QASMT

Mathematics IA

Parallels and Parallelograms

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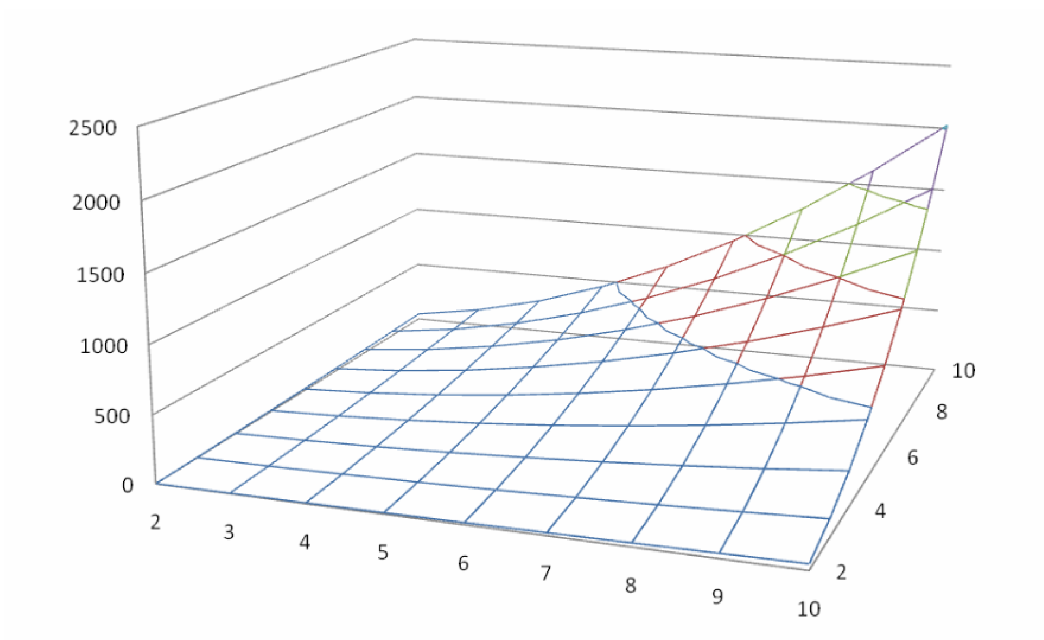

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This internal assessment will investigate the relationship between vertical transversals, horizontal lines and parallelograms. Vertical transversals are lines that intersect horizontal lines. To create parallelograms two or more parallel vertical transversals needs to intersect with two or more horizontal lines. This is shown in figure 1.1 and figure 1.2.

Figure 1.1

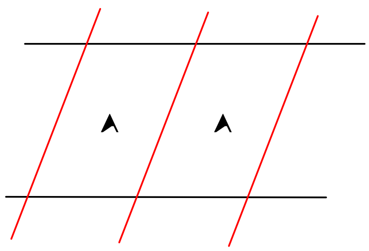
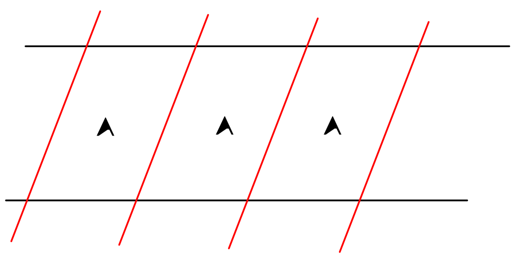


Figure 1.2

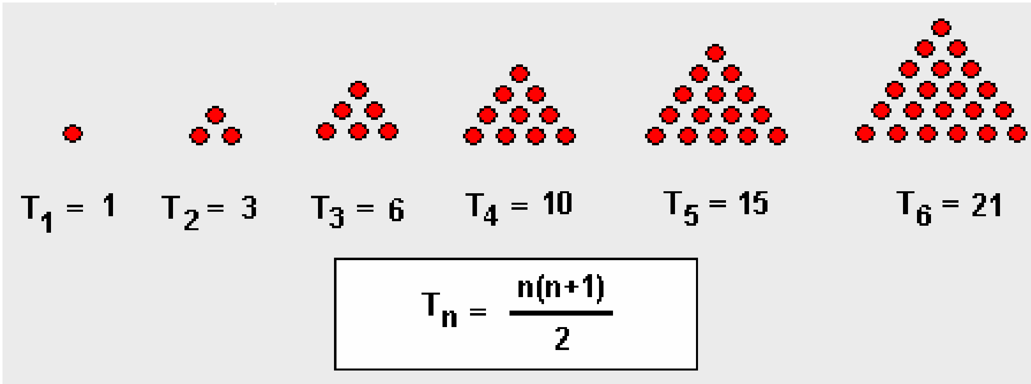


These above figures demonstrate how vertical transversals (red) intersect with horizontal lines (black) to create parallelograms. These parallelograms are demonstrated in figure 1.1, A_1, A_2, A_1 U A_2. Furthermore, the parallelograms are illustrated in figure 1.2, A_1, A_2, A_3, A1 U A_2, A_2 U A_3, and A1 U A_2 U A_3. If parallel transversals are continually added an increasing number of parallelograms would be formed and a general formula can be deduced.

Vertical Transversals	**Parallelograms**	**Notation**
2	1	A_1
3	3	A_1, A_2, A_1 U A_2
4	6	A_1, A_2, A_3, A_1 U A_2, A_2 U A_3, A_1 U A_2 U A_3
5	10	A_1, A_2, A_3, A_4, A_1 U A_2, A_2 U A_3, A_3 U A_4, A_1 U A_2 U A_3 , A_2 U A_2 U A_3, A_1 U A_2 U A_3 U A_4
6	15	A_1, A_2, A_3, A_4, A_5, A_1 U A_2, A_2 U A_3, A_3 U A_4, A_4 U A_5, A_1 U A_2 U A_3, A_2 U A_3 U A_4, A_3 U A_4 U A_5, A_1 U A_2 U A_3 U A_4, A_2 U A_3 U A_4 U A_5, A_1 U A_2 U A_3 U A_4 U A_5
7	21	A_1, A_2, A_3, A_4, A_5, A_6, A_1 U A_2, A_2 U A_3, A_3 U A_4, A_4 U A_5, A_5 U A_6, A_1 U A_2 U A_3, A_2 U A_3 U A_4, A_3 U A_4 U A_5, A_4 U A_5 U A_6, A_1 U A_2 U A_3 U A_4, A_2 U A_3 U A_4 U A_5, A_3 U A_4 U A_5 U A_6, A_1 U A_2 U A_3 U A_4 U A_5, A_2 U A_3 U A_4 U A_5 U A_6, A_1 U A_2 U A_3 U A_4 U A_5 U A_6

The general formula needs to be deduced from the patterns that are seen in the table and previously discovered maths formula. To discover the relationship between parallelograms and the number of vertical transversals a similar sequence of numbers needs to investigated. A similar sequence of numbers is present in Pascal's theory of triangular numbers. Therefore this sequence of numbers needs to be investigated to determine its similarity to the relationship between vertical transversals and the number of parallelograms formed. This sequence of numbers and the general formula of triangular numbers is shown in figure 2.1.

Figure 2.1



The relationship between T_n and n is demonstrated in the picture above and the table below.

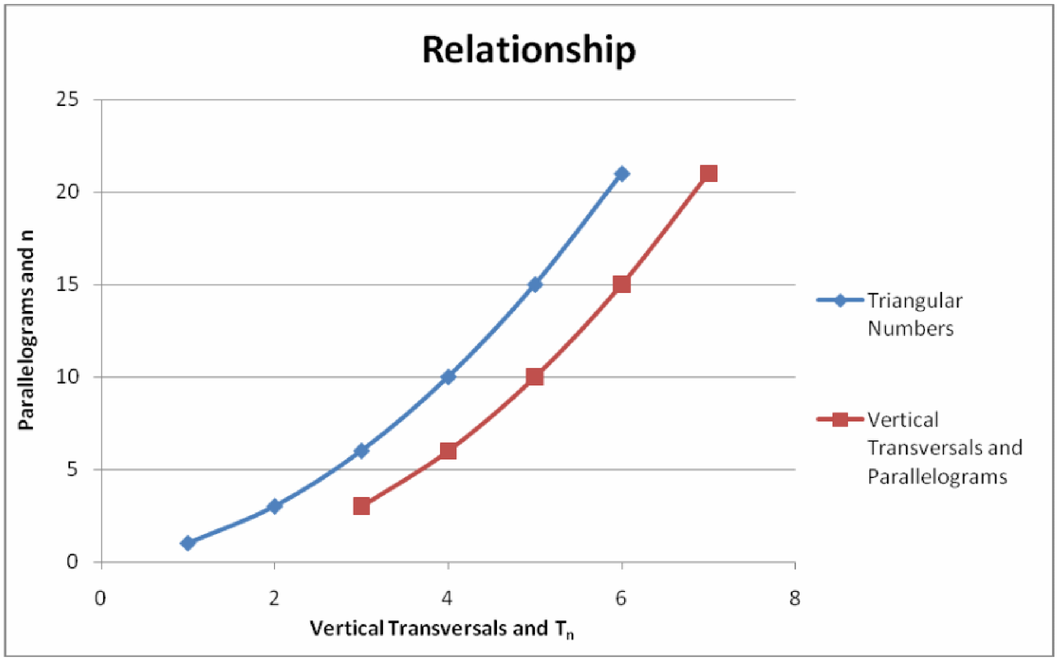
T_n	n
1	1
2	3
3	6
4	10
5	15
6	21

The relationship between the number of vertical transversals and the number of parallelograms formed is shown below.

Vertical Transversals	Parallelograms formed
2	1
3	3
4	6
5	10
6	15
7	21

The similarity of these sequences is demonstrated in graph 1.1.

Graph 1.1



The relationship between triangular numbers and the number of parallelograms formed when the number of vertical transversals is increased is clearly shown by the similarity of both the lines in the graph above. This similarity must indicate that the general formula of Pascal's theory of triangular numbers is similar to the general formula of the relationship between the number of vertical transversals and the number of parallelograms formed.

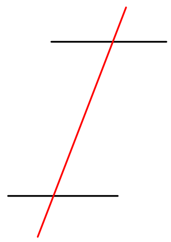
To discover the general formula of the relationship of the number of vertical transversals and the number of parallelograms, it is necessary to test the general formula of triangular numbers (shown in figure 2.1) on the relationship between the number of vertical transversal and the number of parallelograms.

$$T(n) = n\left[\frac{n+1}{2}\right]$$

□ $3 = 3\left[\frac{3+1}{2}\right]$

□ $3 = 6$

This proves that the general formula for triangular numbers does not directly apply to the general formula for the relationship between the number of vertical transversal and the number of parallelograms. This is because the general state for triangular numbers start at $T_n = 1$ whereas the general formula for the relationship between the number of vertical transversal and the number of parallelograms needs to start at 2, this is because a parallelogram is not formed when there is only one vertical transversal, shown below.



To combat this effect the equation needs to be changed to $T(n) = n\left[\frac{n-1}{2}\right]$, this is because a parallelogram is only created when there is two or more vertical transversal.

 The general formula is: $P = v\left[\frac{v-1}{2}\right]$.

Where v is the number of vertical transversal and P is the number of parallelograms formed.
However, this equation assumes that the number of horizontal lines stays constant at 1.

This formula is proved below.

$$P = v\left[\frac{v-1}{2}\right]$$

$$3 = 3\left[\frac{3-1}{2}\right]$$

3 = 3

Furthermore,

$$6 = 4\left[\frac{4-1}{2}\right]$$

$$6 = 6$$

The above examples justify that $P = v\left[\frac{v-1}{2}\right]$ is the general formula for the relationship between the number of vertical transversals and the number of parallelograms. However, to develop a general formula that encompasses the variables of the number of vertical transversals and horizontal lines, an equation needs to be developed that deduces the number of parallelograms that is formed when horizontal lines vary. This equation needs to assume that the number of vertical transversals remain constant. The following figures demonstrate how additional parallelograms are formed when an increasing number of horizontal lines are added.

Figure 3.1

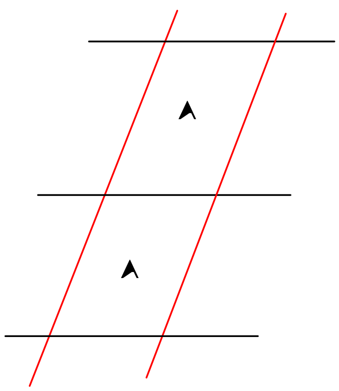
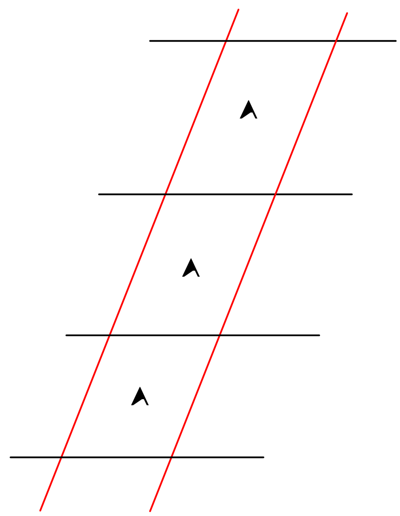


Figure 3.2



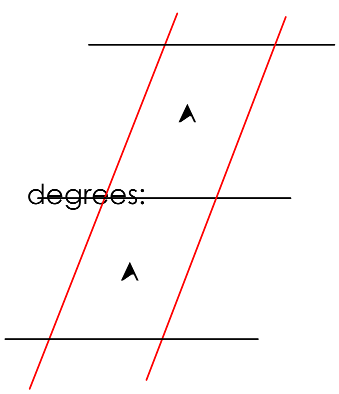
Figures 3.1 and 3.2 demonstrate how horizontal lines (black) are added to a constant number of vertical transversals to form an increasing number of parallelograms. This relationship is extrapolated in the table below.

Horizontal lines	Parallelograms	Notation
2	1	A_1
3	3	A_1, A_2, A_1 U A_2
4	6	A_1, A_2, A_3, A_1 U A_2, A_2 U A_3, A_1 U A_2 U A_3
5	10	A_1, A_2, A_3, A_4, A_1 U A_2, A_2 U A_3, A_3 U A_4, A_1 U A_2 U A_3 , A_2 U A_2 U A_3, A_1 U A_2 U A_3 U A_4
6	15	A_1, A_2, A_3, A_4, A_5, A_1 U A_2, A_2 U A_3, A_3 U A_4, A_4 U A_5, A_1 U A_2 U A_3, A_2 U A_3 U A_4, A_3 U A_4 U A_5, A_1 U A_2 U A_3 U A_4, A_2 U A_3

		U $\blacktriangle_4$ U $\blacktriangle_5$, $\blacktriangle_1$ U $\blacktriangle_2$ U $\blacktriangle_3$ U $\blacktriangle_4$ U $\blacktriangle_5$
7	21	$\blacktriangle_1$, $\blacktriangle_2$, $\blacktriangle_3$, $\blacktriangle_4$, $\blacktriangle_5$, $\blacktriangle_6$, $\blacktriangle_1$ U $\blacktriangle_2$, $\blacktriangle_2$ U $\blacktriangle_3$, $\blacktriangle_3$ U $\blacktriangle_4$, $\blacktriangle_4$ U $\blacktriangle_5$, $\blacktriangle_5$ U $\blacktriangle_6$, $\blacktriangle_1$ U $\blacktriangle_2$ U $\blacktriangle_3$, $\blacktriangle_2$ U $\blacktriangle_3$ U $\blacktriangle_4$, $\blacktriangle_3$ U $\blacktriangle_4$ U $\blacktriangle_5$, $\blacktriangle_4$ U $\blacktriangle_5$ U $\blacktriangle_6$, $\blacktriangle_1$ U $\blacktriangle_2$ U $\blacktriangle_3$ U $\blacktriangle_4$, $\blacktriangle_2$ U $\blacktriangle_3$ U $\blacktriangle_4$ U $\blacktriangle_5$, $\blacktriangle_3$ U $\blacktriangle_4$ U $\blacktriangle_5$ U $\blacktriangle_6$, $\blacktriangle_1$ U $\blacktriangle_2$ U $\blacktriangle_3$ U $\blacktriangle_4$ U $\blacktriangle_5$, $\blacktriangle_2$ U $\blacktriangle_3$ U $\blacktriangle_4$ U $\blacktriangle_5$ U $\blacktriangle_6$, $\blacktriangle_1$ U $\blacktriangle_2$ U $\blacktriangle_3$ U $\blacktriangle_4$ U $\blacktriangle_5$ U $\blacktriangle_6$

The general formula for this relationship would be identical to the general formula for the relationship between the number of vertical transversals and the number of parallelograms. This is due to the relationship of the number of horizontal lines and the number of parallelograms is the same as the relationship between the number of vertical transversals and the number of parallelograms. This is demonstrated in figure 4.1.

Figure 4.1



Rotated 90 degrees:

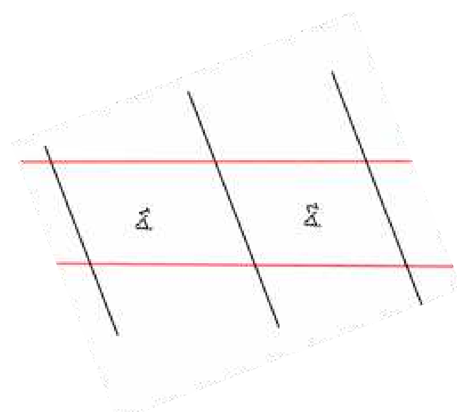
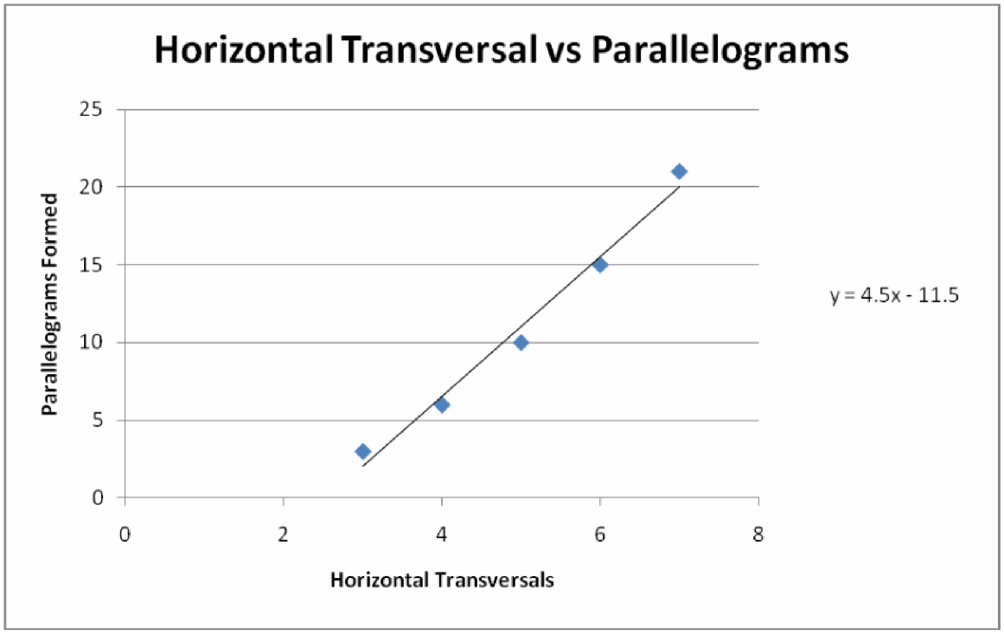


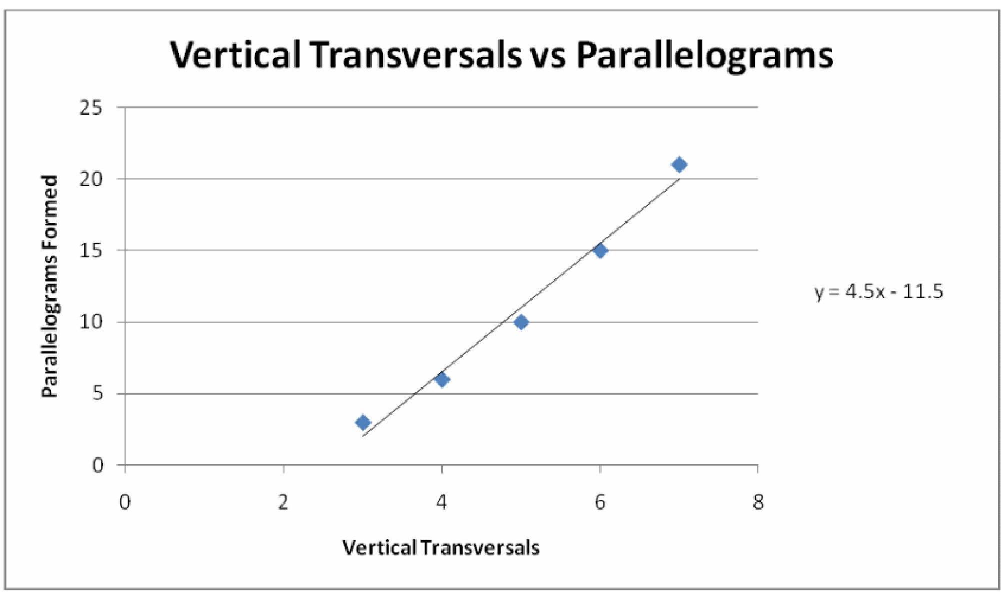
Figure 4.1 clearly illustrates how horizontal lines are identical to vertical transversals. Furthermore, this relationship is further clarified in graphs 2.1 and 2.2 by the identical equations on both graphs. Therefore, from these two observations, the same general formula that was used in the relationship between the number of vertical transversals and the number of parallelograms can be used in the relationship between horizontal lines and the number of parallelograms.

Graph



2.1

Graph 2.2



- The general formula for the relationship between the number of horizontal lines and the number of parallelograms is $P = h\left[\frac{h-1}{2}\right]$.

Where P is the number of parallelograms formed and h is the number of horizontal lines. This general formula does not encompass the variable of vertical transversals. The equation assumes that the number of vertical transversals stays constant at 2. To incorporate the variable of the number of vertical transversals it is imperative to merge both general statements. To do so, we need to multiply both equations together. This is because as a new vertical or horizontal transversal is added, another group of parallelograms is added. Therefore, adding another horizontal or vertical transversal effectively multiplies the number of parallelograms.

- The General formula that encompasses both variables of the number horizontal and vertical transversals is $P = \left[v\left[\frac{v-1}{2}\right]\right]\left[h\left[\frac{h-1}{2}\right]\right]$

Where P is the number of parallelograms produced, v is the number of vertical transversals and h is the number of horizontal lines.

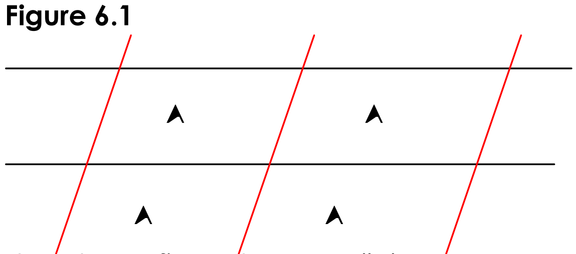
The formula encompasses both variables of the number horizontal and vertical transversals was incorporated into an excel spread sheet. The results are shown in figure 5.1.

Figure 5.1

		Vertical Transversals								
		2	3	4	5	6	7	8	9	10
Horizontal lines	2	1	3	6	10	15	21	28	36	45
	3	3	9	18	30	45	63	84	108	135
	4	6	18	36	60	90	126	168	216	270
	5	10	30	60	100	150	210	280	360	450
	6	15	45	90	150	225	315	420	540	675
	7	21	63	126	210	315	441	588	756	945
	8	28	84	168	280	420	588	784	1008	1260
	9	36	108	216	360	540	756	1008	1296	1620
	10	45	135	270	450	675	945	1260	1620	2025

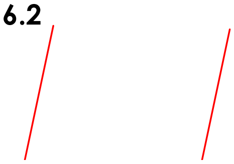
To further validate this formula the numbers that were produced from figure 4.1 need to be tested. This is shown in figure 6.1 and 6.2.

Figure 6.1



The above figure has parallelograms $\blacktriangle_1$, $\blacktriangle_2$, $\blacktriangle_3$, $\blacktriangle_4$, $\blacktriangle_1 \cup \blacktriangle_2$, $\blacktriangle_3 \cup \blacktriangle_4$, $\blacktriangle_1 \cup \blacktriangle_3$, $\blacktriangle_2 \cup \blacktriangle_4$ and $\blacktriangle_1 \cup \blacktriangle_2 \cup \blacktriangle_3 \cup \blacktriangle_4$. In total, figure 6.1 has 9 parallelograms; this coincides with the general formula's answer.

Figure 6.2



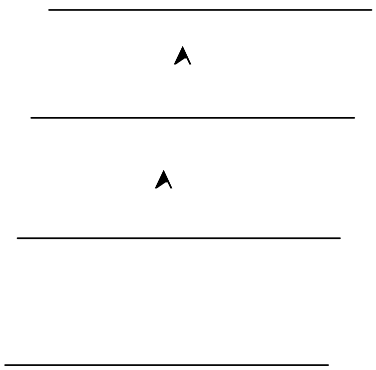


Figure 6.2 has parallelograms ▲$_1$, ▲$_2$, ▲$_3$, ▲1 U ▲$_2$, ▲$_2$ U ▲$_3$, and ▲1 U ▲$_2$ U ▲$_3$. In total, figure 6.2 has 6 parallelograms. This coincides with the general formula's answer.

 Therefore the general statement for the relationship between vertical transversals, horizontal lines and the number of parallelograms is

$$P = \left[v\left[\frac{v-1}{2}\right]\right]\left[h\left[\frac{h-1}{2}\right]\right]$$

Where P is the number of parallelograms produced, v is the number of vertical transversals and h is the number of horizontal lines.

However, this equation can only be used when $v \geq 2$ and $h \geq 2$. This is because a parallelogram is only created when 2 or more of each transversal are present. This is further demonstrated in figures 7.1 and 7.2.

Figure 7.1

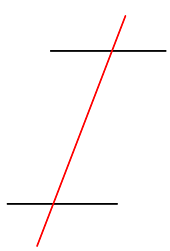
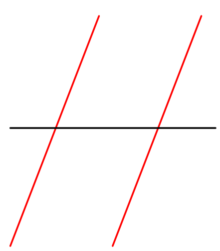


Figure 7.2



Furthermore, the variables, v and h have to be natural numbers. The equation does not work when the variables are fractions, negative numbers or imaginary numbers.

The generalisation of the formula that was found was arrived at because a varying number of vertical transversals, v intersect with a constant number of horizontal lines to produce $\blacktriangle_v$ parallelograms. Furthermore, a varying number of horizontal lines, h, intersect with a constant number of vertical transversals to produce $\blacktriangle_h$ parallelograms. Therefore a varying number of vertical transversals, v, intersect with a varying number of horizontal lines, h, to produce $\blacktriangle_v \times \blacktriangle_h$ parallelograms.

Bibliography

http://upload.wikimedia.org/wikipedia/commons/3/33/Números_triangulares.png

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