Maths SL

Type I Portfolio

TOPIC: Odd & Even Functions.

AIM: To investigate the symmetry of odd and even functions.

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Certain functions are classified as "odd" or "even" functions on the basis of their symmetry in the Cartesian coordinate plane and corresponding algebraic properties. The purpose of this investigation is to explore the characteristics of these functions and transformations of these functions.

Algebraically, even and odd functions are defined as follows:

- An even function is defined as a function $f(x)$ for which $f(-x) = f(x)$.
- An odd function is defined as a function $f(x)$ for which $f(-x) = -f(x)$.

Each elementary function can be classified as odd, even or neither. This is illustrated by some examples below.

even	odd	neither
$f(x) = \lvert x \rvert$ $f(x) = \sqrt{1-x^2}$ $f(x) = x^2$	$f(x) = x^3$ $f(x) = x$ $f(x) = \frac{1}{x}$	$f(x) = \sqrt{x}$ $f(x) = 1$ $f(x) = [\![x]\!]$ $f(x) = 2^x$

To verify whether a given function is even, odd or neither, you have to plug $-x$ for x and simplify. If the result obtained is exactly the same as the formula given, then the function is even ($f(-x) = f(x)$). If the result is the opposite of the formula, then the function is odd ($f(-x) = -f(x)$). Finally, when the result is utterly different from the formula given, the function is neither even nor odd.

The examples will be now followed by calculations.

$f(x) = |x|$
$f(-x) = |x|$ => *As the absolute value*
$f(x) = f(-x)$ *cannot be negative*
even function

$f(x) = \sqrt{(1-x^2)}$
$f(-x) = \sqrt{[1-(-x^2)]}$ => *As raising $-x$ to the*
$f(x) = f(-x)$ *power of 2 gives x^2*
even function

$f(x) = x^2$
$f(-x) = (-x)^2$ => *As above*
$f(-x) = x^2$
even function

$f(x) = x^3$
$f(-x) = (-x)^3$ => *As raising $-x$ to the*
$f(-x) = -x^3$ *power of 3 gives $-x^3$*
odd function

$f(x) = x$
$f(-x) = -x$
odd function

$f(x) = (1/x)$

$f(-x) = [1/(-x)]$ => *As dividing by –x changes the sign*

$f(-x) = -(1/x)$

odd function

$f(x) = \sqrt{x}$

$f(-x) = \sqrt{-x}$ => *As it is impossible to square a negative number*

neither

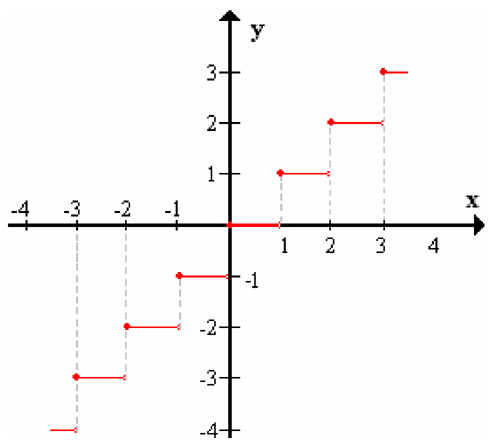
$f(x) = 1$

$f(-x) = 1$ => *As there is no x to substitute*

neither

$f(x) = [\![x]\!]$

neither



$f(x) = 2^x$

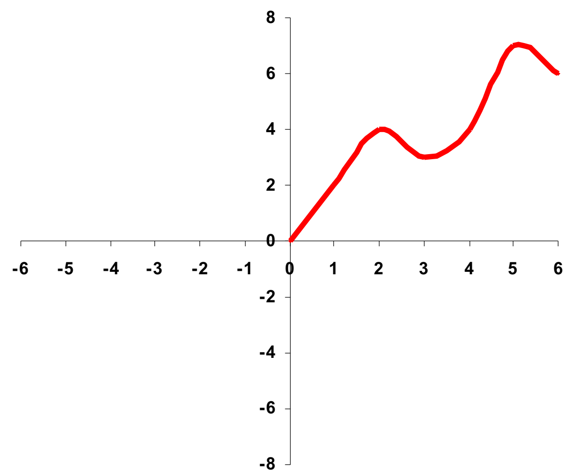
$f(-x) = 2^{-x}$

$f(-x) = (1/2)^x$

neither

As shown above, even functions are symmetrical about the y-axis, while odd functions are symmetrical about the origin.

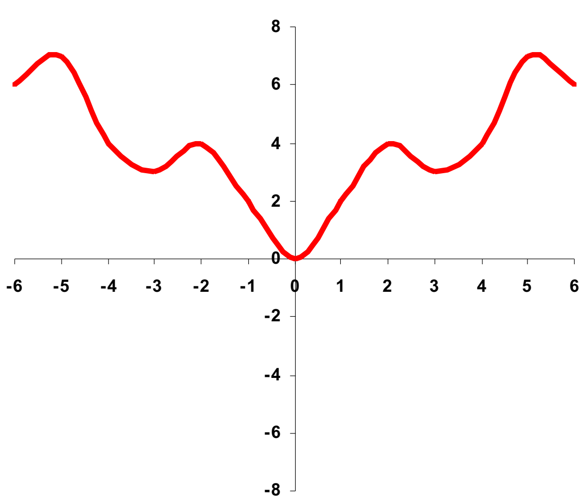
I would like to consider a certain graph of a function $f(x)$ which is shown below.



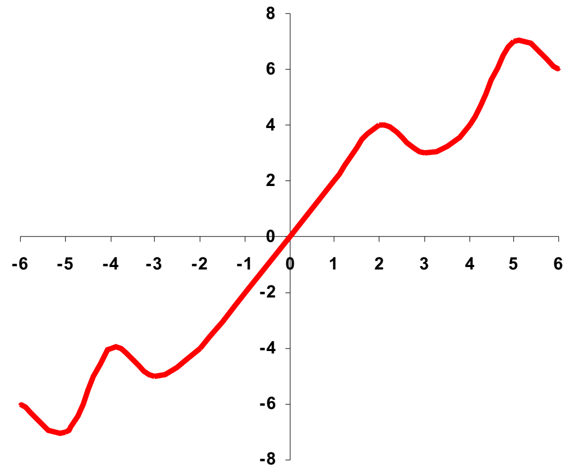
We may assume that the function $f(x)$ is an even function. The complete graph for this function will look as follows:

The range of the completed function will be therefore the same for both graphs:

$$0 \leq f(x) \leq 7$$



We may also assume that the function $f(x)$ is an odd function. The complete graph for this function will look as follows:



The range of the completed function will be therefore different for both graphs. For $f(x)$ as an odd function it will reach:

$$-7 \leq f(x) \leq 7$$

If we consider an unknown function $g(\mathrm{x})$ which is even, certain other functions will be also even for $k \neq 0$, and thus certain conclusions can be drawn from it.

A function:

$$g(\mathrm{x} + k)$$

will not be an even function. The algebraic transformation $g(\mathrm{x} + k)$ causes a translation of $-k$ units left parallel to the x-axis.

A function:

$$g(\mathrm{x}) + k$$

will be an even function. The algebraic transformation $g(\mathrm{x}) + k$ causes a translation of k units parallel to the y-axis.

A function:

$$g(k\mathrm{x})$$

will be an even function. The algebraic transformation $g(k\mathrm{x})$ causes a stretch parallel to the x-axis by a scale factor of $(1/a)$.

A function:

$$kg(\mathrm{x})$$

will be an even function. The algebraic transformation $\mathrm{k}g(\mathrm{x})$ causes a stretch parallel to the y-axis by a scale factor of a.

As stated above, this may be generalised and conclusions about transformations of even functions may be drawn. These are:

- an even function is symmetrical about the y-axis;
- an even function cannot be one-to-one function;
- the function remains even, although the scale factor a of an even function is changed;
- when an even function is translated parallel to the x-axis, after translation the function is no longer even;
- when an even function is translated parallel to the y-axis, after translation the function remains even.

If we consider an unknown function $h(\mathrm{x})$ which is odd, certain other functions will be also odd for $k \neq 0$, and thus certain conclusions can be drawn from it.

A function:

$$h(\mathrm{x} + k)$$

will not be an odd function. The algebraic transformation $h(\mathrm{x} + k)$ causes a translation of $-k$ units left parallel to the x-axis.

A function:

$$h(\mathrm{x}) + k$$

will not be an odd function. The algebraic transformation $h(\mathrm{x}) + k$ causes a translation of k units parallel to the y-axis.

A function:

$$h(k\mathrm{x})$$

will be an odd function. The algebraic transformation $h(k\mathrm{x})$ causes a stretch parallel to the x-axis by a scale factor of $(1/a)$.

A function:

$$kh(\mathrm{x})$$

will be an odd function. The algebraic transformation $\mathrm{k}g(\mathrm{x})$ causes a stretch parallel to the y-axis by a scale factor of a.

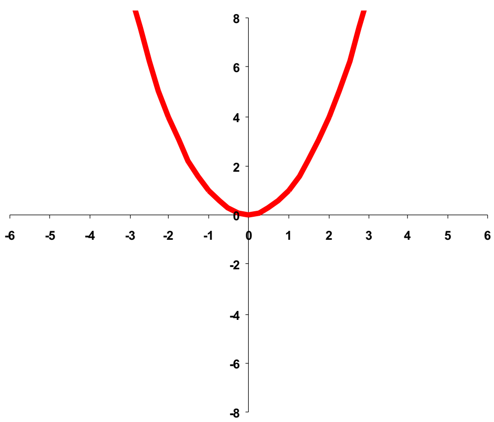
As stated above, this may be generalised and conclusions about transformations of odd functions may be drawn. These are:

- an odd function is symmetrical about the origin;
- the function remains odd, although the scale factor a of an odd function is changed;
- when an odd function is translated parallel to the x-axis, after translation the function is no longer odd;
- when an odd function is translated parallel to the y-axis, after translation the function is no longer odd.

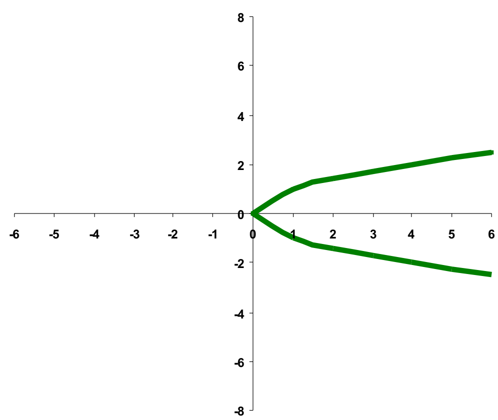
If we have an inverse of any even function, the graph obtained will not be a function. There will be two possible arguments for each value.

The inverse is illustrated by an example below:

$f(x) = x^2$



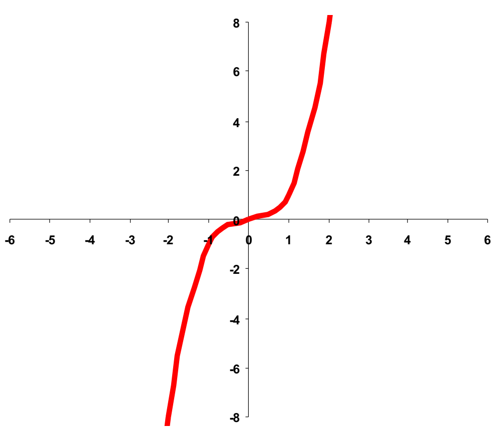
$f^{1}(x) = x^2$



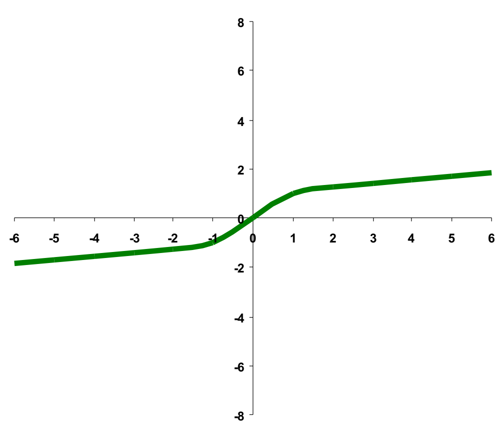
If we have an inverse of any odd function, the graph obtained will also be a function. In some cases the reflection will cover the function.

The inverse is illustrated by an example below:

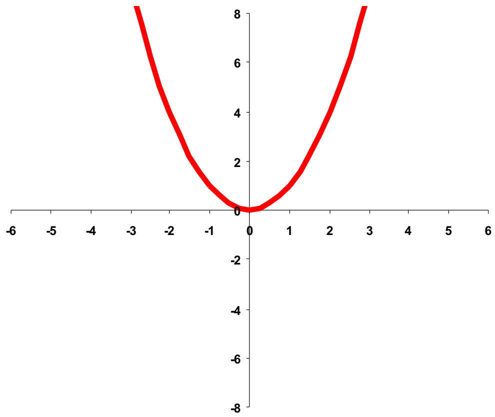
$f(x) = x^3$



$f^{1}(x) = x^3$



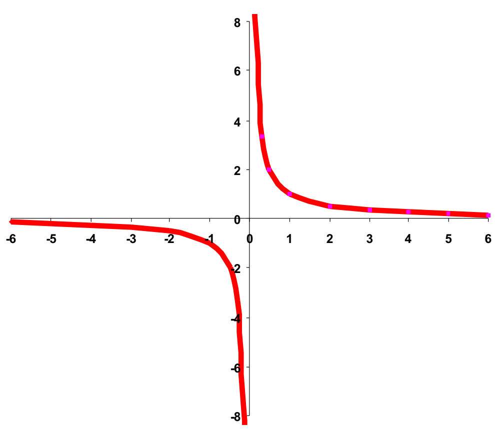
It is possible for an even function to have a domain $D = \{x : x \in \Re\}$, but restricted range. An example of such a function may be already sketched graph of $f(x) = x^2$.



The domain of a function $f(x) = x^2$ is $D = \{x : x \in \Re\}$, while the range is $0 \leq f(x) \leq \infty$.

It is impossible for any negative number to be included. The explanation can be found on page 2.

It is possible for an odd function to have a domain $D = \{x : x \in \Re\}$, but restricted range. An example of such a function may be already sketched graph of $f(x) = \frac{1}{x}$



The domain of a function $f(x) = \frac{1}{x}$ is $D = \{x : x \in \Re\}$, while the range is $0 \leq f(x) \leq \infty$.

It is an asymptote, therefore the curves will never reach zero. In this case zero is the only number that must be excluded from the range.

To sum up, I am to gather all information and conclusions I managed to draw.

even functions	odd functions
1. defined as a function $f(x)$ for which $f(-x) = f(x)$. 2. symmetrical about the y-axis; 3. cannot be one-to-one function; 4. the function remains even, although the scale factor *a* of an even function is changed; 5. when an even function is translated parallel to the *x*-axis, after translation the function is no longer even; 6. when an even function is translated parallel to the *y*-axis, after translation the function remains even; 7. the inverse of any even function will not be a function; 8. It is possible for an even function to have a domain $D = \{x : x \in \Re\}$, but restricted range.	1. $f(x)$ for which $f(-x) = -f(x)$; 2. symmetrical about the origin; 3. the function remains odd, although the scale factor *a* of an odd function is changed; 4. when an odd function is translated parallel to the *x*-axis, after translation the function is no longer odd; 5. when an odd function is translated parallel to the *y*-axis, after translation the function is no longer odd; 6. an inverse of any odd function will also be a function; 7. It is possible for an odd function to have a domain $D = \{x : x \in \Re\}$, but restricted range;

SOURCES:

1. Robert Smedley, Garry Wiseman. *Mathematics Standard Level.* United Kingdom, Oxford Univeristy Press, 2004.
2. http://www.mathematicshelpcentral.com/lecture_notes/precalculus_algebra_folder/odd_and_even_functions.htm
3. http://en.wikipedia.org/wiki/Even_and_odd_functions
4. http://www.purplemath.com/modules/fcnnot3.htm