MATHS COURSEWORK

POPULATION TRENDS IN CHINA

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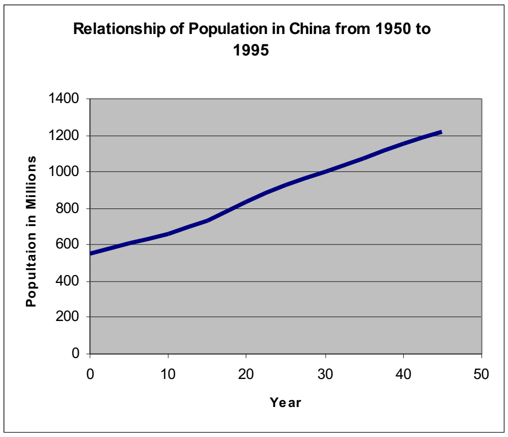
1. The aim in this coursework is to investigate the different functions that best model the population of China from 1950 to 1995. The following table shows the population of China between these years:

Year	1950	1955	1960	1965	1970	1975	1980	1985	1990	1995
Population in Millions	554.8	609.0	657.5	729.2	830.7	927.8	998.9	1070.0	1155.3	1220.5

The relevant variables in this investigation are the population in millions in different years. The parameter is the initial population growth.

The data points from the table above are shown in the graph below, using Microsoft Excel showing the population in China from 1950 to 1995 (presenting the years from 0 being 1950, to 45 being 1995):



What I can observe according to the graph beside is that as the years pass, the population of China (in millions) increases gradually.

The functions which could model the behaviour of the graph can be any of the following:

· **$y = mx+c$**

The graph appears linear therefore we could use the above model, however it is geometric, as it is a population, therefore one of the following three may be better:

· **$y = ar^{(n-1)}$**
· **$y = p\,(1+r)^{n}$**
· **$y = ae^{(kt)}$**

These can be used as they include in the equation population factors, where the year can be included in them as well as interpreting the graph with them.

By using the model of **$y = ar^{(n-1)}$** which could be used in other terms as: **$y = ar^{t}$** (t for time) I may develop this model with my data.

Again, I am referring to the years in terms of being from year 0 to 45 instead of from 1950 to 1995.
When **t**=0 then **r** to the power of 0 will be 1, meaning **a** will equal 554.8
If I place other numbers from the data I can identify a more accurate response from what the equation would be.
By placing 10 in t and 657.5 in y the calculations would be the following:
$657.5 = 554.8*r^{10}$
$1.186 = r^{10}$
$R = 10\sqrt{1.186}$
$R = 1.017$

MODEL: $y = 554.8*1.017^{t}$

I introduced the data given to me in excel in the following way:

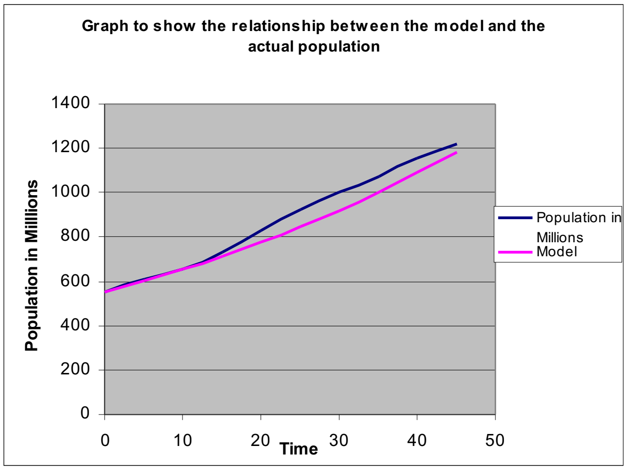
Years	Year	Population in Millions	Model
1950	0	554.8	554.8
1955	5	609	603.6
1960	10	657.5	656.7
1965	15	729.2	714.4
1970	20	830.7	777.2
1975	25	927.8	845.6
1980	30	998.9	920.0
1985	35	1070	1000.9
1990	40	1155.3	1088.9
1995	45	1220.5	1184.6

I introduced this model to see how accurate it would be in reference to the original data.

This model was introduced in the first cell and dragged down so that all cells followed the same rule.

Model	**Model**
554.8	=554.8*(1.017^B2)
603.5889	=554.8*(1.017^B3)
656.6682	=554.8*(1.017^B4)
714.4153	=554.8*(1.017^B5)
777.2407	=554.8*(1.017^B6)
845.5909	=554.8*(1.017^B7)
919.9517	=554.8*(1.017^B8)
1000.852	=554.8*(1.017^B9)
1088.866	=554.8*(1.017^B10)
1184.621	=554.8*(1.017^B11)

Then, I created a graph to compare my model in relation to the original graph of the original data:



Here we can see that my model has a more precise relation with the original in the first years, but later the results start to become more separate and create another geometric line which doesn't relate with the original.

As my model presented different results seen in the graph, I will now try to change mu constants to have a more precise result in relation to the original.

I thought that when placing another constant in "to the power of" so that my results could be more accurate, meaning that my formula would now be using a system similar to the **y = a**e $^{(kt)}$ formula by having two upper constants, although the e in this case would be another constant implemented by me.

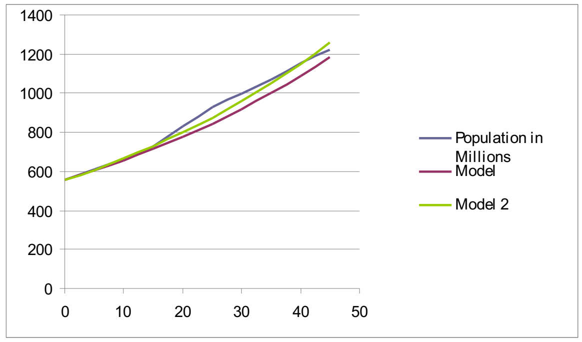
By using trial and error I started to place different numbers into the constants of mi formula so that they would then be as accurate as possible being now:

Constant r
1.016

These constants refer to my column of Model 2.

Constant k
1.15

Years	Year	Population in Millions	Model 2
1950	0	554.8	554.8
1955	5	609	607.8
1960	10	657.5	665.9
1965	15	729.2	729.5
1970	20	830.7	799.3
1975	25	927.8	875.7
1980	30	998.9	959.3
1985	35	1070	1051.0
1990	40	1155.3	1151.5
1995	45	1220.5	1261.5



In the graph we can see that my model 2 in relation to the first model is more near to what the original data is, although it isn't as accurate as it should.

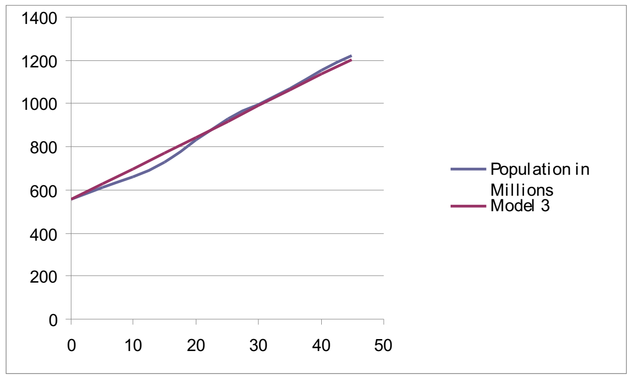
Now I will use the formula of **y=mx+c** which concentrates in linear graphs. I will use this formula using P= mt+c, where:
P= Population in millions
C= 554.8 (where c is the initial population at time 0)
M= constant (gradient of line)
T= time (years)

The final model would be: Pop^{n} = 14.5t+554.8

I tried this formula and by trial and error, finally achieved the best model which fits the original one, which almost fits the original line.

Constant M
14.5

Years	Year	Population in Millions	Model 3
1950	0	554.8	554.8
1955	5	609	627.3
1960	10	657.5	699.8
1965	15	729.2	772.3
1970	20	830.7	844.8
1975	25	927.8	917.3
1980	30	998.9	989.8
1985	35	1070	1062.3
1990	40	1155.3	1134.8
1995	45	1220.5	1207.3



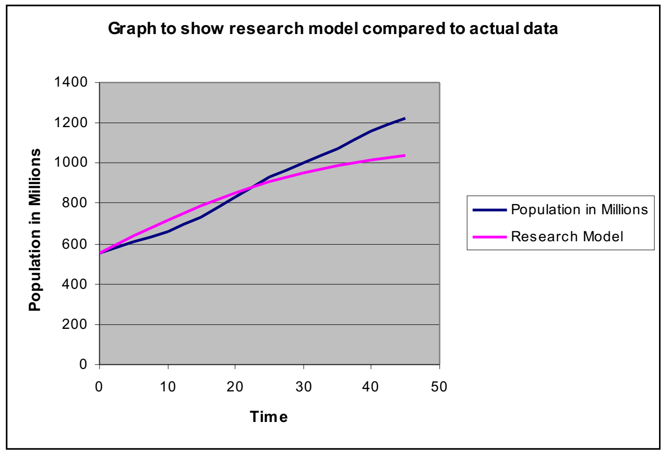
2. A research suggests that the population, *P* at time *t* can be modeled by: $P(t) = K/1+Le^{-Mt}$ where *K, L* and *M* are parameters.

Firstly, to find what *K* is I will place 0 into *t* (time).
Through mathematical knowledge we may say that e^{-Mt} will equal to 1, because when $t=0$ *M* will also be 0 when multiplied together, and any number to the power of 0 will always equal to 1.
When $t=0$, the population is 554.8, meaning the equation would be:
$554.8 = K / 1+ L$ (*1)
If we say $L = 1$, then *K* must equal 2*554.8, which is 1109.6
Being the final equation: $P(t) = 1109.6 / 1+ L$

By using Microsoft Excel I tried estimating the possible values of *K, L* and *M* by entering the data and plotting graphs.
Being:

K	L	M
1109,6	1	0,06

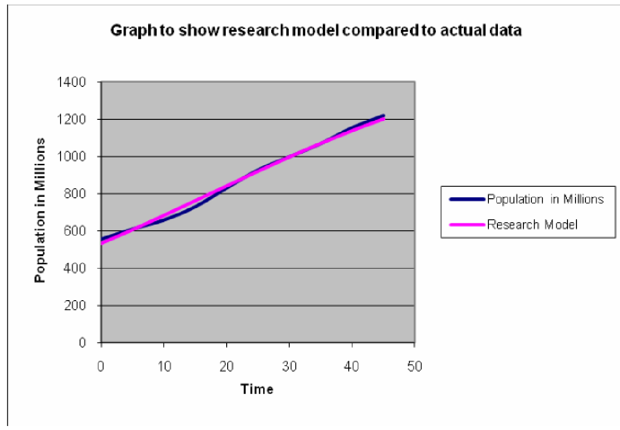
Years	Year	Population in Millions	Research Model
1950	0	554,8	554,8
1955	5	609	637,4
1960	10	657,5	716,4
1965	15	729,2	788,9
1970	20	830,7	852,8
1975	25	927,8	907,2
1980	30	998,9	952,2
1985	35	1070	988,5
1990	40	1155,3	1017,3
1995	45	1220,5	1039,7



The graph shows that the research model line does not fit the actual data; therefore I will try other numbers for *K*, *L* and *M* so that it becomes more accurate.
I changed my constants using trial and error and finally got a model which fits better:

K	L	M
1600	2	0,04

Years	Year	Population in Millions	Research Model
1950	0	554,8	533,3
1955	5	609	606,6
1960	10	657,5	683,6
1965	15	729,2	762,8
1970	20	830,7	842,7
1975	25	927,8	921,8
1980	30	998,9	998,5
1985	35	1070	1071,5
1990	40	1155,3	1139,8
1995	45	1220,5	1202,5

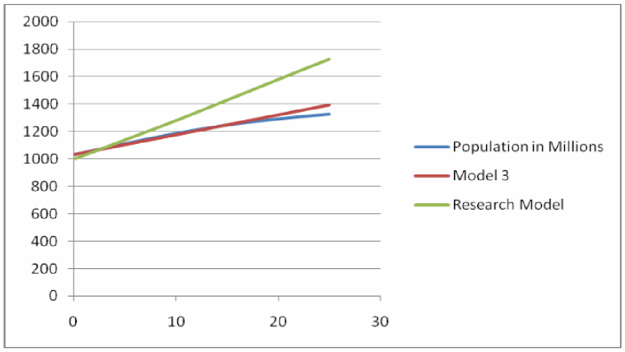


The model would be: 1600/1+e$^{-0.04t}$.
In China, families by law are allowed to have a maximum of one child per family only; therefore this could explain why this population trend tends to be almost linear.

3. Another table was given to me with the population in China in millions from year 1983 to 2008:

Year	1983	1992	1997	2000	2003	2005	2008
Population in Millions	1030.1	1171.7	1236.3	1267.4	1292.3	1307.6	1327.7

The research model increases too quickly and doesn't fit with the IMF data, whereas my model 3 fits better in relation to this line, but it also increases slightly in the end.



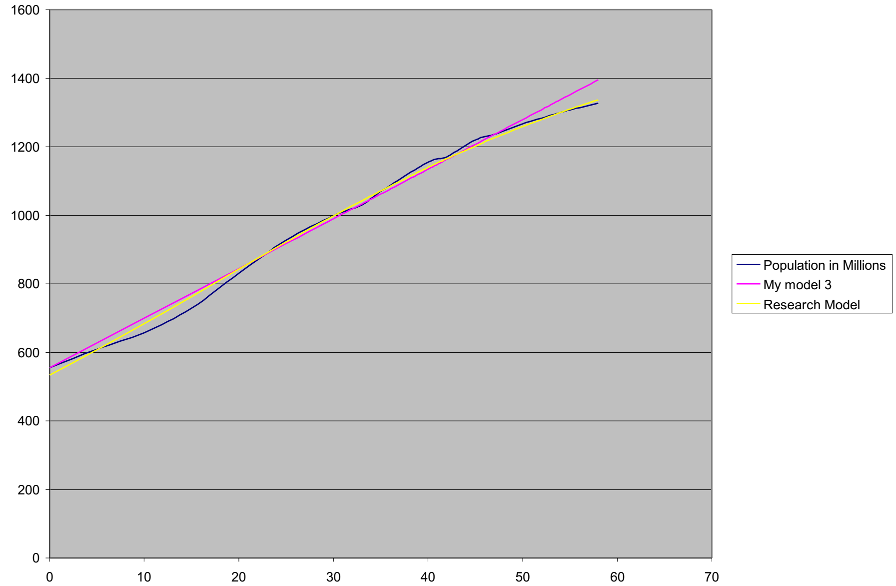
Year	Year	Population in Millions	My model 3	Research Model
1983	0	1030,1	1030,1	1000,0
1992	9	1171,7	1160,6	1252,4
1997	14	1236,3	1233,1	1400,3
2000	17	1267,4	1276,6	1490,1
2003	20	1292,3	1320,1	1580,1
2005	22	1307,6	1349,1	1639,7
2008	25	1327,7	1392,6	1728,4

I finally modified my final model and the research model so that they could fit the IMF data as accurately as possible, which applies to the given data from 1950 to 2008.

Years	Years	Population in Millions	My model 3	Research Model
1950	0	554.8	554.8	533.3
1955	5	609	627.3	606.6
1960	10	657.5	699.8	683.6
1965	15	729.2	772.3	762.8
1970	20	830.7	844.8	842.7
1975	25	927.8	917.3	921.8
1980	30	998.9	989.8	998.5
1983	33	1030.1	1033.3	1042.8
1985	35	1070	1062.3	1071.5
1990	40	1155.3	1134.8	1139.8
1992	42	1171.7	1163.8	1165.5
1995	45	1220.5	1207.3	1202.5
1997	47	1236.3	1236.3	1225.9
2000	50	1267.4	1279.8	1259.2
2003	53	1292.3	1323.3	1290.3
2005	55	1307.6	1352.3	1309.8
2008	58	1327.7	1395.8	1337.2

The graph shown more underneath, demostrates how my final model and the research model have been modified so that they can slightly fit the IMF data. As you can see, I finally obtained a very precise data which fits the actual line thorugh trial and error. Below is also shown how I modified my models, where you can see the formula used for each one in relation to all the IMF data.

Years	Years	Population in Millions	My model 3	Research Model
1950	0	554.8	=14.5*B2+$C$2	=1600/(1+2*2.71828^(-0.04*B2))
1955	5	609	=14.5*B3+$C$2	=1600/(1+2*2.71828^(-0.04*B3))
1960	10	657.5	=14.5*B4+$C$2	=1600/(1+2*2.71828^(-0.04*B4))
1965	15	729.2	=14.5*B5+$C$2	=1600/(1+2*2.71828^(-0.04*B5))
1970	20	830.7	=14.5*B6+$C$2	=1600/(1+2*2.71828^(-0.04*B6))
1975	25	927.8	=14.5*B7+$C$2	=1600/(1+2*2.71828^(-0.04*B7))
1980	30	998.9	=14.5*B8+$C$2	=1600/(1+2*2.71828^(-0.04*B8))
1983	33	1030.1	=14.5*B9+$C$2	=1600/(1+2*2.71828^(-0.04*B9))
1985	35	1070	=14.5*B10+$C$2	=1600/(1+2*2.71828^(-0.04*B10))
1990	40	1155.3	=14.5*B11+$C$2	=1600/(1+2*2.71828^(-0.04*B11))
1992	42	1171.7	=14.5*B12+$C$2	=1600/(1+2*2.71828^(-0.04*B12))
1995	45	1220.5	=14.5*B13+$C$2	=1600/(1+2*2.71828^(-0.04*B13))
1997	47	1236.3	=14.5*B14+$C$2	=1600/(1+2*2.71828^(-0.04*B14))
2000	50	1267.4	=14.5*B15+$C$2	=1600/(1+2*2.71828^(-0.04*B15))
2003	53	1292.3	=14.5*B16+$C$2	=1600/(1+2*2.71828^(-0.04*B16))
2005	55	1307.6	=14.5*B17+$C$2	=1600/(1+2*2.71828^(-0.04*B17))
2008	58	1327.7	=14.5*B18+$C$2	=1600/(1+2*2.71828^(-0.04*B18))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Population in Millions
My model 3
Research Model