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**Introduction**

In this investigation quartic functions will be explored, in particular quartic functions with a unique "W" shape. These quartic functions will have two points of inflection, which will be referred to as Point Q and Point R. When Line QR is drawn it intercepts the quartic function twice more; those interceptions will be referred to as Point P and Point S. The ratio of PQ:QR:RS will be investigated and any findings will be formally proven. This investigation will include the given function $f(x) = x^4 - 8x^3 + 6x^2 - 2x + 24$ and the chosen function $f(x) = x^4 + 3x^3 - 5x$ . These functions will be depicted with graphs.

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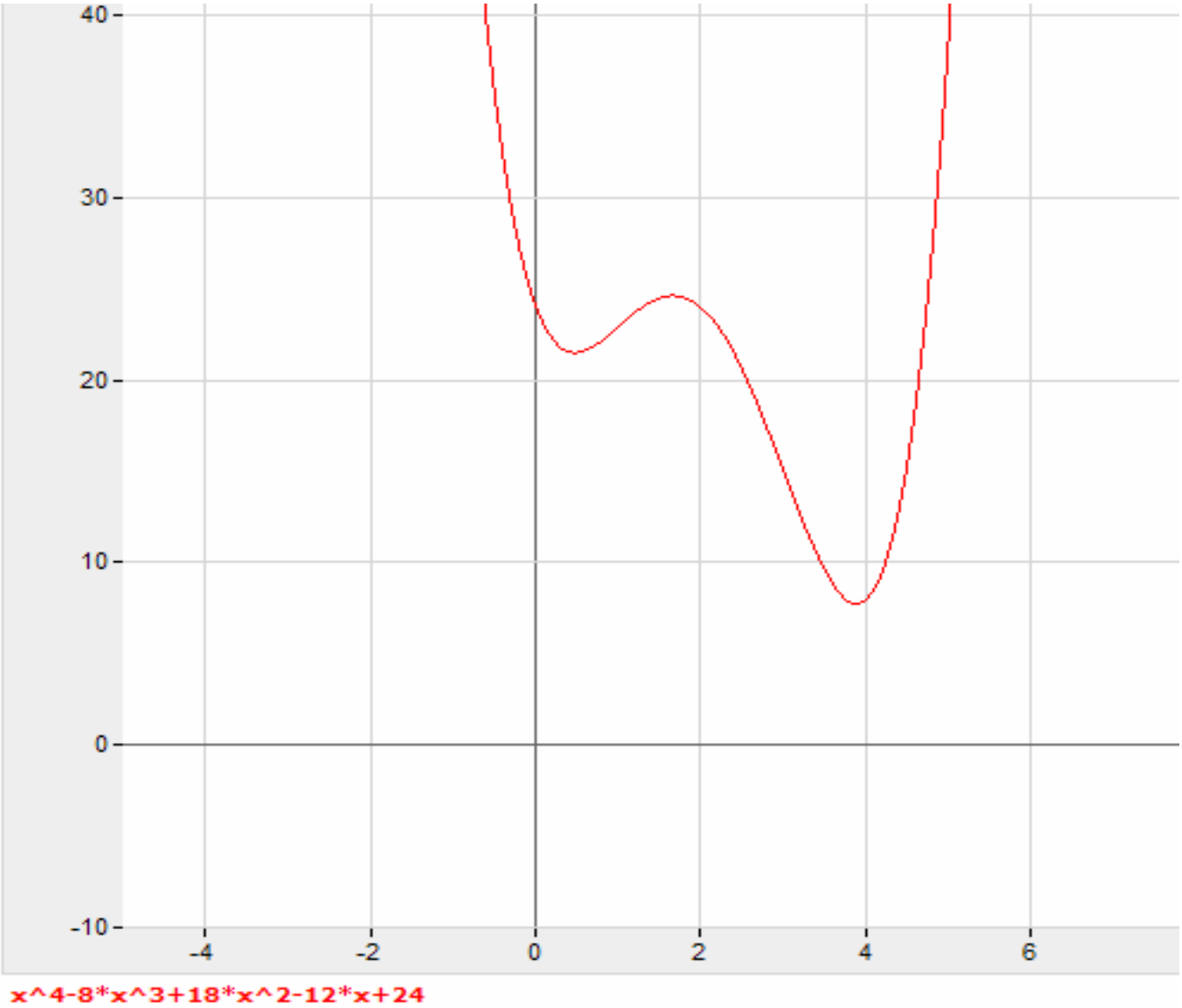
**Method**

**[redacted]. G[redacted]e [redacted](x) = $x^4 - 8x^3 + 18x^2 - 12x + 24$.**



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**2. Find the coordinates of the points of inflection Q and R. Determine the points of inflection S, where the line QR intersects the cubic function again, and compute the ratio Q:QR:RS.**

<u>Find Q and R</u>

Find First Derivative:

$$f(x) = x^4 - 8x^3 + 18x^2 - 12x + 24$$

$$f'(x) = 4x^3 - 24x^2 + 36x - 12$$

Find Second Derivative

$$f''(x) = 12x^2 - 48x + 36$$

Set to Zero

$$0 = 12x^2 - 48x + 36$$

Apply Quadratic Equation

$$x = \frac{-(-48) \pm \sqrt{(-48)^2 - 4(12)(36)}}{2(12)}$$

$$x = \frac{48 \pm \sqrt{2304 - 1728}}{24}$$

$$x = \frac{48 \pm \sqrt{576}}{24}$$

$$x = \frac{48 + 24}{24}; x = \frac{48 - 24}{24}$$

x = 1, 3

Concavity Checks

$f''(0) = 36$ Pos Positive

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$f(2) = -2$ negative — A switch from positive to negative indicates change in direction.

$f(4) = 36$ positive — A switch from negative to positive indicates change in direction.

Find Y

To find Q $f(1) = 23$

To find R $f(3) = 3^4 - 8(3)^3 + 6(3)^2 - 2(3) + 24 = 5$

Inflection points are:

***Q (1, 23)*** ***R(3, 5)***

<u>Line QR</u>

Equation of a Line Passing through Two Points:

$y - y_1 = (y_2 - y_2)/(x_2 - x_1) * (x - x_1)$

Plug in Known Values

$y - 5 = (23-5)/(1-3) * (x-3)$

$\mathbf{y = -4x + 27}$

<u>Find P and S</u>

Set f(x) Equal to g(x) and Then to Zero

$-4x + 27 = x^4 - 8x^3 + 6x^2 - 2x + 24$

$x^4 - 8x^3 + 6x^2 - 8x - 3 = 0$

Divide by Root 1

$x^3 - 7x^2 + 1x + 3 = 0$

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Divide by Root 3

$x^2 - 4x - 1 = 0$

Apply to Quadratic Equation

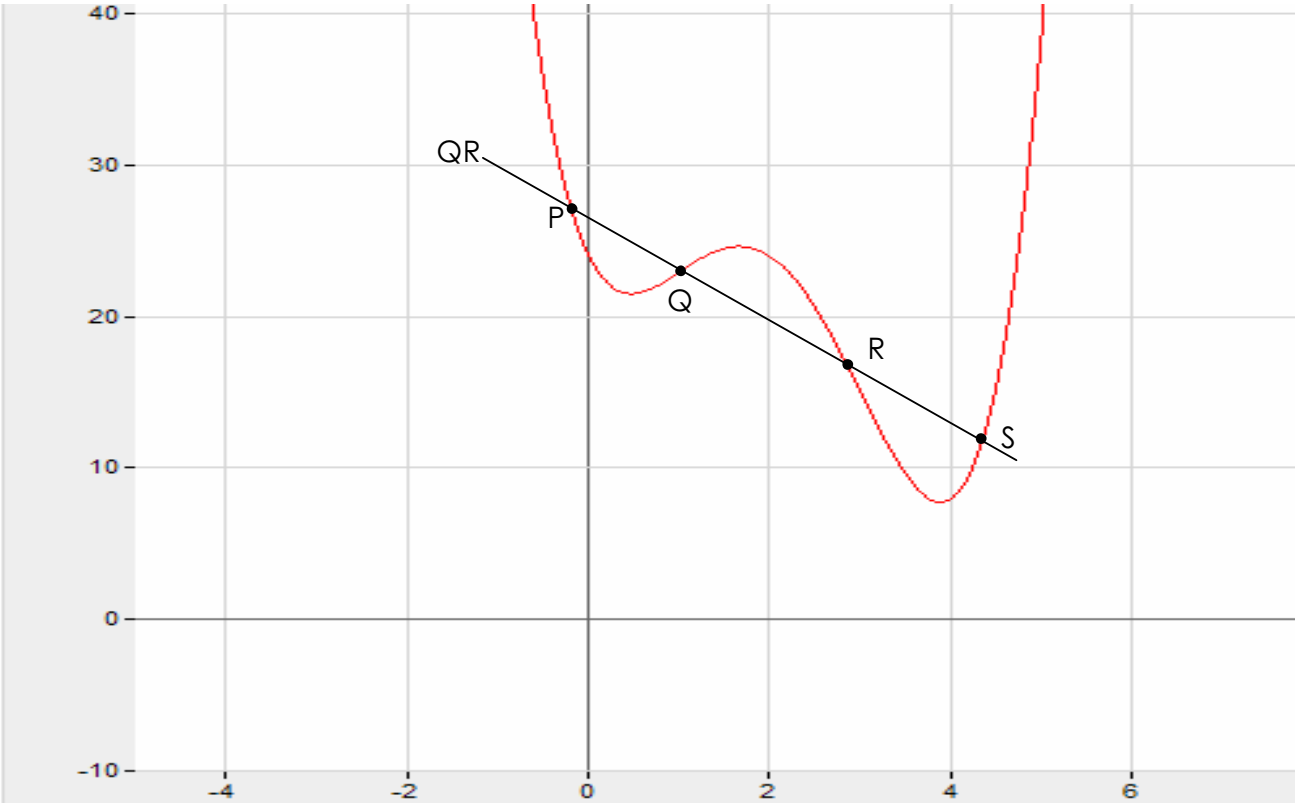
$X_{1,2} = 2 \pm \sqrt{5}$

Find Y

To find P $(2 - \sqrt{5}) = (-.236)^4 - 8(-.236)^3 + 18(-.236)^2 - 12(-.236) + 24 =$ 27.9

To find S $(2 + \sqrt{5}) = (.236)^4 - 8(.236)^3 + 18(.236)^2 - 12(.236) + 24 =$ 22.1

**P = (-.236, 27.9)** ***S = (.236, 22.1)***



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Find the Ratio PQ:QR:RS

Use the Distance Formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

PQ:

$d = \sqrt{((1 - (-.236))^2 + (23 - 27.9)^2)}$

**d = 1.24**

QR:

$d = \sqrt{((3 - 1)^2 + (15 - 23)^2)}$

**d = 2**

RS:

$d = \sqrt{((.236 - 3)^2 + (22.1 - 15)^2)}$

**d = 1.24**

**PQ:QR:RS = 1.24:2:1.24**

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**3. Simplify this and so that PQ = [illegible] correct to your answers.**

<u>Divide by PQ</u>

*([illegible]/[illegible]):(2/[illegible]):([illegible]/[illegible])*

***[illegible]:[illegible]***

*When PQ is set to equal, GR becomes [illegible], this number is known as "[illegible]" (Φ) or the "[illegible] ratio", as it is [illegible] to [illegible] in two [illegible] in nature, such as [illegible] in [illegible] shells or in the construction of the [illegible].*

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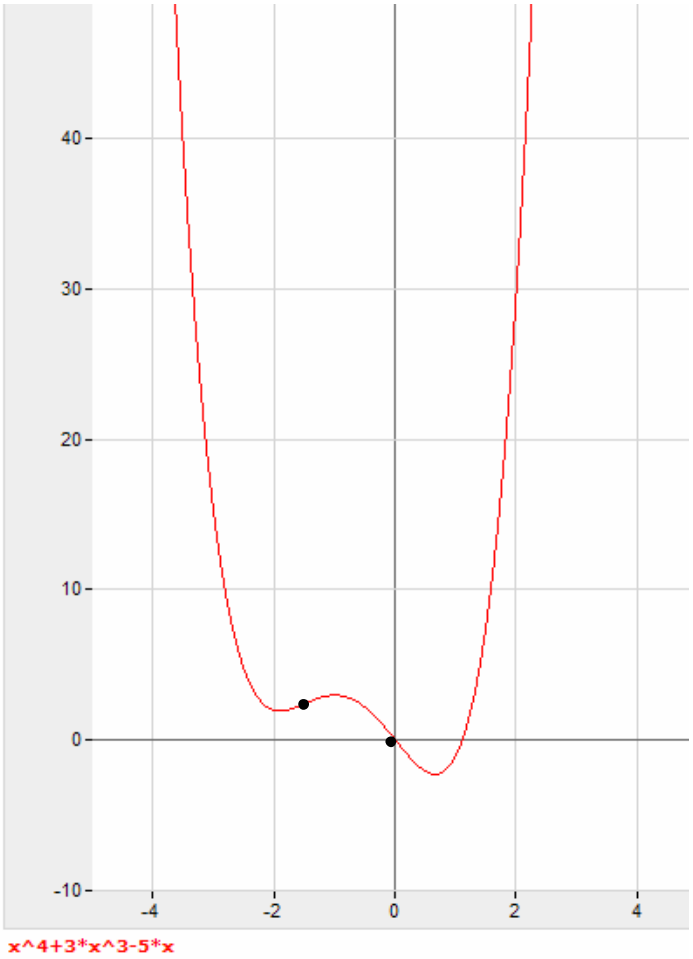
**4. Choose a polynomial function with a "W" shape and investigate the same items.**

Choose a function

$f(x) = x^4 + 3x^3 - 5x$




Find Q and R

Find First Derivative:

$f(x) = x^4 + 3x^3 - 5x$

$f'(x) = 4x^3 + 9x^2 - 5$

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Find Second Derivative

$f''(x) = 12x^2 + 18x$

Set to Zero

$0 = 12x^2 + 18x$

Apply Quadratic Equation

$x = -1.5, 0$

Concavity Checks

| | | |
|---|---|---|
| $f''(-2) = 12$ | positive | As we move from positive to negative inflection points change direction. |
| $f''(-1) = -6$ | negative | As we move from negative to positive inflection points change direction. |
| $f''(1) = 30$ | positive | |

Find Y

| | |
|---|---|
| Point Q | $f(-1.5) = (-1.5)^4 + 3(-1.5)^3 - 5(-1.5) = 2.4$ |
| Point X | $f(0) = 0$ |

Inflection points are:

**Q (-1.5, 2.4)  X (0, 0)**

<u>Line QR</u>

Equation of a Line Passing through Two Points:

$y - y_1 = (y_2 - y_2)/(x_2 - x_1) * (x - x_1)$

Plug in Known Values

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$y - 0 = (2.4-0)/(-1.5-0) * (x-0)$

**$y = -1.6x$**

Find P and S

Set f(x) Equal to g(x) and Then to Zero

$-1.6x = x^4 + 3x^3 - 5x$

$x^4 + 3x^3 - 3.4x = 0$

Reduce to a Quadratic and Apply to Quadratic Equation

$X_{1,2} = -2.7, 1$

Find Y

To find P $f(-2.7) = (-2.7)^4 + 3(-2.7)^3 - 5(-2.7) = 7.6$

To find S $f(1) = (1)^4 + 3(1)^3 - 5(1) = -1$

**P = (-2.7, 7.6)** ***S = (1, -1)***

Find the Ratio PQ:QR:RS

Use the Distance Formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

PQ:

$PQ = \sqrt{((-1.5- (-2.7))^2 + (2.4 - 7.6)^2)}$

**$PQ = 5.34$**

QR:

$QR = \sqrt{((0- (-1.5))^2 + (0 - 2.4)^2)}$

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***d = 8.97***

RS:

*d = √ ((1 - 0)² + (.1 – 0) ²)*

***d = 5.34***

**PQ:QR:RS = 5.34:8.97:5.34**

<u>Divide by PQ</u>

*(5.34/5.34):(8.97/5.34):(5.34/5.34)*

***1:1.68:1***

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**5. Form a conjecture about how many generic functions**

Starting With the General Equation:

$f(x) = ax^4 + bx^3 + cx^2 + dx + e$

Take the First Derivative

$f'(x) = 4ax3 + 3bx2 + 2cx + d$

Take the Second Derivative

$f''(X) = 12ax2 + 6bx + 2c = 0$

Set the Two Derivatives Equal To Each Other (Horizontal Inflection Points Occur When Either the First or Second Derivative Equals Zero)

$4ax3 + 3bx2 + 2cx + d = 12ax2 + 6bx + 2c$

Set It Equal to Zero

$4ax3 + 12ax2 + 3bx2 + 6bx + 2cx + 2c + d = 0$

This equation will be the derivative which we will use to find horizontal inflection points, or, equivalently, the zero point.

As stated by original equation $f(x) = x^4 – 8x^3 + 18x^2 – 12x + 24$ and chosen equation $f(x) = x^4 + 3x^3 -5x$, generic equations will consist of four terms. This is proven by now "W" state of generic will contract the polynomials are exactly 1.68. The term is not exactly this polynomial, therefore "w" state will no longer represent. However, the derivative polynomial represent in generic equation, two distinct inflection points, in state represent, and does not have to less than "w" state generic function.

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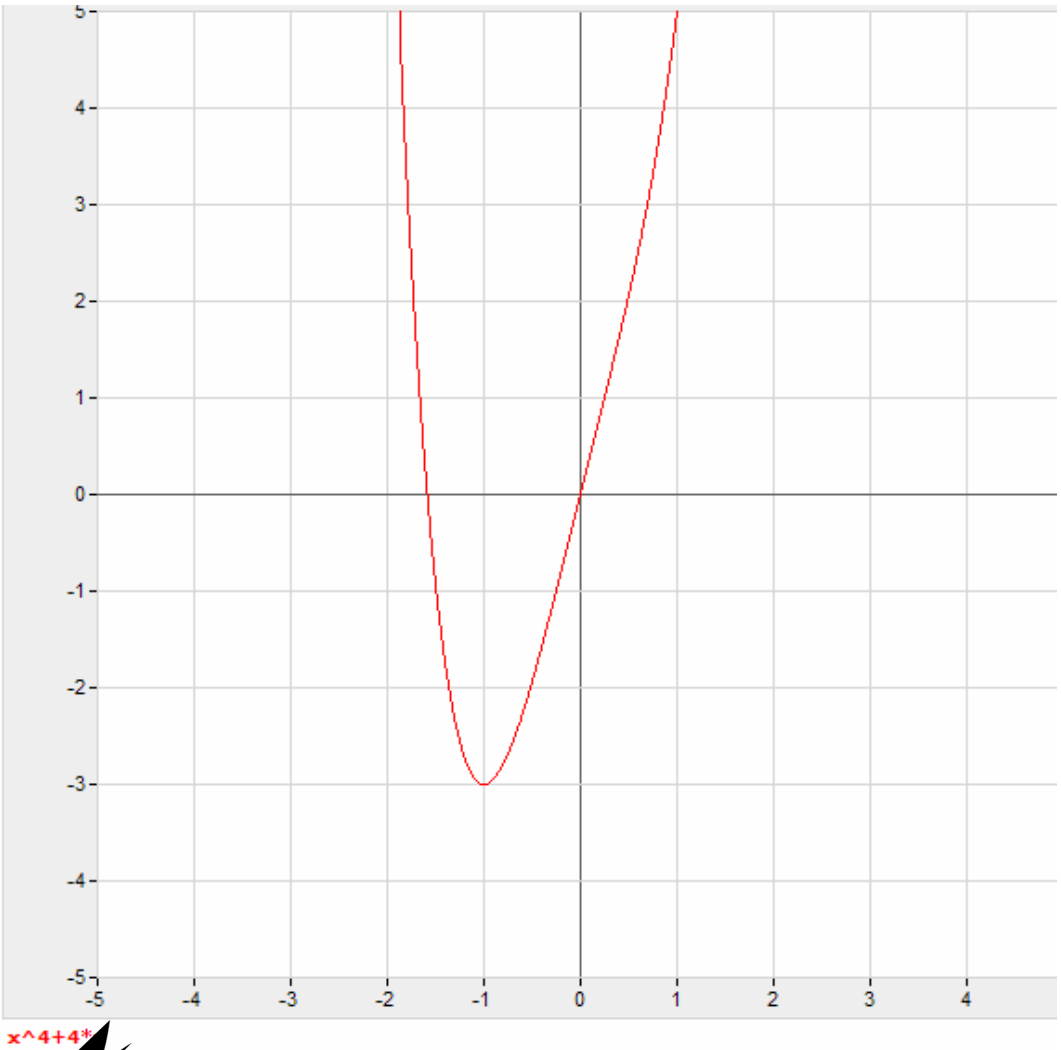
**6. Extend this investigation to other quartic functions that are not strictly of a "W" shape.**

[illegible] quartic functions not strictly of a "W" shape, the minimum proportion of 0.68 does not exist. For example, take the quartic




equation $y = f(x) = x^4 + 4x$.

As can be seen, the quartic is not strictly of a "W" shape, so it does not have a central min. For these types of quartic equations, the

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Those who are directions to the right, the left, to begin coordinate to the
certain is no other to the right of which to coincide to.