Mathematics Standard Level Portfolio by Emanuel Hausmann

Content

Parallels and Parallelograms

In this portfolio task I considered the number of parallelograms formed by intersecting parallel lines. The general statement for the overall pattern is $C = \frac{(h-1)(t-1)ht}{4}$, $(h, t \in \mathbb{N})$ where C is the count, h the number of horizontals and t the number of transversals. In the following I'll explain how I arrived at this generalization.

Figure 1 below shows a pair of horizontal parallel lines and a pair of parallel transversals. One parallelogram (A_1) is formed.

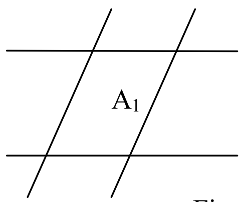


Figure 1

A third parallel transversal is added to the diagram as shown in Figure 2. Three parallelograms are formed: A_1, A_2 and A_1 □ A_2.

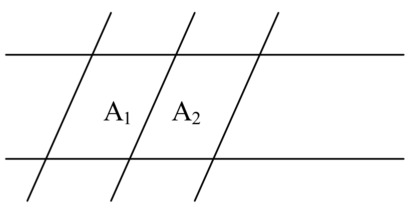


Figure 2

Task 1: ***Show that six parallelograms are formed when a fourth transversal is added to Figure 2. List all parallelograms, using set notation.***

If a fourth transversal is added to Figure 2, six parallelograms are formed like shown in Figure 3. The six parallelograms are: A_1, A_2, A_3, A_1 □ A_2, A_2 □ A_3 and A_1 □ A_2 □ A_3.

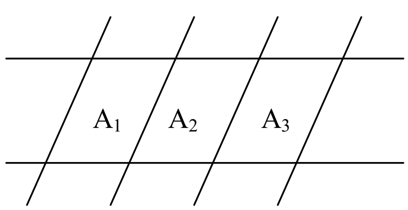


Figure 3

Task 2: ***Repeat the process with 5, 6 and 7 transversals. Show your re sults in a table. Use technology to find a relation between the number of transversals and the number of parallelograms. Develop a general statement, and test its validity.***

Figure 4 shows that ten parallelograms are formed if a further transversal is added to figure 3. These parallelograms are: A_1, A_2, A_3, A_4, A_1 □ A_2, A_2 □ A_3, A_3 □ A_4, A_1 □ A_2 □ A_3, A_2 □ A_3 □ A_4 and A_1 □ A_2 □ A_3 □ A_4.

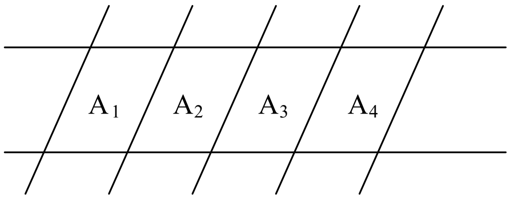


Figure 4

By looking at Figure 5 we can see that 15 parallelograms are formed if the diagram has 6 transversals: The 15 parallelograms are:
A_1, A_2, A_3, A_4, A_5,
A_1 □ A_2, A_2 □ A_3, A_3 □ A_4, A_4 □ A_5,
A_1 □ A_2 □ A_3, A_2 □ A_3 □ A_4, A_3 □ A_4 □ A_5,
A_1 □ A_2 □ A_3 □ A_4, A_2 □ A_3 □ A_4 □ A_5 and
A_1 □ A_2 □ A_3 □ A_4 □ A_5.

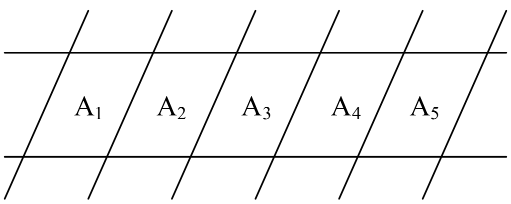


Figure 5

At 7^{th} transversal is added to the diagram as shown in Figure 5. 21 Parallelograms are formed:
A_1, A_2, A_3, A_4, A_5, A_6,
A_1 □ A_2, A_2 □ A_3, A_3 □ A_4, A_4 □ A_5, A_5 □ A_6,
A_1 □ A_2 □ A_3, A_2 □ A_3 □ A_4, A_3 □ A_4 □ A_5, A_4 □ A_5 □ A_6,
A_1 □ A_2 □ A_3 □ A_4, A_2 □ A_3 □ A_4 □ A_5, A_3 □ A_4 □ A_5 □ A_6,
A_1 □ A_2 □ A_3 □ A_4 □ A_5, A_2 □ A_3 □ A_4 □ A_5 □ A_6 and A_1 □ A_2 □ A_3 □ A_4 □ A_5 □ A_6.

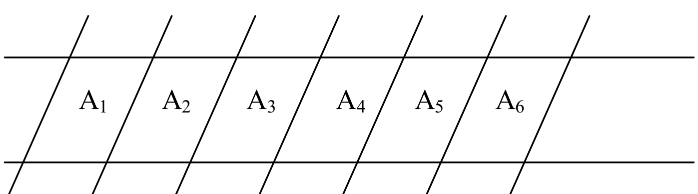


Figure 6

Task 3: ***Next consider the number of parallelograms formed by three horizontal parallel lines intersected by parallel transversals. Develop and test another general statement for this case.***

m	n	a_n
2	1	0
2	2	1
2	3	3
2	4	6
2	5	10
2	6	15
2	7	21

Figure 7

Legend to Figure 7:

$m = number\ of\ horizontal\ lines$

$n = number\ of\ parallel\ transversals$

$a_n = number\ of\ parallelograms\ formed$

Figure 7 shows that there must always be added 1 to the previous term in order to find out the new term u_n . Because the gaps between the numbers are always one smaller than n, it was found out that the formula must include $(n-1)$. In order to find out u_n , $(n-1)$must be added to u, which is one smaller than u_n.

m	n	a_n	S_n
2	1	0	0
2	2	1	1
2	3	3	4
2	4	6	10
2	5	10	20
2	6	15	35
2	7	21	56

Figure 8

Legend to Figure 8:

$m = number\ of\ horizontal\ lines$

$n = number\ of\ parallel\ transversals$

$a_n = number\ of\ parallelograms\ formed$

$S_n = sum\ of\ paralllograms\ formed$

Note that if we define u_6 as the set of all parallelograms that can be formed using six traversals and one parallelogram and u_7 as the set of all parallelograms that can be formed using seven traversals, the following is the case:

$$u_7 = u_6 \cup \{A_6, A_5 \cup A_6, A_4 \cup A_5 \cup A_6, A_3 \cup A_4 \cup A_5 \cup A_6, A_2 \cup A_3 \cup A_4 \cup A_5 \cup A_6, A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5 \cup A_6\} = 15 + 6 = 21$$

One can derive the following general formula for the set of all fields which can be formed using two horizontal lines and n transversals:

$$S_n = S_{n-1} \cup \{A_n, A_{n-1} \cup A_n, \ldots, A_1 \cup A_2 \cup \ldots \cup A_{n-1} \cup A_n\} \text{ with } S_1 = \{\} \text{ and where } n \in \mathbb{N}\backslash\{0\}$$

The number of elements in S_n which will be called u_n can be expressed as follows:

$$u_n = u_{n-1} + n - 1 \text{ with } u_1 = 0 \text{ and where } n \in \mathbb{N}\backslash\{0\}$$

This can be simplified:

$$u = 2 - 1 = 1$$

$$u_3 = 2 - 1 + 3 - 1 = 1 + 2$$

$$u_4 = 2 - 1 + 3 - 1 + 4 - 1 = 1 + 2 + 3$$

$$u_n = 2 - 1 + 3 - 1 + 4 - 1 + \cdots + n - 2 + n - 1$$

$$= 1 + 2 + 3 + \cdots + n - 3 + n - 2 + n - 1$$

$$= \sum_{i=1}^{n-1} i = \frac{1}{2} * (n-1) * \big((n-1) + 1\big)$$

$$= \frac{1}{2} * (n-1) * n$$

The general statement therefore is $u_n = \frac{1}{2}\, n\,(n-1)$. u_n is called the n^{th} term or the general term of the sequence.

In order to find a relationship between the number of transversals and the number of parallelograms I used the computer programme Windows Office Excel 2007 and proved my general statement like shown in Figure 9 and Figure 10.

n	a_n
0	0
1	0
2	1
3	3
4	6
5	10
6	15
7	21
8	28
9	36
10	45
11	55
12	66
13	78
14	91
15	105

Figure 9

Legend to Figure 9:

$n = number\ of\ parallel\ transversals$

$a_n = number\ of\ parallelograms\ formed$

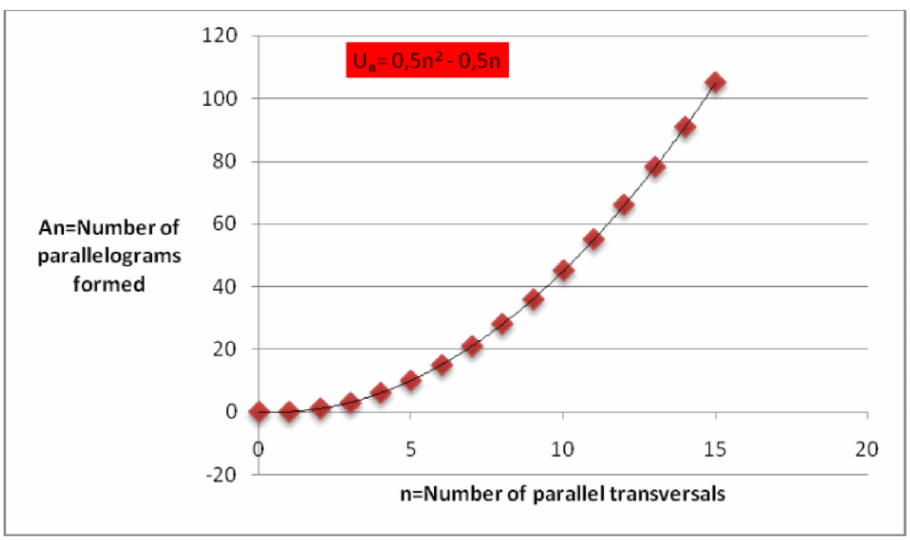


Figure 10

To test the validity of the general statement it can be shown what happens if we have 9 transversals. By using the general term formula we would get 36 parallelograms:

General term: $u_n = \frac{1}{2}\, n\,(n-1)$

Number of transversals $n = 9$

Therefore: $u_9 = \frac{1}{2}\, 9\,(9-1) = 4.5\,(8) = 36$

To back up the general statement Figure 11shows the 36 parallelograms:
A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8,
A_1 □ A_2, A_2 □ A_3, A_3 □ A_4, A_4 □ A_5, A_5 □ A_6, A_6 □ A_7, A_7 □ A_8,
A_1 □ A_2 □ A_3, A_2 □ A_3 □ A_4, A_3 □ A_4 □ A_5, A_4 □ A_5 □ A_6 , A_5 □ A_6 □ A_7 , A_6 □ A_7 □ A_8 ,
A_1 □ A_2 □ A_3 □ A_4, A_2 □ A_3 □ A_4 □ A_5, A_3 □ A_4 □ A_5 □ A_6 , A_4 □ A_5 □ A_6 □ A_7 , A_5 □ A_6 □ A_7 □ A_8 ,
A_1 □ A_2 □ A_3 □ A_4 □ A_5, A_2 □ A_3 □ A_4 □ A_5 □ A_6, A_3 □ A_4 □ A_5 □ A_6 □ A_7, A_4 □ A_5 □ A_6 □ A_7 □ A_8,
A_1 □ A_2 □ A_3 □ A_4 □ A_5 □ A_6, A_2 □ A_3 □ A_4 □ A_5 □ A_6 □ A_7, A_3 □ A_4 □ A_5 □ A_6 □ A_7 □ A_8,
A_1 □ A_2 □ A_3 □ A_4 □ A_5 □ A_6 □ A_7, A_2 □ A_3 □ A_4 □ A_5 □ A_6 □ A_7 □ A_8 and
A_1 □ A_2 □ A_3 □ A_4 □ A_5 □ A_6 □ A_7 □ A_8.

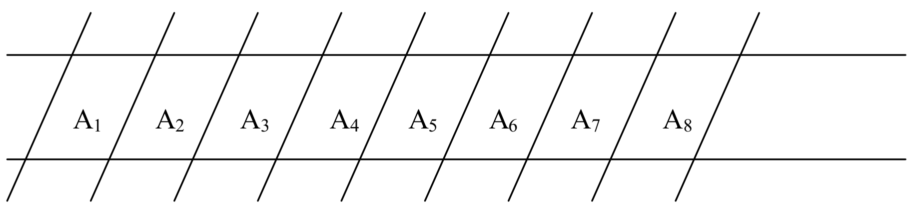


Figure 11

To verify the general statement Figure 12 illustrates how many parallelograms are formed if we have 11 transversals and a pair of horizontal lines.

General term: $u_n = \frac{1}{2}\, n\,(n-1)$

Number of transversals n = 11

Therefore: $u_{11} = \frac{1}{2}\, 11\,(11-1) = 5.5\,(10) = 55$

The so formed parallelograms are: A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9, A_{10},
A_1 □ A_2, A_2 □ A_3, A_3 □ A_4, A_4 □ A_5, A_5 □ A_6, A_6 □ A_7, A_7 □ A_8, A_8 □ A_9, A_9 □ A_{10},
A_1 □ A_2 □ A_3, A_2 □ A_3 □ A_4, A_3 □ A_4 □ A_5, A_4 □ A_5 □ A_6 , A_5 □ A_6 □ A_7 , A_6 □ A_7 □ A_8 , A_7 □ A_8 □ A_9, A_8 □ A_9 □ A_{10},
A_1 □ A_2 □ A_3 □ A_4, A_2 □ A_3 □ A_4 □ A_5, A_3 □ A_4 □ A_5 □ A_6 , A_4 □ A_5 □ A_6 □ A_7 , A_5 □ A_6 □ A_7 □ A_8 , A_6 □ A_7 □ A_8 □ A_9 , A_7 □ A_8 □ A_9 □ A_{10} ,
A_1 □ A_2 □ A_3 □ A_4 □ A_5, A_2 □ A_3 □ A_4 □ A_5 □ A_6, A_3 □ A_4 □ A_5 □ A_6 □ A_7, A_4 □ A_5 □ A_6 □ A_7 □ A_8, A_5 □ A_6 □ A_7 □ A_8 □ A_9, A_6 □ A_7 □ A_8 □ A_9 □ A_{10},

$A_1 \square A_2 \square A_3 \square A_4 \square A_5 \square A_6$, $A_2 \square A_3 \square A_4 \square A_5 \square A_6 \square A_7$, $A_3 \square A_4 \square A_5 \square A_6 \square A_7 \square A_8$, $A_4 \square A_5 \square A_6 \square A_7 \square A_8 \square A_9$, $A_5 \square A_6 \square A_7 \square A_8 \square A_9 \square A_{10}$,
$A_1 \square A_2 \square A_3 \square A_4 \square A_5 \square A_6 \square A_7$, $A_2 \square A_3 \square A_4 \square A_5 \square A_6 \square A_7 \square A_8$, $A_3 \square A_4 \square A_5 \square A_6 \square A_7 \square A_8 \square A_9$, $A_4 \square A_5 \square A_6 \square A_7 \square A_8 \square A_9 \square A_{10}$,
$A_1 \square A_2 \square A_3 \square A_4 \square A_5 \square A_6 \square A_7 \square A_8$, $A_2 \square A_3 \square A_4 \square A_5 \square A_6 \square A_7 \square A_8 \square A_9$, $A_3 \square A_4 \square A_5 \square A_6 \square A_7 \square A_8 \square A_9 \square A_{10}$,
$A_1 \square A_2 \square A_3 \square A_4 \square A_5 \square A_6 \square A_7 \square A_8 \square A_9$, $A_2 \square A_3 \square A_4 \square A_5 \square A_6 \square A_7 \square A_8 \square A_9 \square A_{10}$ and
$A_1 \square A_2 \square A_3 \square A_4 \square A_5 \square A_6 \square A_7 \square A_8 \square A_9 \square A_{10}$.

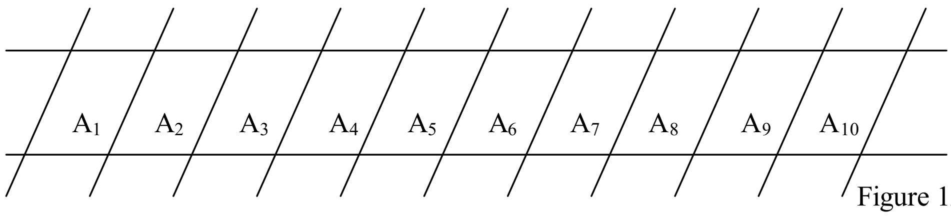


Figure 12

Figure 13 below shows 3 horizontal parallel lines and a pair of parallel transversals. Three parallelograms are formed: A_1, A_2 and $A_1 \square A_2$.

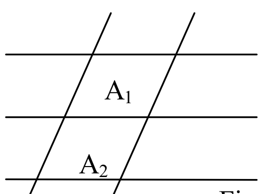
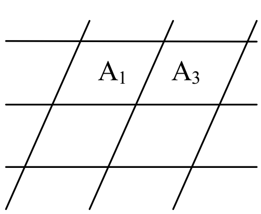


Figure 13

If a third parallel transversal is added to the diagram as shown in Figure 13, nine parallelograms are formed. These parallelograms are:
A_1, A_2, A_3, A_4,
$A_1 \square A_2$, $A_3 \square A_4$, $A_1 \square A_3$, $A_2 \square A_4$ and
$A_1 \square A_2 \square A_3 \square A_4$



A_2 A_4

Figure 14

A further transversal is added to Figure 14. Now we have four transversals and nine parallelograms are created. The nine parallelograms are:

A_1, A_2, A_3, A_4, A_5, A_6,

A_1 □ A_2, A_3 □ A_4, A_5 □ A_6, A_1 □ A_3, A_3 □ A_5, A_2 □ A_4, A_4 □ A_6,

A_1 □ A_2 □ A_3 □ A_4, A_3 □ A_4 □ A_5 □ A_6 and

A_1 □ A_2 □ A_3 □ A_4 □ A_5 □ A_6.

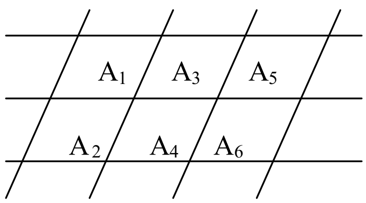


Figure 15

If we have five transversals 30 parallelograms are formed like shown in Figure 16. They are:

A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8,

A_1 □ A_2, A_3 □ A_4, A_5 □ A_6, A_7 □ A_8, A_1 □ A_3, A_3 □ A_5, A_5 □ A_7, A_2 □ A_4, A_4 □ A_6, A_6 □ A_8,

A_1 □ A_3 □ A_5, A_3 □ A_5 □ A_7, A_2 □ A_4 □ A_6, A_4 □ A_6 □ A_8,

A_1 □ A_2 □ A_3 □ A_4, A_3 □ A_4 □ A_5 □ A_6, A_5 □ A_6 □ A_7 □ A_8, A_1 □ A_3 □ A_5 □ A_7, A_2 □ A_4 □ A_6 □ A_8,

A_1 □ A_2 □ A_3 □ A_4 □ A_5 □ A_6, A_3 □ A_4 □ A_5 □ A_6 □ A_7 □ A_8 and

A_1 □ A_2 □ A_3 □ A_4 □ A_5 □ A_6 □ A_7 □ A_8.

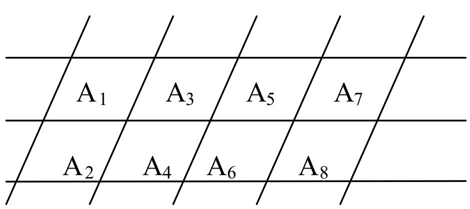


Figure 16

If we have six transversals 45 parallelograms are formed like shown in Figure 17. They are:

A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9, A_{10},

A_1 □ A_2, A_3 □ A_4, A_5 □ A_6, A_7 □ A_8, A_9 □ A_{10}, A_1 □ A_3, A_3 □ A_5, A_5 □ A_7, A_7 □ A_9, A_2 □ A_4, A_4 □ A_6, A_6 □ A_8, A_8 □ A_{10},

A_1 □ A_3 □ A_5, A_3 □ A_5 □ A_7, A_5 □ A_7 □ A_9, A_2 □ A_4 □ A_6, A_4 □ A_6 □ A_8, A_6 □ A_8 □ A_{10},

A_1 □ A_2 □ A_3 □ A_4, A_3 □ A_4 □ A_5 □ A_6, A_5 □ A_6 □ A_7 □ A_8, A_7 □ A_8 □ A_9 □ A_{10},

$A_1 \square A_3 \square A_5 \square A_7$, $A_3 \square A_5 \square A_7 \square A_9$, $A_2 \square A_4 \square A_6 \square A_8$, $A_4 \square A_6 \square A_8 \square A_{10}$,
$A_1 \square A_3 \square A_5 \square A_7 \square A_9$, $A_2 \square A_4 \square A_6 \square A_8 \square A_{10}$,
$A_1 \square A_2 \square A_3 \square A_4 \square A_5 \square A_6$, $A_3 \square A_4 \square A_5 \square A_6 \square A_7 \square A_8$, $A_5 \square A_6 \square A_7 \square A_8 \square A_9 \square A_{10}$,
$A_1 \square A_2 \square A_3 \square A_4 \square A_5 \square A_6 \square A_7 \square A_8$, $A_3 \square A_4 \square A_5 \square A_6 \square A_7 \square A_8 \square A_9 \square A_{10}$ and $A_1 \square A_2 \square A_3 \square A_4 \square A_5 \square A_6 \square A_7 \square A_8 \square A_9 \square A_{10}$.

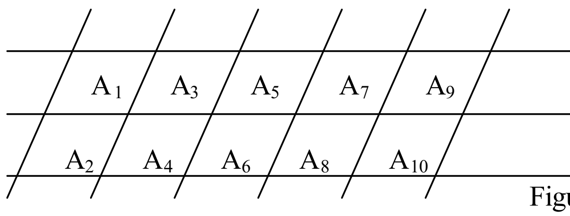


Figure 17

If we have six transversals 63 parallelograms are formed like shown in Figure 18. They are:
A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9, A_{10}, A_{11}, A_{12},
$A_1 \square A_2$, $A_3 \square A_4$, $A_5 \square A_6$, $A_7 \square A_8$, $A_9 \square A_{10}$, $A_{11} \square A_{12}$, $A_1 \square A_3$, $A_3 \square A_5$, $A_5 \square A_7$, $A_7 \square A_9$, $A_9 \square A_{11}$, $A_2 \square A_4$, $A_4 \square A_6$, $A_6 \square A_8$, $A_8 \square A_{10}$, $A_{10} \square A_{12}$,
$A_1 \square A_3 \square A_5$, $A_3 \square A_5 \square A_7$, $A_5 \square A_7 \square A_9$, $A_7 \square A_9 \square A_{11}$, $A_2 \square A_4 \square A_6$, $A_4 \square A_6 \square A_8$, $A_6 \square A_8 \square A_{10}$, $A_8 \square A_{10} \square A_{12}$,
$A_1 \square A_2 \square A_3 \square A_4$, $A_3 \square A_4 \square A_5 \square A_6$, $A_5 \square A_6 \square A_7 \square A_8$, $A_7 \square A_8 \square A_9 \square A_{10}$, $A_9 \square A_{10} \square A_{11} \square A_{12}$,$A_1 \square A_3 \square A_5 \square A_7$, $A_3 \square A_5 \square A_7 \square A_9$, $A_5 \square A_7 \square A_9 \square A_{11}$, $A_2 \square A_4 \square A_6 \square A_8$, $A_4 \square A_6 \square A_8 \square A_{10}$, $A_6 \square A_8 \square A_{10} \square A_{12}$,
$A_1 \square A_3 \square A_5 \square A_7 \square A_9$, $A_3 \square A_5 \square A_7 \square A_9 \square A_{11}$, $A_2 \square A_4 \square A_6 \square A_8 \square A_{10}$, $A_4 \square A_6 \square A_8 \square A_{10} \square A_{12}$,
$A_1 \square A_3 \square A_5 \square A_7 \square A_9 \square A_{11}$, $A_2 \square A_4 \square A_6 \square A_8 \square A_{10} \square A_{12}$, $A_1 \square A_2 \square A_3 \square A_4 \square A_5 \square A_6$, $A_3 \square A_4 \square A_5 \square A_6 \square A_7 \square A_8$, $A_5 \square A_6 \square A_7 \square A_8 \square A_9 \square A_{10}$, $A_7 \square A_8 \square A_9 \square A_{10} \square A_{11} \square A_{12}$,
$A_1 \square A_2 \square A_3 \square A_4 \square A_5 \square A_6 \square A_7 \square A_8$, $A_3 \square A_4 \square A_5 \square A_6 \square A_7 \square A_8 \square A_9 \square A_{10}$, $A_5 \square A_6 \square A_7 \square A_8 \square A_9 \square A_{10} \square A_{11} \square A_{12}$, $A_1 \square A_2 \square A_3 \square A_4 \square A_5 \square A_6 \square A_7 \square A_8 \square A_9 \square A_{10}$, $A_3 \square A_4 \square A_5 \square A_6 \square A_7 \square A_8 \square A_9 \square A_{10} \square A_{11} \square A_{12}$ and $A_1 \square A_2 \square A_3 \square A_4 \square A_5 \square A_6 \square A_7 \square A_8 \square A_9 \square A_{10} \square A_{11} \square A_{12}$.

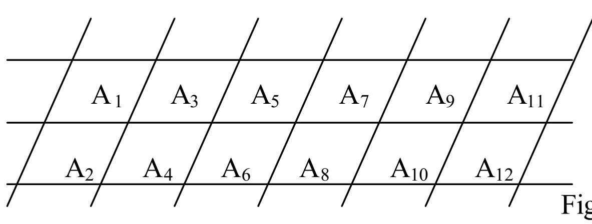


Figure 1 8

m	n	a_n
3	1	0
3	2	3
3	3	9
3	4	18
3	5	30
3	6	45
3	7	63

+3 +6 +9 +12 +15 +18

Figure 19

Legend to Figure 19:

$m = \text{ number of horizontal lines}$

$n = \text{number of parallel transversals}$

$a_n = \text{ number of parallelograms formed}$

By looking at Figure 16 I realized that there are always gaps by 3 in order to get the next u : 3,6,9,12,15,18...

All these gaps are always three times bigger than n. Therefore: $(n-1) \times 3$

In order to find out u_n we have to add $(n-1) \times 3$ to u_{n-1}. This is called a dependent variable because we have to know u to be able to work out u_n.

To get the new general statement for the number of parallelograms formed by three horizontal parallel lines intersected by parallel transversals I used the statement developed for the relationship between the number of transversals and the number of parallelograms and multiplied it by 3 becasue we have got three horizontals.

Now we have $u_n = u_{n-1} + (n - 1) \times 3$.

To test my general statement for the number of parallelograms formed by three horizontal parallel lines intersected by parallel transversals I give two examples:

Example 1:

General statement: $u_n = u_{n-1} + (n - 1) \times 3$

Number of transversals: $n = 5$

$$\therefore u_5 = u_{5-1} + (5 - 1) \times 3$$
$$= u_4 + (4) \times 3$$
$$= 18 + 12$$
$$= 30$$

Figure 16 underlines that my example is right.

Example 2:

General statement: $u_n = u_{n-1} + (n - 1) \times 3$

Number of transversals: $n = 7$

$$\therefore u_7 = u_{7-1} + (7 - 1) \times 3$$
$$= u_6 + (6) \times 3$$
$$= 45 + 18$$
$$= 63$$

Looking at Figure 18 the correctness of example 2 can be seen.

Task 4: ***Now extend your results to m horizontal parallel lines intersected by n parallel transversals.***

The number of parallelograms visible, when h horizontals and t transversals are intersected, $u_n(h,t)$ can be determined considering the following:
The overall sum of parallelograms is equal to the sum of the number of parallelograms for of all possible sizes. The maximum size of a parallelogram is $(h-1)\times(t-1)$.
My Example Figure 20 examines the case that there are five vertical and four horizontal lines:

Subpart size (in fields)		Number of possible parallelograms	Σ
width (x)	height (y)		
1	1	$q_{11}=12=4*3=(5-1)(4-1)$	12
1	2	$q_{12}=8=4*2=(5-1)(4-2)$	20
1	3	$q_{13}=4=4*1=(5-1)(4-3)$	24
2	1	$q_{21}=8=2*4=(5-2)(4-1)$	32
2	2	$q_{22}=6=3*2=(5-2)(4-2)$	38
2	3	$q_{23}=4=2*2=(5-3)(4-2)$	42
3	1	$q_{31}=6=2*3=(5-3)(4-1)$	48
3	2	$q_{32}=4=2*2=(5-3)(4-2)$	52
3	3	$q_{33}=2=2*1=(5-3)(4-3)$	54
4	1	$q_{41}=3=1*3=(5-4)(4-1)$	57
4	2	$q_{42}=2=1*2=(5-4)(4-2)$	59
4	3	$q_{43}=1=1*1=(5-4)(4-3)$	60

Figure 20

The example shows that the number of parallelograms (q_{xy}) of a certain size $(x\times y)$ can be determined using the following rule:
$q_{xy}=(h-x)(t-y)$
If one adds all q $u_n(h,t)$ is obtained:

$$u_n(h,t) = \sum_{x=1}^{h-1}\left(\sum_{y=1}^{t-1} q_{xy}\right) = \sum_{x=1}^{h-1}\left(\sum_{y=1}^{t-1}(h-x)(t-y)\right)$$

$$= \sum_{x=1}^{h-1}\left((h-x) * \sum_{y=1}^{t-1}(t-y)\right)$$

$$= \sum_{y=1}^{t-1}(t-y) * \sum_{x=1}^{h-1}(h-x)$$

$$\sum_{i=1}^{n}(n+1) - i = \sum_{i=1}^{n} i$$

$$\therefore u_n(h,v) = \sum_{y=1}^{t-1}\big((t-1)+1-y\big) * \sum_{x=1}^{h-1}\big((h-1)+1-x\big)$$

$$= \sum_{y=1}^{t-1}(y) * \sum_{x=1}^{h-1}(x)$$

$$\sum_{i=1}^{n-1} i = \frac{1}{2} * (n-1) * \big((n-1)+1\big)$$

$$= \frac{1}{2} * (n-1) * n$$

$$\therefore u_n(h,t) = \left(\tfrac{1}{2} * (v-1) * t\right)\left(\tfrac{1}{2} * (h-1) * h\right)$$

$$= \frac{(h-1)(t-1)ht}{4}$$

To verify my general statement I'll show how it works by using two examples.

Example 3:

General statement: $u_n = \frac{(h-1)(t-1)ht}{4}$

Number of transversals: $t = 5$

Number of horizontal lines: h$= 3$

$$u_n = \frac{(3-1)(5-1)3 \times 5}{4}$$

$$u_n = \frac{(2)(4) \times 15}{4}$$

$$u_n = \frac{8 \times 15}{4}$$

$$u_n = \frac{120}{4}$$

$u_n = 30$

Figure 17 also verifies that my statement works.

Example 4:

General statement: $u_n = \frac{(h-1)(t-1)ht}{4}$

Number of transversals: $t = 5$

Number of horizontal lines: h=2

$$u_n = \frac{(h-1)(t-1)ht}{4}$$

$$u_n = \frac{(2-1)(5-1)2 \times 5}{4}$$
$$u_n = \frac{(1)(4) \times 10}{4}$$
$$u_n = \frac{4 \times 10}{4}$$
$$u_n = \frac{40}{4}$$
$$u_n = 10$$

Figure 4 underlines the correctness of my general statement.

Scope and Limitations:

By looking at all my work I made some observations in order to find out the scope and limitations:

a. We cannot have negative numbers.
b. There are no negative transversals or horizontals .
c. There are no transversals or horizontals with a fraction.

$\therefore h, t \in \mathbb{N}$

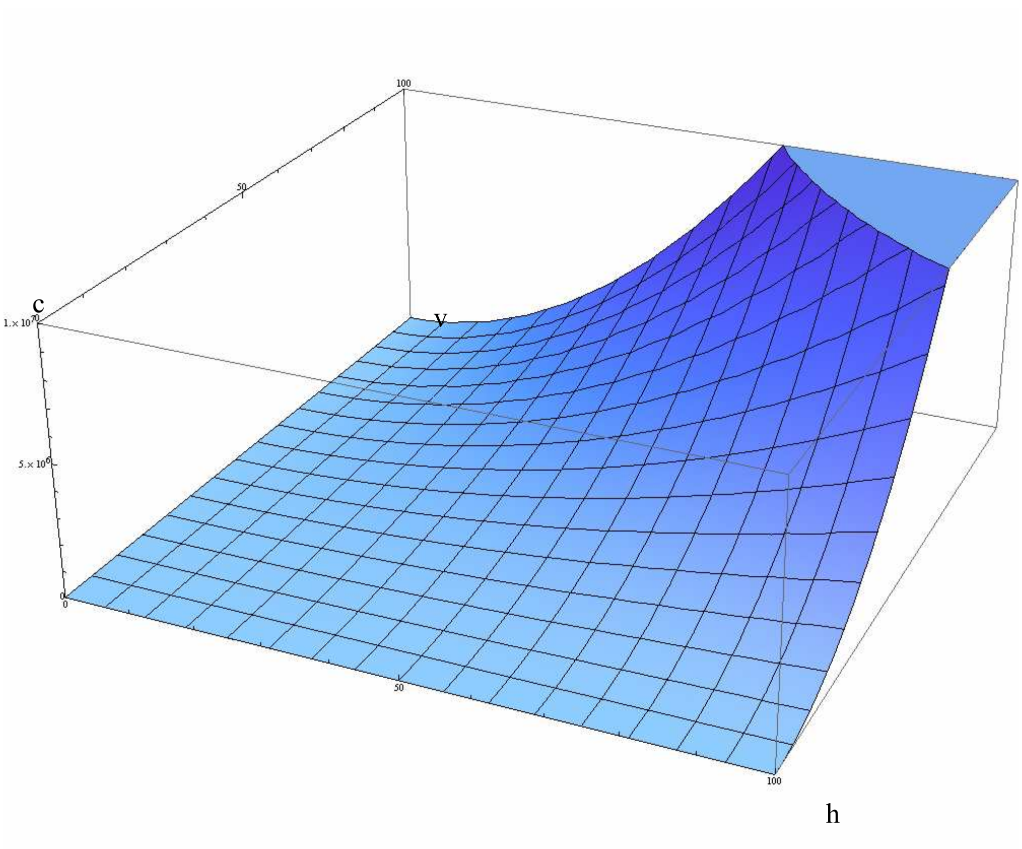


Figure 21

Figure 21 shows the graph I have plotted in order to visualize the general statement and its limitations.

The following sheet (Figure 22) shows a part of the the table I have used to plot the graph.