The aim of this essay is to model the course of a viral illness and its treatment.
When viral particles of a certain virus enter the human body, they replicate rapidly. In about four hours the number of viral particles has doubled. The immune system does not respond until there are about 1 million viral particles in the body.

The first response of the immune system is fever and the rise in temperature lowers the rate at which the viral particles replicate to 160% every four hours. The immune system tough can only eliminate about 50 000 viral particles per hour. If the number of viral particles, however, reaches 10^{12}, the person dies.

For a person infected with 10 000 viral particles we could determine how long it will take for the immune system response to begin. As we have an ordered set of numbers and a common ratio we could consider this to be a geometric sequence. The form of a geometric sequence is $a, ar, ar^2, ar^3, ...$ So $U_n = ar^{n-1}$, where a is the initial term of the sequence and r is the common ratio.

In this case $U_n = a \times r^{\frac{t_1}{4}}$ where U_n is the number of viral particles approximately 10^6, a is the initial phase of the illness respectively 10 000 viral particles, r is the growth of rate of 200% every four hours and, t_1 is the time in hours where fever begins.
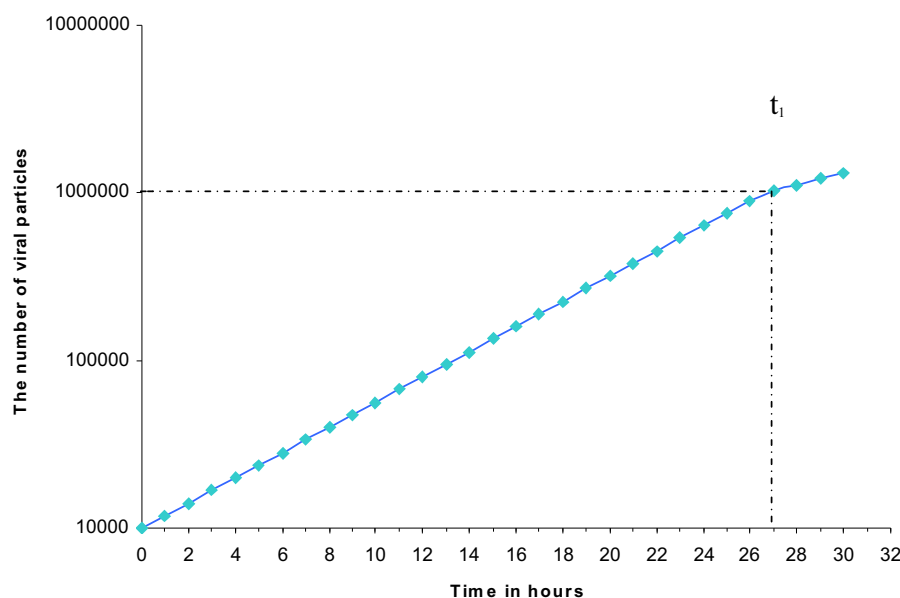
To determine how long it will take for the body's immune system response to begin we can substitute with the values into the formula:

$$10^6 = 10^4 \times 2^{\frac{t_1}{4}} \quad 2^{\frac{t_1}{4}} = 10^2 \quad \Rightarrow \quad \frac{t_1}{4} \times \ln 2 = 2 \ln 10 \Rightarrow \frac{t_1}{4} = \frac{2 \ln 10}{\ln 2} \Rightarrow t_1 = \frac{8 \ln 10}{\ln 2}$$

$$\ln \left(2^{\frac{t_1}{4}} \right) = \ln 10^2 \qquad t_1 = 26.5...$$
$$t_1 \approx 26.5$$

Hence it will take around 27 hours for the immune system response to begin.

In order to determine how long it will be before the patient dies if the infection is left untreated, we have to consider that after the fever begins at t_1, r will be 160% every four hours and the immune system will eliminate 50 000 viral particles per hour. Therefore the model should be developed into: $U_n = 10^6 \times r^{\frac{t_0 - t_1}{4}} - 5 \times 10^4 \left| t_0 - t_1 \right|$ where t_0 is the approximate time where the patient dies without any treatment.

$$10^{12} = 10^6 \times 1.6^{\frac{t_0 - t_1}{4}} - 5 \times 10^4 (t_0 - t_1)$$
$$t_0 = 144.1...$$
$$t_0 \approx 144$$

Hence if the patient receives any treatment it will take about 144 hour before he dies.
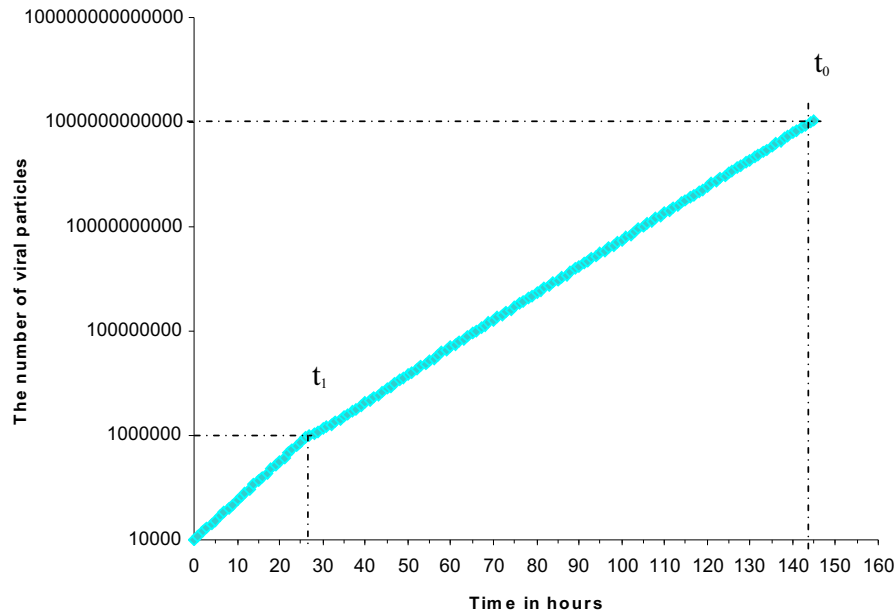

An antiviral medication can be administrated as soon as a person seeks medical attention. The medication does not affect the growth of rate of the viruses but together with the immune system response can eliminate 1.2 million viral particles per hour. But if the patient is to make full recovery effective medication must be administrated before the number of viral particles reaches 9 to 10 million

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Let's take a look at the course of the viral illness between 1 and 12 millions without any medication
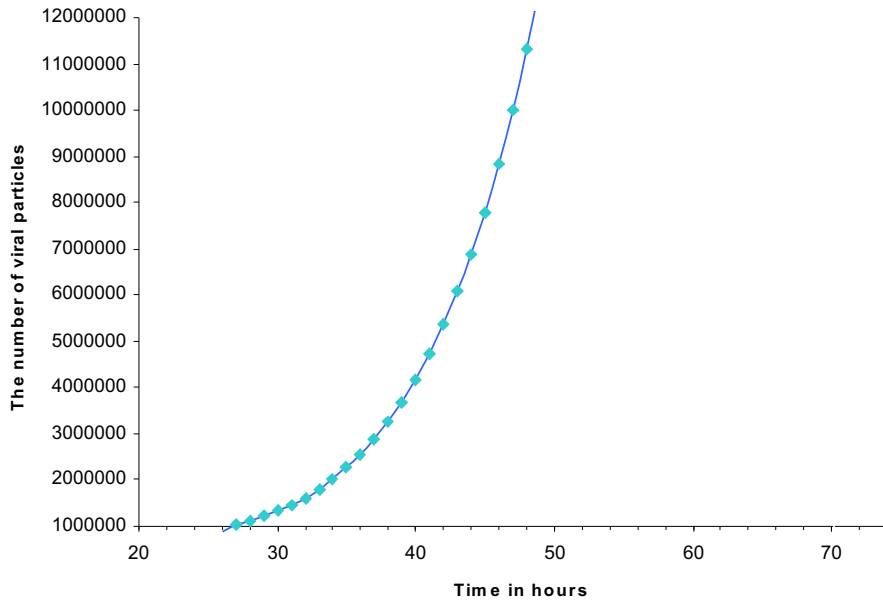

And then let's assume that the medication is administrated when the number of viral particles reaches about 9 millions
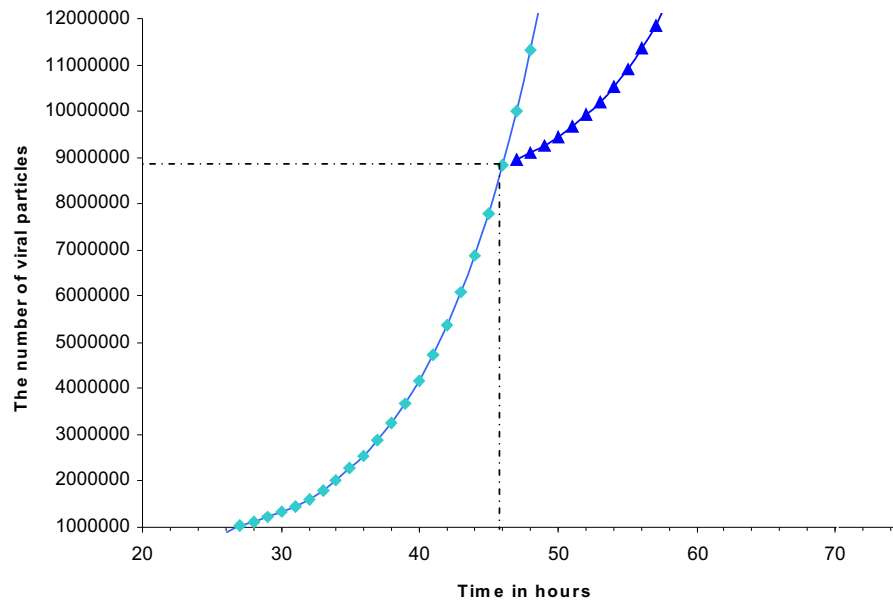

The growth of particles is obviously greater than number of particles that the medication and immune system response can eliminate.

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If we assume that the medication is administrated somewhere around 8 millions we could see the difference. The patient will be able to make full recovery.
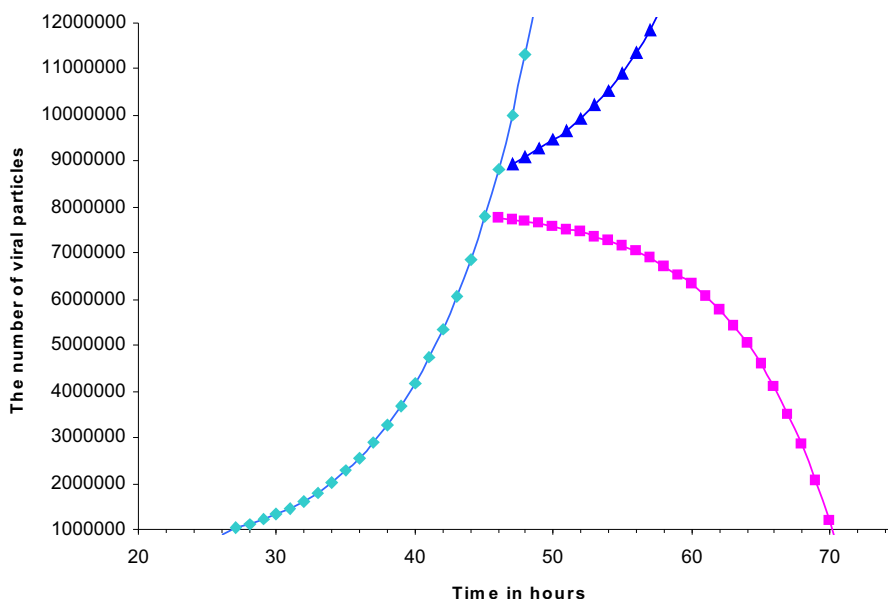
Another way to explain it will be that the derivative of the curve from 9 to 10 millions is greater than the curve and therefore the medication not being enough.

$\Delta y > 1.2 \times 10^6$

$\Delta y < 1.2 \times 10^6$

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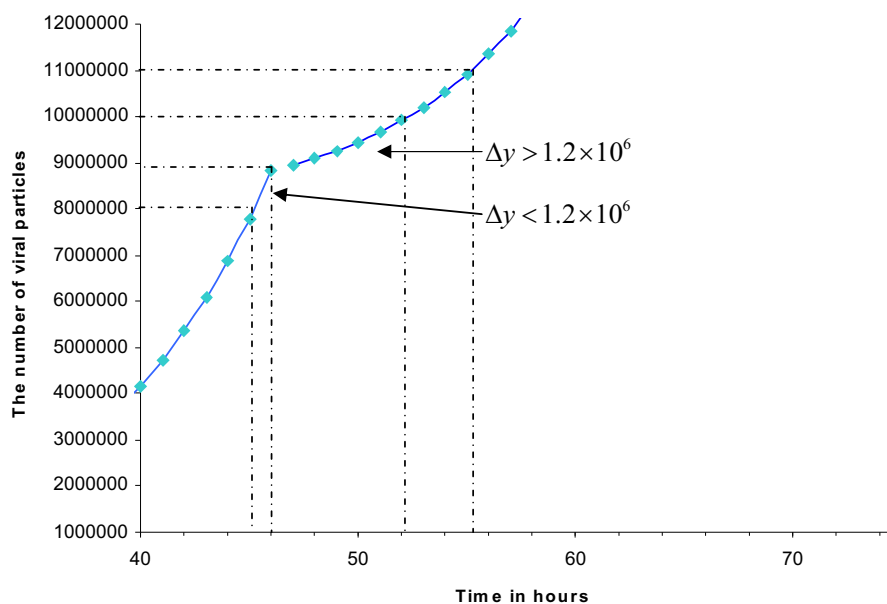
Doctors have calculated that the patient needs at least 90 micrograms of medication to begin and maintain the rate of 1.2 million viral particles. Over a four-hour time period of continuous intravenous dosing we will have around $22.5\ \mu g$. Then 2.5% of this is eliminated by the kidney. Therefore we will have around $22\ \mu g$ every hour. In order to maintain the rate of elimination we should add to $22\ \mu g$ around $3\ \mu g$ each hour. Hence the amount entering the body should be around $25\ \mu g$ per hour.

$$\frac{90}{4} = 22.5$$

$$22.5 - 0.025 \times 22.5 \approx 22 \mu g$$

$$M_0 = 25$$

$$M_1 = M_0 - 0.025 \times M_0 = 24.37...$$

$$M_2 = M_1 + 25 - 0.025 \left| M_1 + 25 \right| = 48.14...$$

$$M_3 = M_2 + 25 - 0.025 \left| M_2 + 25 \right| = 71.31...$$

$$M_4 = M_3 + 25 - 0.025 \left| M_3 + 25 \right| = 93.90...$$

time (h)	medication (µg)
0	25
1	24,375
2	48,141
3	71,312
4	93,904

$M_n = M_{n-1} + a - 0.025 \left| M_{n-1} + a \right|$ where a is the amount of medication entering the body every hour and M_n is the amount of medication at n hours.

As the kidneys will eliminate about 2.5% of this medication per hours that means that every four hours they will eliminate 10% of this medication.

Verification using the previous model:
$$M_4 = 93.9...$$

$$M_5 = M_4 + 2.5 - 0.025 \times \left| M_4 + 2.5 \right| = 93.99...$$

$$M_6 = M_5 + 25 - 0.025 \left| M_5 + 25 \right| = 94.08...$$

$$M_7 = M_6 + 25 - 0.025 \left| M_6 + 25 \right| = 94.16...$$

$$M_8 = M_7 + 25 - 0.025 \left| M_7 + 25 \right| = 94.25...$$

$M_4 = 93.9... \Rightarrow 93.9... \times 0.025 \approx 2.35 \Rightarrow a$ should be greater than 2.35 in order for the dosage D to slightly increase rather than decrease. $\Rightarrow D = 2.5 \times 4 = 10$

Hence for the patient to maintain at least 90 micrograms of the medication in his system, a dosage of about $10 \mu g$ per four hours should be administrated.

We could use a spreadsheet in order to verify it. We could have different values of a, where a is the amount of medication per hour.

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a =1.6		a =2.3		a =2.5	
time (h)	medication (µg)	time (h)	medication (µg)	time (h)	medication (µg)
4	93,904	4	93,904	4	93,904
5	93,116	5	93,799	5	93,994
6	92,348	6	93,696	6	94,082
7	91,600	7	93,597	7	94,167
8	90,870	8	93,499	8	94,250
9	90,158	9	93,404	9	94,332
10	89,464	10	93,312	10	94,411
11	88,787	11	93,221	11	94,488
12	88,128	12	93,133	12	94,563
13	87,485	13	93,047	13	94,637
14	86,857	14	92,964	14	94,708
15	86,246	15	92,882	15	94,778
16	85,650	16	92,803	16	94,846
17	85,069	17	92,725	17	94,913
18	84,502	18	92,649	18	94,977
19	83,949	19	92,576	19	95,040
20	83,411	20	92,504	20	95,102
21	82,885	21	92,434	21	95,162
22	82,373	22	92,365	22	95,220
23	81,874	23	92,299	23	95,277
24	81,387	24	92,234	24	95,333
25	80,912	25	92,170	25	95,387
26	80,450	26	92,109	26	95,440
27	79,998	27	92,048	27	95,491
28	79,558	28	91,990	28	95,541

From the spreadsheet we could conclude that it is more convenient to have an amount of medication per hour greater than the amount eliminated by the kidneys.

The last possible time from the onset of infection to start the regimen of medication is approximately 46 hours. It will take about 25 hours to clear the viral particles from the patient's system.

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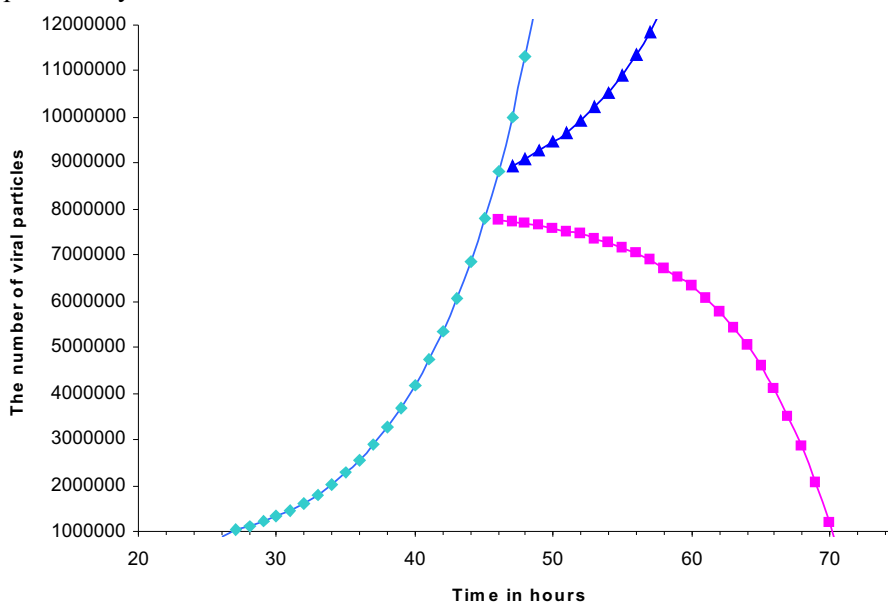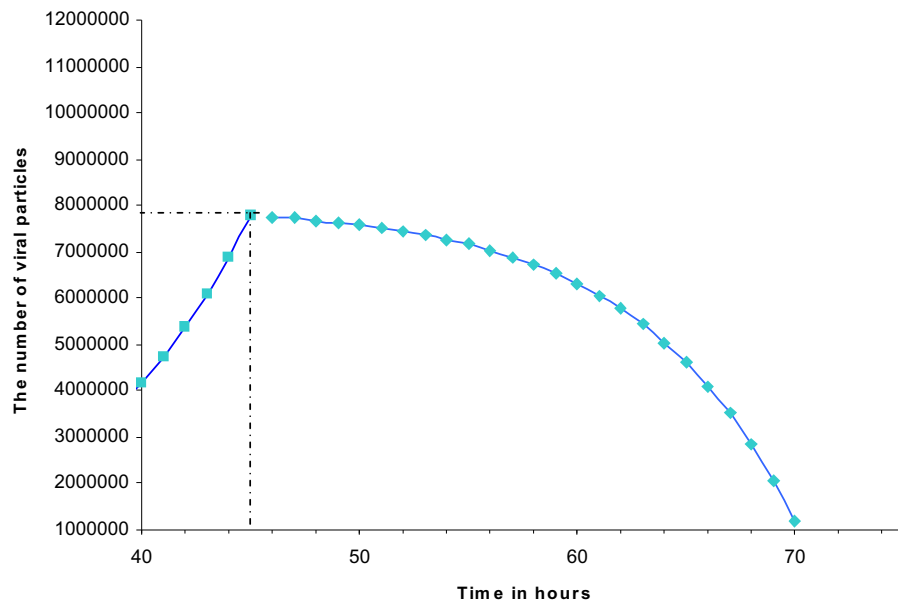
This graph shows the entire treatment regimen from the time the treatment begins until the viral particles are eliminated from the last possible time to start the regimen of medication.