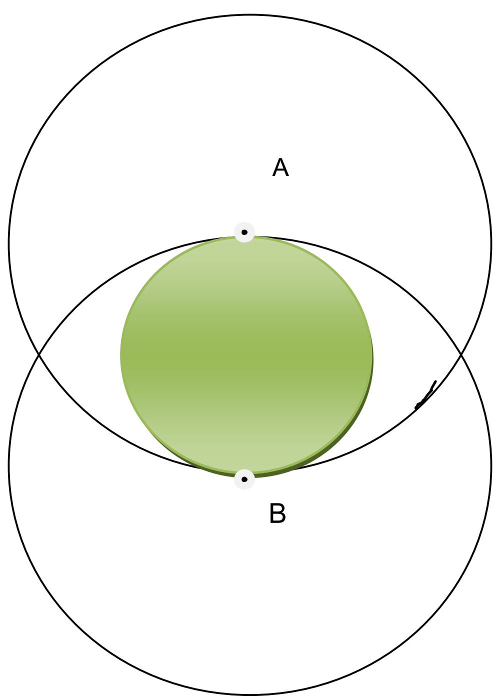
Moss's Egg

The following formulas were used in the solving of the questions of this assessment piece:

Area of a Circle	$\boldsymbol{A = \pi r^2}$
Circumference of a Circle	$C = 2\pi r$
Area of a Sector	$A = \frac{\theta}{360}\pi r^2$
Arc Length	$l = \frac{\theta}{180}\pi r$

<u>*Working Out and Explanation*</u>

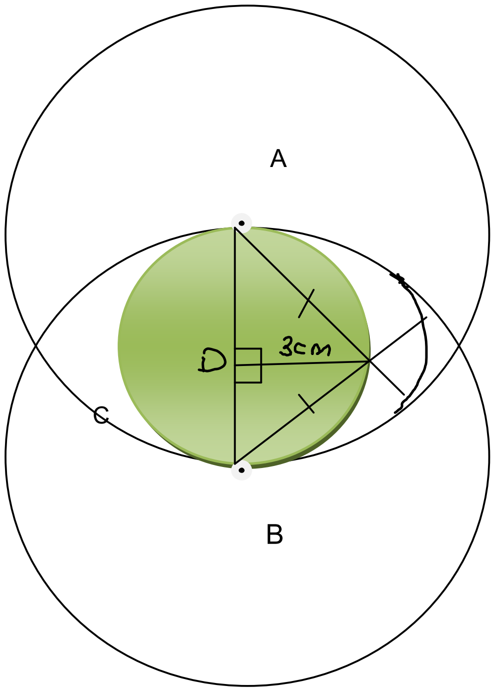
Task -1- Find the area of the shaded region inside the two circles shown below. The two large circles have a radius of 6cm. Their centres are A and B.



From the information given above, we know that the radii of the two larger circles are 6 cm in length. We define the radius of the circle as a straight line extending from the centre of a circle to its circumference. Since we know that points A and B are the centres of the two large circles, we can conclude that this is the length of the two points from A to B is 6 cm also, since point B is along the circumference of the top larger circle, and vice versa. From lengths A to B is the therefore the diameter of the smaller circle between the two larger ones, and thus we can conclude that the radius of the smaller circle is 3 cm. The area of the small circle can therefore be calculated using the formula indicated: $\boldsymbol{A = \pi r^2}$, where A equals the area and r is the radius. Thus: $A = \pi\ (3^2) = 9\ \pi$

$\approx 28.3\ cm^2$

Task -2- The same circles are shown below. Find the area and perimeter of the triangle ABC.



a) In order to determine the area of triangle ABC, we must adopt the formula: $\frac{1}{2}bh$, where b is the base and h is the perpendicular height. From the information we are given, we know that the base of the triangle is 6 cm in length, as we know that this is the diameter of the small circle and that the base extends to its ends (A to B). What we don't know however, is the triangle's perpendicular height, and we must refer to the dynamics of the triangle to determine this. It is noticed that chords AC and BC both meet at the same location, the point which links the radius of the small circle (C) to its base (D). Therefore, triangle ABC is an isosceles triangle. Thus we see that point C extending directly below to the base triangle ABC is the radius of the small circle, which equals 3 cm. Therefore, to calculate the area, we simply substitute the information we have to the formula above:

Area of triangle ABC $= \frac{1}{2}bh = \frac{1}{2} \times 6 \times 3$

$= 9\ \text{cm}^2$

b) The properties of this triangle tell us that the perpendicular line of an isosceles triangle meets the base at angles of 90^0 and thus we have two right angles triangles. Therefore, we can find the lengths of the two sides using Pythagoras' theorem. AC and BC are both considered as hypotenuses of their right angled triangles. We know that the perpendicular height of the triangle is 3 cm and AD and BD are 3 cm as well. Thus,

$AC^2 = 3^3 + 3^2 = 18$, $\quad BC^2 = 3^2 + 3^2 = 18$

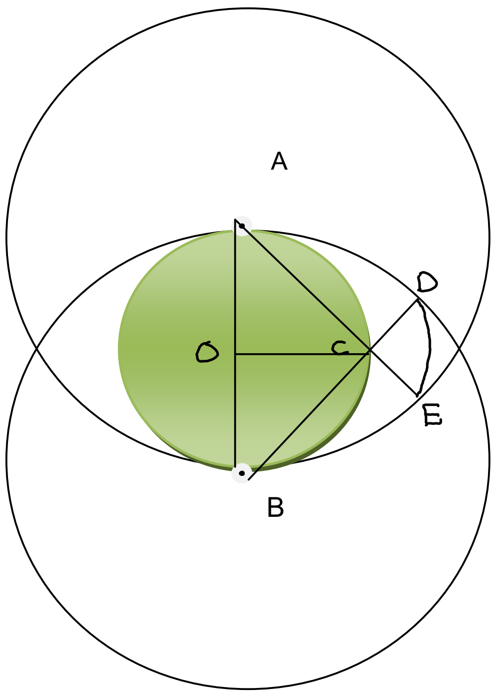
$AC = \pm\sqrt{18} \;\; = 3\sqrt{2}$, $\quad BC = \pm\sqrt{18} \;\; = 3\sqrt{2}$

The perimeter of triangle ABC is the sum of AB + AC + BC

$$P = 3\sqrt{2} + 3\sqrt{2} + 6$$
$$= 6\sqrt{2} + 6$$
$$\approx 14.5\ \text{cm}$$

* ***Note:*** We reject the negative values for the $\pm$ figures above as there cannot be a negative value for a geometric shape thus we only apply the positive value.

Task -3- Find the area enclosed by the sector DCE shown below.



In order to find the area of sector DCE, the formula: $A = \frac{\theta}{360}\pi r^2$ where r is radius. In this case, both CD and CE can be counted as a radius, as like AC and BC, they are the same length. We can see this by subtracting the bigger line to the smaller one. Thus, AE – AC = BD – BC

This can be seen in the following way: CD = 6 - $3\sqrt{2}$ **≈ 1.76 cm**, CE = 6 - $3\sqrt{2}$ **≈ 1.76cm**

Therefore, 6- $3\sqrt{2}$ will be used as the value for *r*.

To solve the area of this sector, all that is now required is angle θ. In order to have this, we must find angles ACO and BCO. Since it has been justified that the triangle is an isosceles triangle, the angles of ACO and BCO are the same. □ = tan $\theta = \frac{3}{3}$ □ = $\theta = \tan^{-1}(\frac{3}{3})$ = 45^0

Thus angle ACE is 90^0 and therefore is right angled. As □ ACB and DCE are vertically opposing angles, □ DCE is also 90^0. We can now substitute the information we have to determine the area of the sector: $A = \frac{\theta}{360}\pi r^2$ where θ is 90^0 and r is 6- $3\sqrt{2}$.

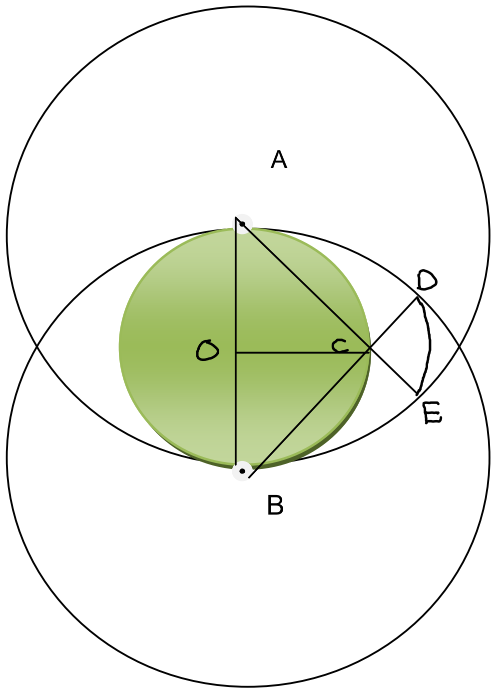
$$A = \frac{90}{360}\pi(6 - 3\sqrt{2})^2 \approx 2.43\text{cm}^2$$

Task -4- Find the area of the sector BAE in the diagram below.

To find sector BAE in the circle, we also must apply the formula: $= \frac{\theta}{360}\pi r^2$. In the diagram displayed below, two internal angles of the two right angled triangles have been found. Therefore, the quickest method to determine □BAE is to subtract 180^0 from the two known angles, as it is known that all internal angles of any triangle is equal to 180^0.

□ □BAE = 180 – 90 – 45

$= 45^{0}$

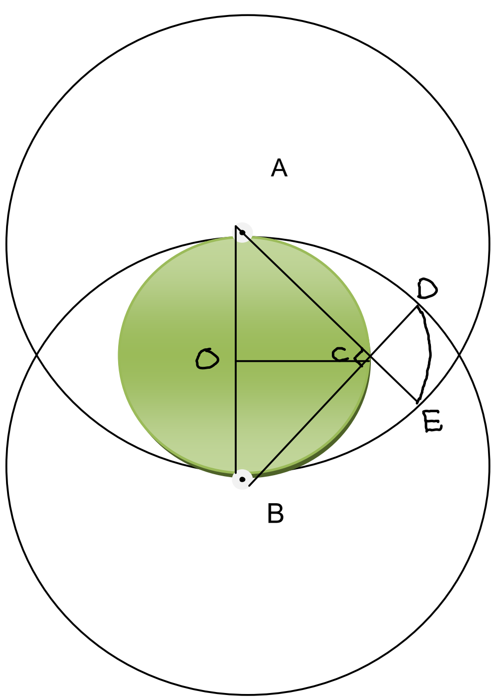


The second value needed in calculating this sector is the radius. □BAE falls into both larger circles of the diagram. The length AB is the radius of these two circles, which we know to be 6cm in length. Therefore, the radius for determining the area of this sector is 6 cm. Thus, we simply substitute into the formula.

$$A = \frac{45}{360}\pi(6)^2$$

$$\approx 14.14 \text{ cm}^2$$

Task -5- Look for the shape of an egg in the enclosed areas of the circles shown below. This is called Moss's Egg. Find the area and perimeter of Moss's Egg.



a) In order to determine the area of Moss's Egg, we must find the areas of the segments it is made up of. The area of the egg is therefore the sum of the semicircle AB, sector DCE, sector ABD, sector BAE as well as the subtraction of the triangle ABC as this area has been included when calculating sectors ABD and EAB. Therefore, the following operations would need to take place when calculating the area of Moss's Egg:

$A_{\text{Moss's Egg}} = \frac{1}{2}$ A small □ AB + A sector BAE+ A sector ABD + A sector DCE – A ΔABC

□ $A_{\text{Moss's Egg}} = \frac{1}{2}9\pi + \frac{45}{360}\pi(6)^2 + \frac{45}{360}\pi(6)^2 + \frac{90}{360}\pi(6 - 3\sqrt{2})^2 - 9$

$\approx 35.8 \text{ cm}^2$

b) Determining the perimeter of Moss's Egg involves similar processes as that in finding its area, as this too, involves breaking up Moss's Egg into its smaller segments. The perimeter of Moss's Egg would therefore be the sum of half the circumference of circle AB as well as the arcs of ABD, BAE and DCE. To find the arc lengths of the sectors above, we employ the formula: $l = \frac{\theta}{180}\pi r$.

Thus, Arc AD $= \frac{45}{180}\pi 6$

Arc BE $= \frac{45}{180}\pi 6$

Arc DE $= \frac{90}{180}\pi(6 - 3\sqrt{2}) = \frac{(6-3\sqrt{2})}{2}\pi$

To find the circumference of the semi circle, we use the formula $C = 2\pi r$ and then divide by two in order to obtain the circumference for only half the circle.

□ $C_{\text{semicircle}} = \frac{2\pi 3}{2} = 3\pi$

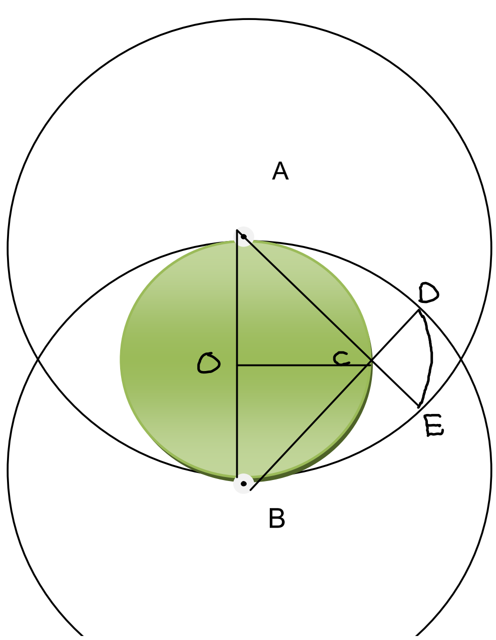
Thus, the perimeter of the egg can be calculated by the following processes:

$C_{\text{semicircle}}$ + Arc AD + Arc BE + Arc DE

□ $P_{\text{Moss's Egg}} = 3\pi + \frac{45}{180}\pi 6 + \frac{45}{180}\pi 6 + \frac{(6-3\sqrt{2})}{2}\pi$

≈ 21.6 cm

Task -6- If the radii of the two large circles shown below are r, find a formula for the area and perimeter of the egg in terms of r.



a) The information that we are provided states that AB equals r. Therefore, it can be said that the radius of the smaller circle equals $\frac{r}{2}$. Through this, we deduce that AC will equal $\frac{\sqrt{2}r}{2}$ by using Pythagoras's Theorem. The radius of sector DCE would then therefore equal to:

Sector DCE $= r - \frac{\sqrt{2}r}{2} = \frac{2-\sqrt{2}}{2}$r

Thus, in order to find a formula for determining the area of Moss's Egg, the following equation must be used, with substituted figures in terms of r.

$A_{\text{Moss's Egg}} = \frac{1}{2}$ A small □ AB + A sector BAE+ A sector ABD + A sector DCE – ΔABC

□ $A_{\text{Moss's Egg}} = \frac{1}{2}\pi(\frac{r}{2})^2 + \frac{45}{360}\pi r^2 + \frac{45}{360}\pi r^2 + \frac{90}{360}\pi(\frac{2-\sqrt{2}}{2}r)^2 - \frac{1}{2}r(\frac{r}{2})$

$= \frac{1}{8}\pi r^2 + \frac{1}{4}\pi r^2 + \frac{3-2\sqrt{2}}{8}\pi r^2 - \frac{1}{4}r^2$

$= r^2(\frac{1}{8}\pi + \frac{1}{4}\pi + \frac{3-2\sqrt{2}}{8}\pi - \frac{1}{4})$

$= r^2(\frac{3-\sqrt{2}}{4}\pi - \frac{1}{4})$

$\approx 0.995r^2$

To test this formula, the radius 6cm can be substituted into *r*, which allows the answer calculated by this formula to be compared with that of question 5a.

$0.995(6)^2 =$ **35.82 cm**

Therefore, the suggested formula has been supported.

b) Much like section A of this question, finding the perimeter of the egg consists of substituting the calculated figures in the previous questions into their relationships in terms of r. Therefore, a formula for the perimeter of Moss's Egg can be calculated using the method displayed below.

$C_{\text{semicircle}}$ + Arc AD + Arc BE + Arc DE

$= \frac{1}{2} \times 2\pi(\frac{r}{2}) + \frac{45}{180}\pi r + \frac{45}{180}\pi r + \frac{90}{180}\pi(\frac{2-\sqrt{2}}{2}r)$

$= \frac{1}{2}\pi r + \frac{1}{2}\pi r + \frac{1}{2} \text{ x } \frac{2-\sqrt{2}}{4}\pi r$

$= \pi r \left(1 + \frac{2-\sqrt{2}}{4}\right)$

$= \pi r\left(\frac{6-\sqrt{2}}{4}\right)$

$\approx 3.6r$

To test this formula, substitute r to 6 and compare with answer calculated from question 5b.

3.6 x 6 = **21.6 cm**

Therefore the formula above has been supported.

Bibliography

Haese, S et al. 2006, *Mathematics For The International Student.* Haese & Harris Publications, Australia