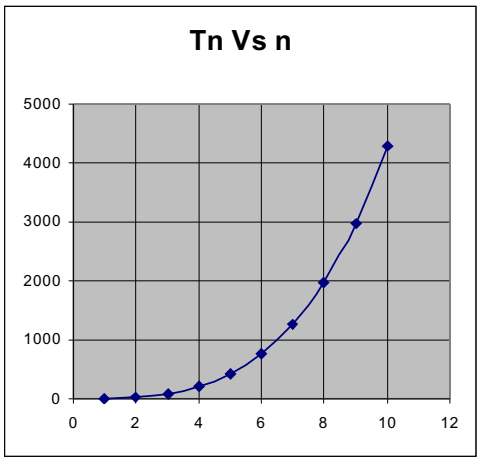
$T_3 = (1 \text{ x } 2 \text{ x } 3) + (2 \text{ x } 3 \text{ x } 4) + (3 \text{ x } 4 \text{ x } 5)$
$= 6 + 24 + 60$
$= 90$

$T_4 = (1 \text{ x } 2 \text{ x } 3) + (2 \text{ x } 3 \text{ x } 4) + (3 \text{ x } 4 \text{ x } 5) + (4 \text{ x } 5 \text{ x } 6)$
$= 6 + 24 + 60 + 120$
$= 210$

$T_5 = (1 \text{ x } 2 \text{ x } 3) + (2 \text{ x } 3 \text{ x } 4) + (3 \text{ x } 4 \text{ x } 5) + (4 \text{ x } 5 \text{ x } 6) + (5 \text{ x } 6 \text{ x } 7)$
$= 6 + 24 + 30 + 120 + 210$
$= 420$

$T_6 = (1 \text{ x } 2 \text{ x } 3) + (2 \text{ x } 3 \text{ x } 4) + (3 \text{ x } 4 \text{ x } 5) + (4 \text{ x } 5 \text{ x } 6) + (5 \text{ x } 6 \text{ x } 7) + (6 \text{ x } 7 \text{ x } 8)$
$= 6 + 24 + 60 + 120 + 210 + 336$
$= 756$

A graph between T_n and *n* showing how T_n changes with a change in *n*.



Marked by Teachers
This document was downloaded from www.markedbyteachers.com

(b).

$T_n = (1 \times 2 \times 3) + (2 \times 3 \times 4) + (3 \times 4 \times 5) + \ldots + n(n + 1)(n + 2)$

$T_n = \sum n(n + 1)(n + 2)$
$= \sum (n^3 + 3n^2 + 2n)$
$= \sum n^3 + 3\sum n^2 + 2\sum n$

We know that $n^4 - (n-1)^4 = n^4 - n^4 + 4n^3 - 6n^2 + 4n - 1$
$= 4n^3 - 6n^2 + 4n - 1$

Thus,

$$
\begin{aligned}
1^4 - 0^4 &= 4(1)^3 - 6(1)^2 + 4(1) - 1 \\
2^4 - 1^4 &= 4(2)^3 - 6(2)^2 + 4(2) - 1 \\
3^4 - 2^4 &= 4(3)^3 - 6(3)^2 + 4(3) - 1 \\
&\;\;\vdots \\
+\; \underline{n^4 - (n-1)^4 = 4(n)^3 - 6(n)^2 + 4(n) - 1} \\
n^4 - 0^4 &= 4(1^3 + 2^3 + \ldots + n^3) - 6(1^2 + 2^2 + \ldots + n^2) + 4(1 + 2 + \ldots + n) - n
\end{aligned}
$$

$\Rightarrow n^4 = 4\sum n^3 - 6\sum n^2 + 4\sum n - n$
$\Rightarrow n^4 = 4\sum n^3 - [6n(n + 1)(2n + 1)/6] + [4n(n + 1)/2] - n$
$\Rightarrow n^4 = 4\sum n^3 - [n(n + 1)(2n + 1)] + [2n(n + 1)] - n$
$\Rightarrow \sum n^3 = [n^4 + n + (2n^3 + 3n^2 + n) - (2n^2 + 2n)]/4$
$= (n^4 + n + 2n^3 + 3n^2 + n - 2n^2 - 2n)/4$
$= (n^4 + 2n^3 + n^2)/4$
$= [n^2(n^2 + 2n + 1)]/4$
$\sum n^3 = [n^2(n + 1)^2]/4$

Hence,

$T_n = \sum n^3 + 3\sum n^2 + 2\sum n$
$= [n^2(n + 1)^2/4] + [3n(n + 1)(2n + 1)/6] + [2n(n + 1)/2]$
$= [n^2(n + 1)^2/4] + [n(n + 1)(2n + 1)/2] + [n(n + 1)]$
$= [n^2(n + 1)^2 + 2n(n + 1)(2n + 1) + 4n(n + 1)]/4$ (taking 4 as the LCM)
$= n(n + 1)[n(n + 1) + 2(2n + 1) + 4]/4$
$= n(n + 1)(n^2 + n + 4n + 2 + 4)/4$
$= n(n + 1)(n^2 + 5n + 6)/4$
$= n(n + 1)(n + 2)(n + 3)/4$

$\boxed{\mathbf{T_n = n(n + 1)(n + 2)(n + 3)/4}}$

Marked by Teachers
This document was downloaded from www.markedbyteachers.com

(c).
$1x2x3 + 2x3x4 + 3x4x5 + \ldots + n(n + 1)(n + 2) = n(n + 1)(n + 2)(n + 3)/4$

When n = 1
LHS = 1x2x3 = 6; RHS = (1 x 2 x 3 x 4)/4 = 6
Thus, T_1 is true.
When n = 2
LHS = 1x2x3 + 2x3x4 = 30; RHS = (2 x 3 x 4 x 5)/4 = 30
Thus, T_2 is true.

Let it be true for n = k.
Thus,
$1x2x3 + 2x3x4 + 3x4x5 + \ldots + k(k + 1)(k + 2) = k(k + 1)(k + 2)(k + 3)/4$ ……...[1]

To prove for n = k + 1,
$\boxed{1x2x3 + 2x3x4 + 3x4x5 + \ldots + (k + 1)(k + 2)(k + 3) = (k + 1)(k + 2)(k + 3)(k + 4)/4}$

<u>LHS</u>
$= 1x2x3 + 2x3x4 + 3x4x5 + \ldots + (k + 1)(k + 2)(k + 3)$
$= 1x2x3 + 2x3x4 + 3x4x5 + \ldots + k(k + 1)(k + 2) + (k + 1)(k + 2)(k + 3)$
$= [k(k + 1)(k + 2)(k + 3)/4] + [(k + 1)(k + 2)(k + 3)]$ ……..From [1]
$= (k + 1)(k + 2)(k + 3)[(k/4) + 1]$
$= (k + 1)(k + 2)(k + 3)(k + 4)/4 =$ <u>RHS</u>

Thus, as T_{k+1} is true, T_k is also true. Since T_1 and T_2 are also true, so T_n is true for all $n \in Z^+$.

(d).
As formulated in (b) above,
$\mathbf{1^3 + 2^3 + 3^3 + \ldots + n^3 = n^2(n + 1)^2/4}$

Marked by Teachers
This document was downloaded from www.markedbyteachers.com

Q4. Consider $U_n = 1x2x3x4 + 2x3x4x5 + 3x4x5x6 + \ldots + n(n + 1)(n + 2)(n + 3)$

(a). Determine several values of U_k, including U_1, U_2, U_3, …, U_6 and note observations.

(b). Formulate a conjecture for a general expression for U_n.

(c). Prove your conjecture by induction.

(d). Using the above result, calculate $1^4 + 2^4 + 3^4 + \ldots + n^4$.

A4.

(a).

$U_n = 1x2x3x4 + 2x3x4x5 + 3x4x5x6 + \ldots + n(n + 1)(n + 2)(n + 3)$

$U_1 = 1x2x3x4$
$= 24$

$U_2 = 1x2x3x4 + 2x3x4x5$
$= 24 + 120$
$= 144$

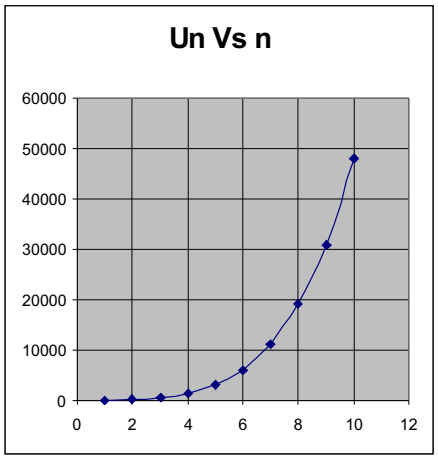
$U_3 = 1x2x3x4 + 2x3x4x5 + 3x4x5x6$
$= 24 + 120 + 360$
$= 504$

$U_4 = 1x2x3x4 + 2x3x4x5 + 3x4x5x6 + 4x5x6x7$
$= 24 + 120 + 360 + 840$
$= 1344$

$U_5 = 1x2x3x4 + 2x3x4x5 + 3x4x5x6 + 4x5x6x7 + 5x6x7x8$
$= 24 + 120 + 360 + 840 + 1680$
$= 3024$

$U_6 = 1x2x3x4 + 2x3x4x5 + 3x4x5x6 + 4x5x6x7 + 5x6x7x8 + 6x7x8x9$
$= 24 + 120 + 360 + 840 + 1680 + 3024$
$= 6048$

A graph between U_n and *n* showing how U_n changes with a change in *n*.



Marked by Teachers
This document was downloaded from www.markedbyteachers.com

(b).

$U_n = (1 \times 2 \times 3 \times 4) + (2 \times 3 \times 4 \times 5) + (3 \times 4 \times 5 \times 6) + \ldots + n(n + 1(n + 2)(n + 3)$

$$\begin{aligned} U_n &= \sum n(n + 1)(n + 2)(n + 3) \\ &= \sum (n^2 + n)(n^2 + 5n + 6) \\ &= \sum (n^4 + 6n^3 + 11n^2 + 6n) \\ &= \sum n^4 + 6\sum n^3 + 11\sum n^2 + 6\sum n \end{aligned}$$

We know that $n^5 - (n-1)^5 = n^5 - n^5 + 5n^4 - 10n^3 + 10n^2 - 5n + 1$
$= 5n^4 - 10n^3 + 10n^2 - 5n + 1$

Thus,

$$\begin{aligned} 1^5 - 0^5 &= 5(1)^4 - 10(1)^3 + 10(1)^2 - 5(1) + 1 \\ 2^5 - 1^5 &= 5(2)^4 - 10(2)^3 + 10(2)^2 - 5(2) + 1 \\ 3^5 - 2^5 &= 5(3)^4 - 10(3)^3 + 10(3)^2 - 5(3) + 1 \\ &\vdots \\ + n^5 - (n-1)^5 &= 5(n)^4 - 10(n)^3 + 10(n)^2 - 5(n) + 1 \end{aligned}$$

$n^5 - 0^5 = 5(1^4+2^4+\ldots+n^4) - 10(1^3+2^3+\ldots+n^3) + 10(1^2+2^2+\ldots+n^2) - 5(1+2+\ldots+n) + n$

$\Rightarrow n^5 = 5\sum n^4 - 10\sum n^3 + 10\sum n^2 - 5\sum n + n$

$\Rightarrow n^5 = 5\sum n^4 - [5n^2(n + 1)^2/2] + [5n(n + 1)(2n + 1)/3] - [5n(n + 1)/2] + n$

$\Rightarrow 5\sum n^4 = n^5 + [5n^2(n + 1)^2/2] - [5n(n + 1)(2n + 1)/3] + [5n(n + 1)/2] - n$

$\Rightarrow \sum n^4 = (6n^5 + 15n^4 + 30n^3 + 15n^2 - 20n^3 - 30n^2 - 10n + 15n^2 + 15n - 6n)/30$
$= (6n^5 + 15n^4 + 10n^3 - n)/30$
$= n(n + 1)(2n + 1)(3n^2 + 3n - 1)/30$

Hence,

$$\begin{aligned} U_n &= \sum n^4 + 6\sum n^3 + 11\sum n^2 + 6\sum n \\ &= (6n^5 + 15n^4 + 10n^3 - n)/30 + 3n^2(n + 1)^2/2 + 11n(n + 1)(2n + 1)/6 + 3n(n + 1) \\ &= (6n^5 + 15n^4 + 10n^3 - n + 45n^4 + 90n^3 + 45n^2 + 110n^3 + 165n^2 + 55n + 90n^2 + 90n)/30 \\ &= (6n^5 + 60n^4 + 210n^3 + 300n^2 + 144n)/30 \\ &= 6n(n^4 + 10n^3 + 35n^2 + 50n + 24)/30 \\ &= n(n^4 + 10n^3 + 35n^2 + 50n + 24)/5 \\ &= n(n + 1)(n + 2)(n + 3)(n + 4)/5 \end{aligned}$$

$\boxed{\mathbf{U_n = n(n + 1)(n + 2)(n + 3)(n + 4)/5}}$

Marked by Teachers
This document was downloaded from www.markedbyteachers.com

(c).
$1x2x3x4 + 2x3x4x5 + 3x4x5x6 +\ldots+ n(n + 1)(n + 2)(n + 3) = n(n + 1)(n + 2)(n + 3)(n + 4)/5$

When n = 1
LHS = 1x2x3x4 = 24; RHS = (1 x 2 x 3 x 4 x 5)/5 = 24
Thus, U_1 is true.
When n = 2
LHS = 1x2x3x4 + 2x3x4x5 = 144; RHS = (2 x 3 x 4 x 5 x 6)/5 = 144
Thus, U_2 is true.

Let it be true for n = k.
Thus,
$1x2x3x4+2x3x4x5+\ldots+k(k + 1)(k + 2)(k + 3) = k(k + 1)(k + 2)(k + 3)(k + 4)/5$ ….[1]

To prove for n = k + 1
$\boxed{1x2x3x4 + 2x3x4x5 +\ldots+ (k + 1)(k + 2)(k + 3)(k + 4) = (k + 1)(k + 2)(k + 3)(k + 4)(k + 5)/5}$

<u>LHS</u>
$= 1x2x3x4 + 2x3x4x5 +\ldots+ (k + 1)(k + 2)(k + 3)(k + 4)$
$= 1x2x3x4 + 2x3x4x5 +\ldots+ k(k + 1)(k + 2)(k + 3) + (k + 1)(k + 2)(k + 3)(k + 4)$
$= [k(k + 1)(k + 2)(k + 3)(k + 4)/5] + (k + 1)(k + 2)(k + 3)(k + 4)$
$= (k + 1)(k + 2)(k + 3)(k + 4)[(k/5) + 1]$ ……..From [1]
$= (k + 1)(k + 2)(k + 3)(k + 4)(k + 5)/5 =$ <u>RHS</u>

Thus, as U_{k+1} is true, U_k is also true. Since U_1 and U_2 are also true, so U_n is true for all $n \in Z^+$.

(d).

As formulated in (b) above,
$1^4 + 2^4 + 3^4 +\ldots+ n^4 = n(n + 1)(2n + 1)(3n^2 + 3n - 1)/30$

Marked by Teachers

This document was downloaded from www.markedbyteachers.com

Q5. With the patterns noted above, can you formulate a conjecture for the series $1^k + 2^k + 3^k + \ldots + n^k$?

A5.
Following the same procedure in the above questions, we know that

$$n^{k+1} - (n-1)^{k+1} = n^{k+1} - [{}^{k+1}C_0.n^{k+1} - {}^{k+1}C_1.n^k + {}^{k+1}C_2.n^{k-1} - {}^{k+1}C_3.n^{k-2} + \ldots + {}^{k+1}C_{k+1}.n^{k-k}.(-1)^{k+1}]$$
$$= n^{k+1} - [n^{k+1} - {}^{k+1}C_1.n^k + {}^{k+1}C_2.n^{k-1} - {}^{k+1}C_3.n^{k-2} + \ldots + (-1)^{k+1}]$$
$$= n^{k+1} - n^{k+1} + {}^{k+1}C_1.n^k - {}^{k+1}C_2.n^{k-1} + {}^{k+1}C_3.n^{k-2} + \ldots - (-1)^{k+1}$$
$$= {}^{k+1}C_1.n^k - {}^{k+1}C_2.n^{k-1} + {}^{k+1}C_3.n^{k-2} + \ldots - (-1)^{k+1}$$
$$= (k+1)n^k - [k(k+1)/2]n^{k-1} + [k(k+1)(k-1)/6]n^{k-2} + \ldots - (-1)^{k+1} \quad \ldots\ldots^{1}$$

Thus,

$$1^{k+1} - 0^{k+1} = (k+1)1^k - [k(k+1)/2]1^{k-1} + [k(k+1)(k-1)/6]1^{k-2} + \ldots - (-1)^{k+1}$$
$$2^{k+1} - 1^{k+1} = (k+1)2^k - [k(k+1)/2]2^{k-1} + [k(k+1)(k-1)/6]2^{k-2} + \ldots - (-1)^{k+1}$$
$$3^{k+1} - 2^{k+1} = (k+1)3^k - [k(k+1)/2]3^{k-1} + [k(k+1)(k-1)/6]3^{k-2} + \ldots - (-1)^{k+1}$$
$$\vdots$$
$$+\underline{n^{k+1} - (n-1)^{k+1} = (k+1)n^k - [k(k+1)/2]n^{k-1} + [k(k+1)(k-1)/6]n^{k-2} + \ldots - (-1)^{k+1}}$$
$$n^{k+1} - 0^{k+1} = (k+1)\sum n^k - [k(k+1)/2]\sum n^{k-1} + [k(k+1)(k-1)/6]\sum n^{k-2} + \ldots - n(-1)^{k+1}$$

$$\Rightarrow n^{k+1} = (k+1)\sum n^k - [k(k+1)/2]\sum n^{k-1} + [k(k+1)(k-1)/6]\sum n^{k-2} + \ldots - n(-1)^{k+1}$$
$$\Rightarrow (k+1)\sum n^k = n^{k+1} + [k(k+1)/2]\sum n^{k-1} - [k(k+1)(k-1)/6]\sum n^{k-2} + \ldots + n(-1)^{k+1}$$
$$\Rightarrow \sum n^k = \frac{n^{k+1} + [k(k+1)/2]\sum n^{k-1} - [k(k+1)(k-1)/6]\sum n^{k-2} + \ldots + n(-1)^{k+1}}{k+1}$$

Hence,

$$\boxed{\mathbf{1^k + 2^k + 3^k + \ldots + n^k = \frac{n^{k+1} + [k(k+1)/2]\sum n^{k-1} - [k(k+1)(k-1)/6]\sum n^{k-2} + \ldots + n(-1)^{k+1}}{k+1}}}$$

---

[1] ${}^{k+1}C_1 = (k+1)!/[(k+1-1)!x1!] = (k+1)k!/k! = k+1$
${}^{k+1}C_2 = (k+1)!/[(k+1-2)!x2!] = k(k+1)(k-1)!/[(k-1)!x2] = k(k+1)/2$
${}^{k+1}C_3 = (k+1)!/[(k+1-3)!x3!] = k(k+1)(k-1)(k-2)!/[(k-2)!x6] = k(k+1)(k-1)/6$

Marked by Teachers
This document was downloaded from www.markedbyteachers.com

CONCLUSION

After going through my portfolio, I hope I have satisfied the basic needs required and have presented the portfolio well. I have learnt a lot of things while doing this investigation. Firstly, my knowledge about sequences and series have been broadened and I am now craving for more knowledge on general formulas of these sequences. There is a lot to learn from this portfolio. We can use the general formulas of S_n, U_n and T_n in our everyday lives. We should observe the beauty of the portfolio, the beauty of the patterns, the way they progress at every stage. I was so inspired that I looked up on the internet for more methods to solve this portfolio. Infact I even studied the binomial theorem and the pascal's triangle deeply as it interested me. While surfing on the internet I found something very interesting. The method I used to find the conjectures of S_n, U_n and T_n has a name and is called the **Telescopic Method**.

REFERENCE:

http://www.xula.edu/math/Research/Colloquium/Past/Colloquium20030320-Paper-SahooP.pdf
http://www.mathpages.com/home/kmath279.htm
http://mathworld.wolfram.com/PowerSum.html
http://mathworld.wolfram.com/BinomialTheorem.html

All graphs have been made with the help of Microsoft Excel 2003
All major calculations have been done with the help of Graphic Display Calculator TI-84 Plus.