HL MATH PORTFOLIO
TYPE 1

PARABOLA INVESTIGATION

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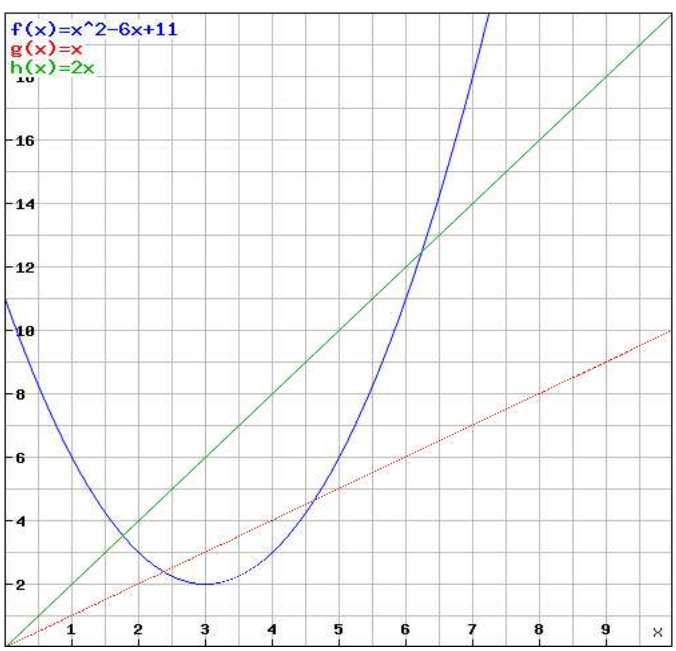
PARABOLA INVESTIGATION

Description

In this task, you will investigate the patterns in the intersections of parabolas and the lines y = x and y = 2x. Then you will be asked to prove your conjectures and to broaden the scope of the investigation to include other lines and other types of polynomials

1) Consider the parabola **x^2 -6x + 11** and the lines y = x and y = 2x

- ✓ .Intersection points of the parabola with the line y=x are x_2 and x3
  x_2: 2.38, x_3: 4.62
- ✓ Intersection points of the parabola with the line y=2x are x3 and x4
  x_1: 1.76 x_4: 6.24



X_2-X_1 = **S_L**
2.38-1.76= 0.39

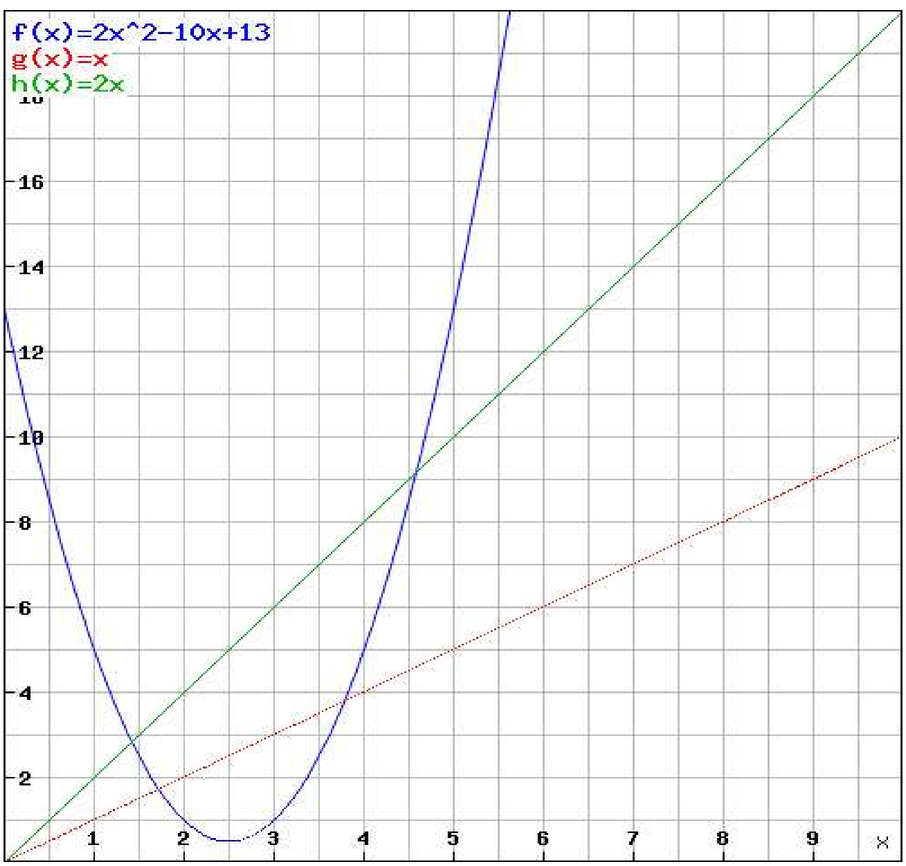
X_4-X_3 = **S_R**
6.24 – 4.62. = 1.39

S_R – S_L = D
1.39- 0.39 = 1

2)Find the values of D for other parabolas of the form **$y= ax^2 + bx + c$, $a>0$,** with vertices in quadrant 1, intersected by the lines y = x and y = 2x. Consider various values of a, beginning with a =1. Make a conjecture about the value of D for these parabolas.

- a= 2, b= -10, c =13

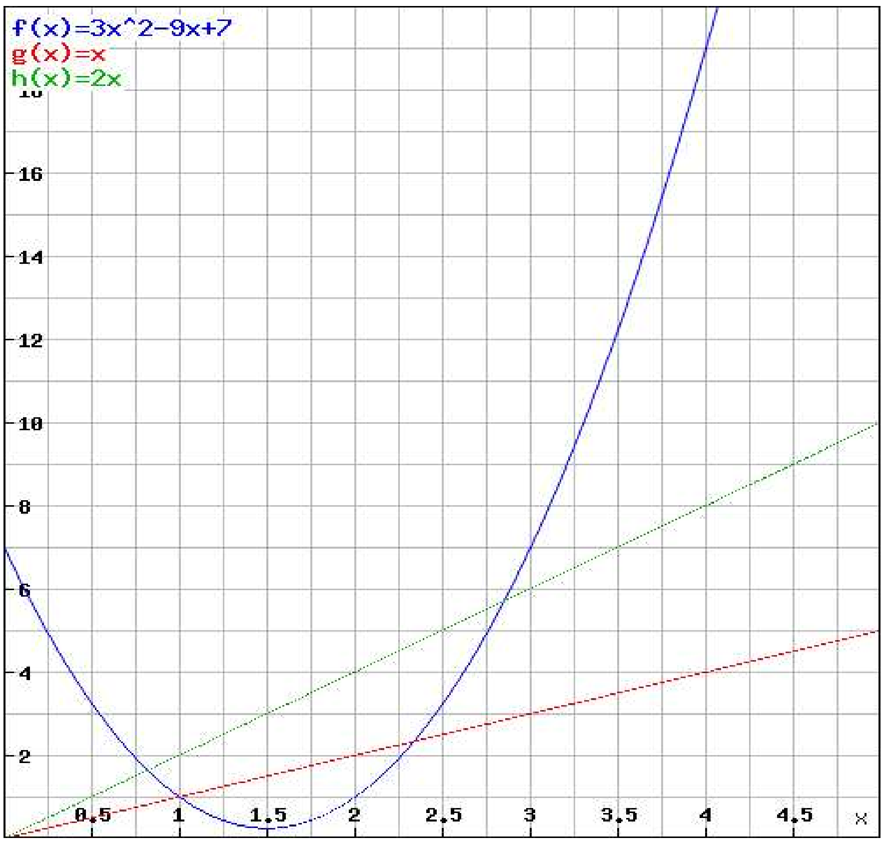
  f(x) = **$2x^2$-10x+13**



X_1: 1.40
X_2: 1.70 X_2-X_1= 0.30
X_3: 3.78
X_4: 4.58 X_4-X_3= 0.80

D= 0.80-0.30 = 0.50
When a = 2, D = 0.5

- **a=3, b= -9, c = 7**
  y = $3x^2$-9x+7



X_1 : 0.82
X_2: 1
X_3 : 2.33
X_4 : 2.85

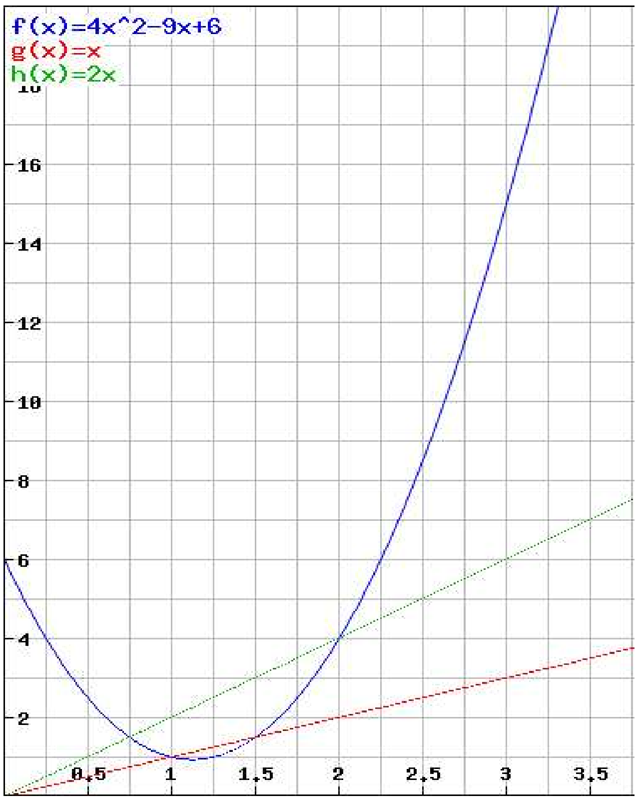
X_2- X_1 = S_L
=1-0.82
= 0.18

X_4-X_3 = S_R
= 2.85-2.33

= 0.52

D= 0.52 – 0.18
=0.33

- a=4 , b=-15 , c= 14



X_1= 0.75
X_2= 1
X_3= 1.5
X_4 =2

S_L = 0.25
S_R= 0.50
D= 0.50 – 0.25
= 0.25

Evaluation

- **For a = 1** ,D value is equal to **1.00**
- For **a = 2** , D value is equal to **0.50**
- For **a= 3** , D value is equal to **0.33**
- For **a= 4** , D value is equal to **0.25**

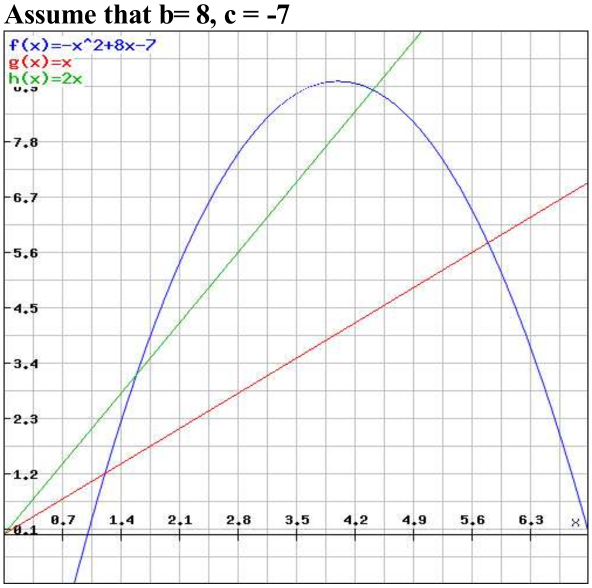
If the pattern of the D values are examined it can be concluded that it is directly proportional with 1/a

Conjecture is D= 1/a

3) Investigate your conjecture for any real value of a , and any placement of the vertex. Refine your conjecture as necessary, and prove it.

At part two , I have examined the a values grater than 1. But at this part as a can take any reel value I will be examining the a values less then 0 to prove whether my conjecture is true or not.

For a = -1
Assume that b= 8, c = -7



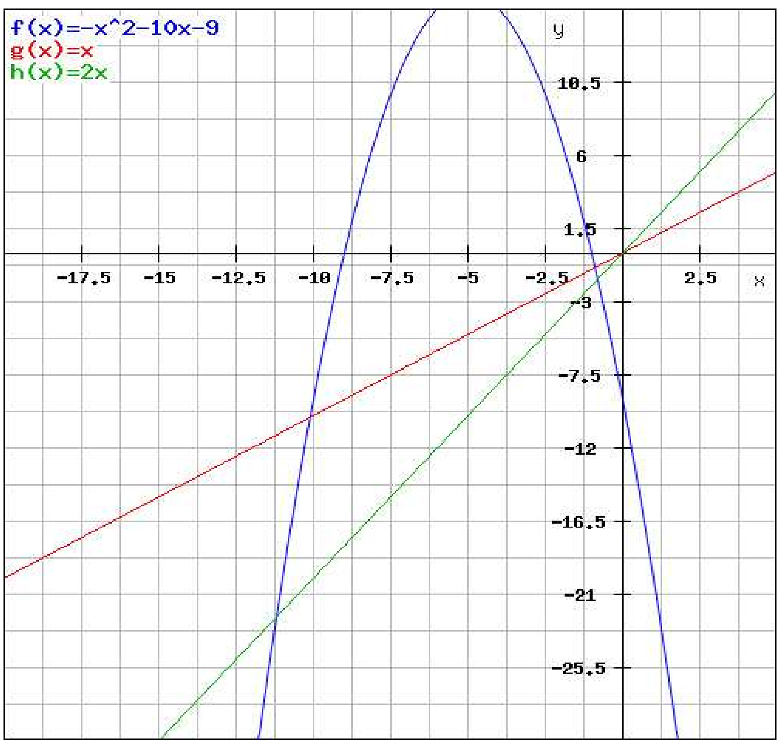
i. $x_1 - x_2 = 1.27 - 1 = 0.27 = S_L$
ii. $x_3 - x_4 = 6 - 4.73 = 1.27 = S_R$

D =1.27-0.27
=1

- The conjecture I made previously was **D= 1 /a.** However it doesn't seem to work for a= -1 as 1/a is equal to -1 in this case but D is equal to 1. So, I will test this conjecture with another reel value of "a".

- As I have to test my conjecture for any placement of vertex, I will be testing it for a parabola with a vertex in the 2nd quadrant:

  ✓ f(x) = $\mathbf{-2x^2-10x-9}$



X_1:-4.5
X2: -5.12
X3:-0.88
X4: -1

SL= -5.12-(-4.5) = -0.62

SR= -1-(-0.88) = -0.12

D= -0.12-(-0.62) =0.50

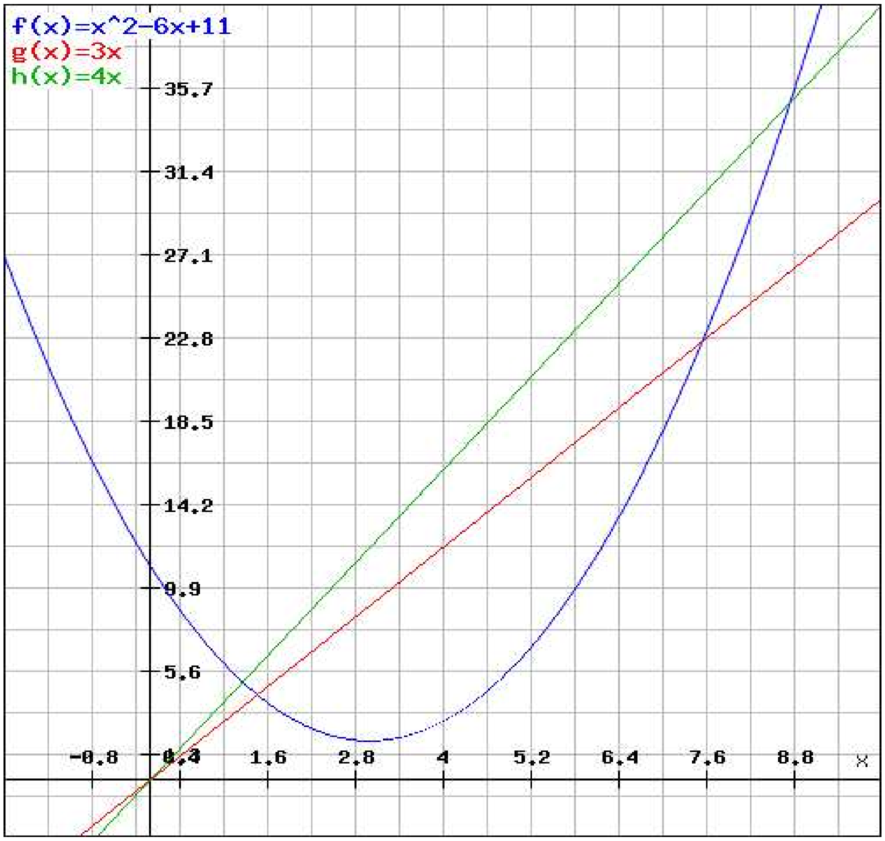
According to my conjecture, D value found is not correct as it must have been 1/a which is -0.50 in this case. It can be concluded that there is a problem with the sign of a in my conjecture as despite "a"'s having a negative sign D is always positive . If I modify my conjecture as **D = 1/ lal** the problem will be solved.

4)Does your conjecture hold if the intersecting lines are changed? Modify your conjecture, if necessary , and prove it.

- Previously I have made a conjecture about the D values of a parabolas intersection of lines y=x and y=2x. To test my cojecture I have worked with several parabolas and modified my conjecture accordingly as **D = 1/ lal .** To generalize this conjecture I must test it's validity by changing the intersecting lines.

  - ✓ Let's test the conjecture for y= 3x and y=4x

  I will continue working on my first parabola which was **x^2 -6x + 11=y**

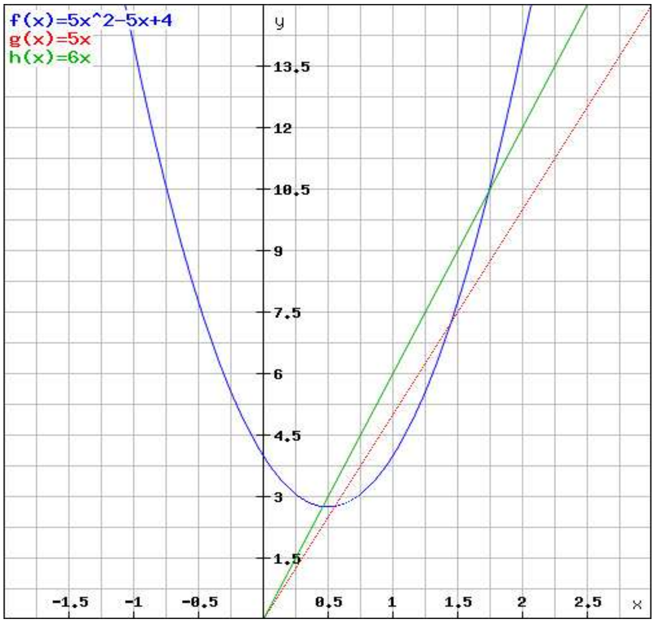


X_1 : 1.26
X_2: 1.46
1.46-1.26 = 0.2 = S_L
X_3: 7.54
X_4: 8.74
8.74 - 7.54 = 1.2 = S_R

D= 1.2 - 0.2 = 1 = 1/ lal

- As it can be seen above, even tough I have changed the intersecting lines into y=3x and y= 4x, my conjecture still worked.

✓ To be sure, I will test my conjecture for the intersecting lines y=5x , y=6x and the parabola 5x^2 **-5x + 4**



X_1: 0.46

X_2: 0.55

0.55- 0.46 = 0.09 = S_L

X_3: 1.45

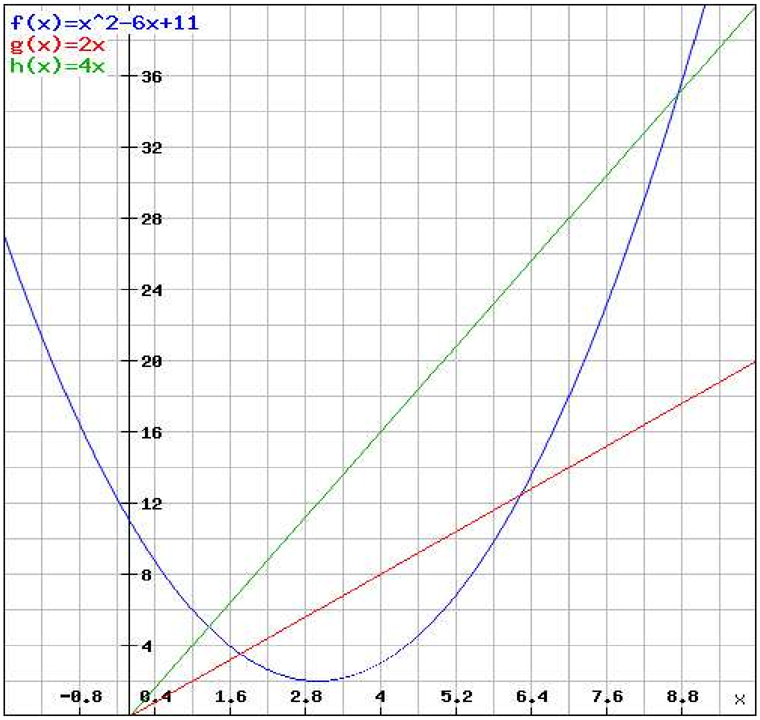
X_4 : 1.74

1.74-1.45 = 0.29 = S_R

D = 0.29-0.09 = 0.20

As 1/ 5 is equal to 0.20. I proved my conjecture to be working with different intersecting lines than y=x and y=2x.

- Both two trials were made with intersecting lines that have 1 difference between their slopes. Now I will test my conjecture for the intersecting lines y= 2x and y= 4x and the parabola **x^2 -6x + 11.**



- X1 : 1.26

  X2: 1.76

  1.76- 1.26 =0.5= SL

  X3: 6.24

  X4: 8.72

  8.74 – 6.24 = 2.5 = SR

  D = 2.5- 0.5 = 2.0

- According to my conjecture D must have been equal to 1 but it is 2 in this case . When I worked with lines which had one difference in their slopes, my conjecture worked, however it doesn't seem to be working with the lines y= 2x and y=4x. It is obvious that I must modify my conjecture in order to involve slope differences of the lines.

$M_2 - M_1 = 4-2 = 2$

Interestingly I found D value to be 2 .

I think if I modify my conjecture as

$D = (m_2 - m_1) / |a|$ it will become more accurate.

<u>Conjecture :</u>

$y=b_1x$ (these are the equations of the lines)

$y=b_2x$

$D=|b_2-b_1|/|a|$

Proof:

$ax^2+(b-b_1)x+c=0$

$ax^2+(b-b_2)x+c=0$

$x_{2,3}=-(b-b1)\pm\sqrt{((b-b1)2-4ac)}/2a$

$x_{1,4}=-(b-b2)\pm\sqrt{((b-b2)2-4ac)}/2a$

$D=|SL-SR|=|(x_2-x_1)-(x_4-x_3)|=|x_2-x_1-x_4+x_3|=|(x_2+x_3)-(x_1+x_4)|$

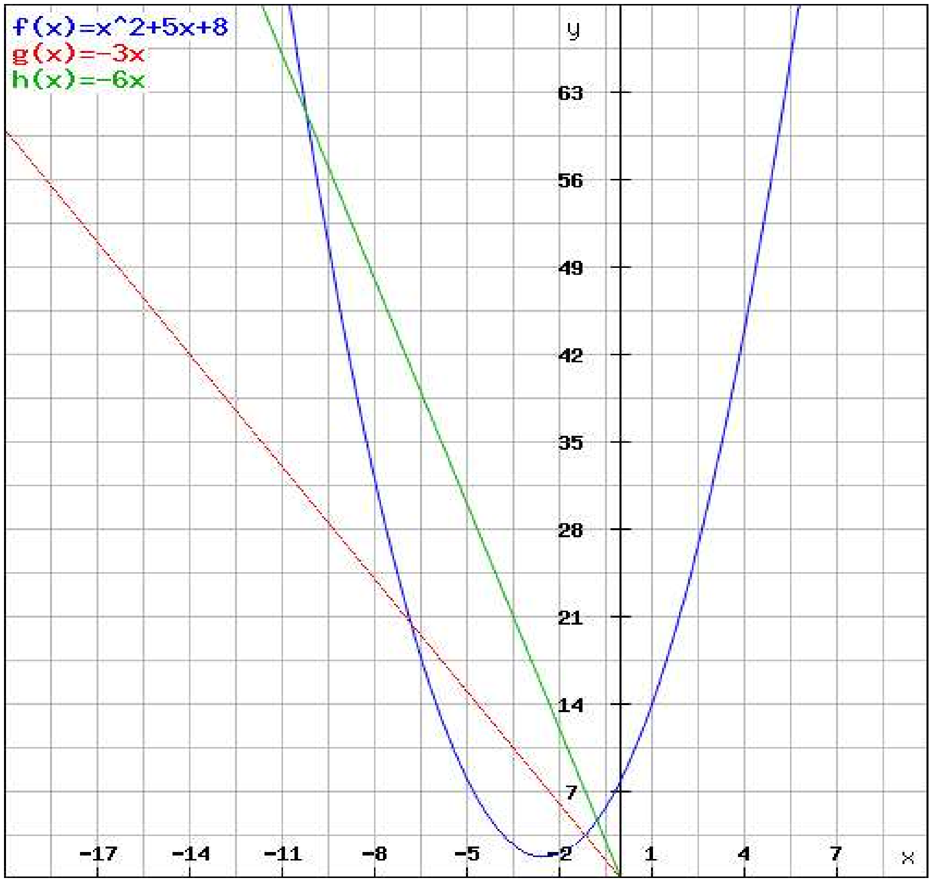
$x_2+x_3=[-(b-b1)/2a]2=-(b-b1)/a$

$x_1+x_4=[-(b-b2)/2a]2=-(b-b2)/a$

$\mathbf{D}=|(-(b-b1)/a)-(-(b-b2)/a)|=\mathbf{|b2-b1|/|a|}$

To test my new conjecture I will work on two other intersecting lines which have negative slope values.

- $y=-3x$, $y=-6x$, $y=\mathbf{x^2+5x+8}$



X1: -0.78

X2: -1.17

SL : -0.39

X3: -6.83

X4 : -10.22

SR : -3.39

l-3.39+0.391 =3.00

$(m_2 - m_1) = 6-3 = 3$

A =1

3/1 = 3.

<u>So ,the conjecture is proved</u>

5)Determine whether a similar conjecture can be made for cubic polinomials

According to the fundamental theorem of algebra:

$ax^3+bx^2+cx+d= a(x-x_1)(x-x_2)(x-x_3)$

$(x-x_1)(x-x_2)(x-x_3)= x^2-xx_1-xx_2+x_1x_2\ (xx_3)$

$=x^3-x^2x_1-x^2x_2-xx_1x_2- x^2x_3+xx_1x_3+xx_2x_3-x_1x_2x_3$

$= a(x^3-(x_1+x_2+x_3)x^2+(x_1x_2+x_2x_3+x_1x_3)x-(x_1x_2x_3))$

$= ax^3-a(x_1+x_2+x_3)x^2+a(x_1x_2+x_2x_3+x_1x_3)x-a(x_1x_2x_3)$

From the proof we can see what each of the coefficient equals:
$a=a$
$b= -a(x_1+x_2+x_3)$
$c= a(x_1x_2+x_2x_3+x_1x_3)$
$d= a(x_1x_2x_3)$

The sum of the roots can be obtained by b:
$b= -a(x_1+x_2+x_3)$
$x_1+x_2+x_3= b/-a = -b/a$

According to the formula
$D=|(x_2+x_3)-(x_1+x_4)|$
The sum of the roots for a cubic polynomial is $-b/a$ so
$D= |(-b/a)-(-b/a)|=0$

6)The conjecture obtained by the cubic polynomials can be applied to higher order polinomials as the roots will cancel out so D=0 will always be true.

Conclusion

- By this investigation, I had a chance to apply the fundamental formulae we have learned in lesson and this, I think played a very important role in making my knowledge about parabolas more permenant. I also learned how to make a conjecture and how to empower it by several different methods. This investigation , I think improved my ability to think critically and independently while providing a practical base for the theoretical knowledge I have learned in the lesson.