

Modelling the course of a viral illness and its treatment

Description (from assignment)

When viral particles of a certain virus enter the human body, they replicate rapidly. In about four hours, the number of viral particles has doubled. The immune system does not respond until there are about 1 billion viral particles in the body.

The first response of the immune system is fever. The rise in temperature lowers the rate at which the viral particles replicate to 160% every four hours, but the immune system can only eliminate these particular viral particles at the rate of about 50 000 viral particles per hour. Often people do not seek medical attention immediately as they think they have a common cold. If the number of viral particles, however, reaches 10^{12} , the person dies.

Modelling infection

1. A patient is infected with 10 000 viral particles. Every four hours, the viral particles double = 200% (see figure 1-1). The viral particles replicate every four hours. So, for every one hour the viral particles will replicate with a value of $2^{\frac{1}{4}} = 1,189$ (see figure 1-2). Therefore, the total amount of viral particles in the body for every one hour, would be equal to the start amount multiplied with 2 to the power of $\frac{1}{4}x$. So, this

phase of illness can be calculated using the model $V_{total}(x) = 10000 \times 2^{\frac{1}{4}x}$

By using the regression on the calculator, putting the values for every four hours the viral particles is replicating, I find the best fit line to be an exponential regression.

ExpReg

- a=10000
- b=0,173
- r=1
- $r^2=1$

$e^{0,173} = 1,189$, so the function of the graph is $V_{total}(x) = 10000 \times 1,189^x$ (see graph 1-1)

- The best fit model for the initial phase of the illness is $V_{total}(x) = 10000 \times 1,189^x$, because this is a growth formula $V_n = V(1 + \frac{P}{100})^x$. So to determine the time for the body's immune response, could be solved as an inequality with a total viral particles amount (V_{total}) of 10^6

$$\begin{aligned} 1000000 &> 10000 \times 1,189^x \\ \frac{1000000}{10000} &> 1,189^x \\ \frac{\log 100}{\log 1,189} &> x \frac{\log 1,189}{\log 1,189} \\ x &> 26,6 \end{aligned}$$

So, the body's immune will response after 26,6 hours (figure 1-3)

- The fever starts when the immune system is responding, which means that the human body will have fever after 26,6 hours. The immune system is responding when the viral particles exceed 1 million and it will eliminate 50000 viral particles per hour. The replication will then decrease from 200% to 160%. (see figure 2-1). This means for every one hour, the replication will increase with a value of $160\% / \text{fourth hour} = 1,6^{\frac{1}{4}}$. So the total amount after the immune system is responding with fever is

$$V_{total} = (10^6 \times 1,6^{\frac{1}{4}x}) - 50000 \text{ (see figure 2-2)}$$

By using the exponential regression, putting in the values (see figure 2-1). I find the function $y = 987792,4 \times e^{0,1137x}$ and by solving $e^{0,1137x} = 1,125$. I will get a function that looks like this $V_n = 987792,4 \times 1,125^x$

So the growth factor after 26,6 hours when the human body has fever is 112,5 % found through regression.. However, the immune system will eliminate 50000 viral particles per hour so I need to subtract 50000 from the total amount. Which means,

$$V_n = (10000 \times 1,125^x) - 50000. \text{ Since a person will die if the number of viral particles}$$

reaches 1 million (10^{12}), we can estimate the time using the model.

$$V_n = (10^6 \times 1,125^x) - 50000$$

$$V_n = 10^{12}, \text{ so}$$

$$10^{12} > (10^6 \times 1,125^x) - 50000$$

$$\frac{10^{12} + 50000}{10^6} > 1,125^x$$

$$\frac{\log 10^9}{\log 1,125} > x \frac{\log 1,125}{\log 1,125}$$

$$x > 117,3$$

117,3 hours is the remaining time after the immune system have responded. So to find the total amount of time the patient have before he dies can be found by adding the time it takes from when the patient is infected to the immune system response adding with the times before it exceed 10^{12} viral particles. Time before death = $117,3 + 26,6 = 143,9$. (see figure 2-3)

Modelling recovery

3. The medication and immune response can together eliminate 1,2 million viral particles. This means that if a person seeks medical attention have to do it before the replication of viral particles exceed 1,2 million. So from 9 to 10 million, around 50 and 51 hours (see figure 3-1). The replication from 50 to 51 hours is around 11 19903 viral particles. However from 51 to 52, the particles are replicating with around 1234795 viral particles. This exceed the number both the medication and immune response can eliminate. This means after the viral particles reaches 10 million, the replications would be greater than the elimination from medication and immune response. Therefore an antiviral medication have to be done before 9 to 10 million viral particles.
4. The medication have to begin with at least 90 micrograms and the kidneys are eliminating 2,5 % of this medication per hour. So to find the initial amount of medication for the first four hours can be find using the formula $y = 90 \times 0,975^x$ where (x) is the number of hours. So by graphing the function and using x-calc=-4 (see graph 4-1) we find the initial dose of medication to be 99,6 micrograms
5. The dosage, D, to maintain at least 90 micrograms of the medication is $(90 + x)0,975^4 = 90$, where x is every hours. So the amount of 9,6 micrograms

$$(90 + x) = \frac{90}{0,975^4}$$
 dosage have to be administered every hours to maintain the medication in the system

$$(90 + x) = 99,6$$

$$x = 99,6 - 90$$

$$x = 9,6$$

Analyzing your models

My models include both precision and accuracy in the result, since I have used values with decimals and exact numbers. This makes me see the small differences in the precision of the values

Applying your model

7. If the patient is a young child instead of an adult, then the models have to be modified carefully. The immune system of a child is weaker than an adult, so this means the immune system would respond later than an adult. Therefore, a child would die earlier of the amount of viral particles than an adult if left untreated, because the body of a child is smaller and the immune system eliminates less particles. The amounts of dosage and medications also have to be reduced, since a body of a child can not handle as much compared to an adult body. So the time to start the regimen of medication also have to start earlier, since a child is smaller than an adult and the body can not handle the same amount of viral particles. So the models of immune response have to be increased and the medication has to be decreased.

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Appendix

Graph 4-1

