International Baccalaureate Mathematics Portfolio - Standard Level Type I

Matrix Powers

Student Names: Nam Vu Nguyen
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School Name: Father Lacombe Senior High School
Teacher: Mrs. Stephanie Gabel

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Nam Vu Nguyen: ______________________________

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Mathematics is the science where the concepts of quantity, structure, space and change is studied. A science where patterns are discovered in numbers, in space, in science, computers, imaginary abstractions, and everything else contained in the universe. It is the type of science that draws conclusions and connections to the world's analytical problems that exists all around us. Mathematics is used to describe the numerous natural phenomena that occur around us every day. Today math is being applied and developed into numerous evolving educational fields, inspiring humans to discover and make use of their mathematical knowledge, which will in turn lead to entirely new discoveries. An example of the usage of mathematics in society today would be the use and manipulation of matrices in the field of computer graphics.

The Matrix theory is a branch of mathematics that focuses on the study of matrices. Originally it is a sub-branch of linear algebra, yet it has grown to cover subjects related to graph theory, algebra, combinatory, and statistics as well. In mathematics, a matrix is a rectangular table of elements, or entries, which may be numbers or, any abstract quantities that can be added and multiplied. Matrices are used to describe linear equations, keep track of the coefficients of linear transformations and to record data that depend on multiple parameters. Matrices can be added, multiplied, and decomposed in various ways, which also makes them a key concept in the field of linear algebra. The purpose of this mathematical paper is to explore and investigate the powers of matrix.

A matrix may be squared or even raised to an integer power. However, for a matrix to be able to be raised to the power the matrix must be a square matrix, meaning that it must be orthogonal in shape and contains the same width and height all around.

The following list describes the effects on a matrix when a matrix is raised to an integer power:

M^{-1}	Creates the inverse of the matrix
M^0	Generates what is called an "Identity Matrix"
M^1	Leaves the matrix unchanged
M^2	Squares the matrix
M^3	Cubes the matrix

Raising a matrix to a power greater than one involves multiplying the matrix by itself a specific number of times.

For example, if $M = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$, and $M^2 = M \bullet M$

$$\therefore M^2 = M \bullet M = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \bullet \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

In order to solve the matrix power above, we multiply the matrix by "n" number of times.

Thus, if $M = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$, and $M^2 = M \bullet M$, then to solve we times the matrix "M" by two times.

To multiply matrices, we take the rows of the left hand matrix and pair it with the column of the right hand matrix. The first step is to take the first row of the left hand matrix, in this case being [2 0], write it as a column and then pair it up with the first column of the right hand matrix, (the second matrix). You then multiply the pairs together and the sum of the products will give a single number which is the first digit of the new matrix.
Therefore the matrix equation being solved will look like:

$$\begin{bmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \bullet \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \\ (2 \bullet 2) + (2 \bullet 0) = 4 \\ (0 \bullet 2) + (0 \bullet 0) = 0 \\ (2 \bullet 2) + (2 \bullet 0) = 4 \\ (0 \bullet 2) + (0 \bullet 0) = 0 \end{bmatrix} = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$$

Another easier way of solving matrices powers is raise the power of the digits inside the matrix
Therefore the matrix solved using the new method is seen as:

$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}^2 = \begin{pmatrix} 2^2 & 0^2 \\ 0^2 & 2^2 \end{pmatrix} = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$$

In order to check if the new method of finding the power of a matrix, we apply the same methods to three more powers of the same matrices, but different values of "n".
The first being:

$$M^n = M^3, \text{where } M = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

Method 1: $$\begin{bmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \bullet \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \\ (2 \bullet 2) + (2 \bullet 0) = 4 \\ (0 \bullet 2) + (0 \bullet 0) = 0 \\ (2 \bullet 2) + (2 \bullet 0) = 4 \\ (0 \bullet 2) + (0 \bullet 0) = 0 \end{bmatrix} = \begin{bmatrix} \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} \bullet \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \\ (4 \bullet 2) + (4 \bullet 0) = 8 \\ (0 \bullet 2) + (0 \bullet 0) = 0 \\ (4 \bullet 2) + (4 \bullet 0) = 8 \\ (0 \bullet 2) + (0 \bullet 0) = 0 \end{bmatrix} = \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix}$$

Method 2: $$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}^3 = \begin{pmatrix} 2^3 & 0^3 \\ 0^3 & 2^3 \end{pmatrix} = \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix}$$

The second:

$$M^n = M^4, \text{where } M = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

Method 1:

$$\begin{bmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \bullet \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \\ (2 \bullet 2) + (2 \bullet 0) = 4 \\ (0 \bullet 2) + (0 \bullet 0) = 0 \\ (2 \bullet 2) + (2 \bullet 0) = 4 \\ (0 \bullet 2) + (0 \bullet 0) = 0 \end{bmatrix} = \begin{bmatrix} \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} \bullet \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \\ (4 \bullet 2) + (4 \bullet 0) = 8 \\ (0 \bullet 2) + (0 \bullet 0) = 0 \\ (4 \bullet 2) + (4 \bullet 0) = 8 \\ (0 \bullet 2) + (0 \bullet 0) = 0 \end{bmatrix} = \begin{bmatrix} \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix} \bullet \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \\ (8 \bullet 2) + (8 \bullet 0) = 8 \\ (0 \bullet 2) + (0 \bullet 0) = 0 \\ (8 \bullet 2) + (8 \bullet 0) = 8 \\ (0 \bullet 2) + (0 \bullet 0) = 0 \end{bmatrix} = \begin{pmatrix} 16 & 0 \\ 0 & 16 \end{pmatrix}$$

Method 2:

$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}^4 = \begin{pmatrix} 2^4 & 0^4 \\ 0^4 & 2^4 \end{pmatrix} = \begin{pmatrix} 16 & 0 \\ 0 & 16 \end{pmatrix}$$

And finally, the Third equation being:

$$M^n = M^5, \text{where } M = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

Method 1:

$$\begin{bmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \bullet \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \\ (2 \bullet 2) + (2 \bullet 0) = 4 \\ (0 \bullet 2) + (0 \bullet 0) = 0 \\ (2 \bullet 2) + (2 \bullet 0) = 4 \\ (0 \bullet 2) + (0 \bullet 0) = 0 \end{bmatrix} = \begin{bmatrix} \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} \bullet \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \\ (4 \bullet 2) + (4 \bullet 0) = 8 \\ (0 \bullet 2) + (0 \bullet 0) = 0 \\ (4 \bullet 2) + (4 \bullet 0) = 8 \\ (0 \bullet 2) + (0 \bullet 0) = 0 \end{bmatrix} = \begin{bmatrix} \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix} \bullet \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \\ (8 \bullet 2) + (8 \bullet 0) = 8 \\ (0 \bullet 2) + (0 \bullet 0) = 0 \\ (8 \bullet 2) + (8 \bullet 0) = 8 \\ (0 \bullet 2) + (0 \bullet 0) = 0 \end{bmatrix} = \begin{bmatrix} \begin{pmatrix} 16 & 0 \\ 0 & 16 \end{pmatrix} \bullet \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \\ (16 \bullet 2) + (16 \bullet 0) = 32 \\ (0 \bullet 2) + (0 \bullet 0) = 0 \\ (16 \bullet 2) + (16 \bullet 0) = 32 \\ (0 \bullet 2) + (0 \bullet 0) = 0 \end{bmatrix} = \begin{pmatrix} 32 & 0 \\ 0 & 32 \end{pmatrix}$$

Method 2:

$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}^5 = \begin{pmatrix} 2^5 & 0^5 \\ 0^5 & 2^5 \end{pmatrix} = \begin{pmatrix} 32 & 0 \\ 0 & 32 \end{pmatrix}$$

Therefore based on the results above, there are two ways of solving the powers of a matrix. The first is multiplying the matrix by "n" times. The second method is simply bringing the individual digits in the matrix by the power of "n". Therefore for n = 10, 20, and 50 we can simply use the second method to determine the power of the matrix at the prescribed "n"

$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}^{10} = \begin{pmatrix} 2^{10} & 0^{10} \\ 0^{10} & 2^{10} \end{pmatrix} = \begin{pmatrix} 1024 & 0 \\ 0 & 1024 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}^{20} = \begin{pmatrix} 2^{20} & 0^{20} \\ 0^{20} & 2^{20} \end{pmatrix} = \begin{pmatrix} 1048576 & 0 \\ 0 & 1048576 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}^{50} = \begin{pmatrix} 2^{50} & 0^{50} \\ 0^{50} & 2^{50} \end{pmatrix} = \begin{pmatrix} 1.12589907_{E}{}^{15} & 0 \\ 0 & 1.12589907_{E}{}^{15} \end{pmatrix}$$

Based on the following data above we concluded that there are two sets of methods to solve the powers of matrices. There is also another topic that should also be brought light upon. There exists a pattern in the end results of the powers of the matrices. The final matrices themselves form an identity matrix where the final non-zero digit is the power of the original non-zero digit, and the zeros remain the same and unchanged. In linear algebra, the identity matrix of size "n" is a n-by-n square matrix with either ones or non-zero digits on the main diagonal and zeros elsewhere. Sometimes the zeros could be replaced by a different number. In this case the power of the non-zero digit, being 2 in this report, replaces the non-zero digit in the identity matrix, and the zeros remain the same. Therefore the general expression of a M^n matrix in terms of "n is simply:

$$\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}^{n} = \begin{pmatrix} k^n & 0^n \\ 0^n & k^n \end{pmatrix}$$

When given the matrices $P = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$ and $S = \begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix}$, we will now test and analyze the pattern observe from before and also modify our pattern conclusion and our methods of solving for the powers of a matrix.

First we shall calculate $P^n \text{and } S^n, \text{for } n = 3, 4, \text{and } 5$ by using a Ti-83 Graphing calculator. To enter a matrix into a Ti-83 Graphing calculator we would need to follow the following steps.

In order to enter the matrix into the Ti-83, press the [MATRIX] key, select [EDIT] and then [MATRIX A].
Next, enter the order of the matrix, in this case the matrix has a order of 2x2 (2 rows by 2 columns), and then being entering the values into the given table. In order to move from position to position use the arrow keys or the [ENTER] key. Once the information has been entered into the table, its can now be accessed using the [MATRIX] function and selecting the matrix under the [NAMES] function.

After entering in the matrix into the calculator for the matrix $P = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$ under the matrix [A], select the matrix and then on the main screen type in ^2, then [ENTER]. The following answer should be displayed as $\begin{pmatrix} 10 & 6 \\ 6 & 10 \end{pmatrix}$. Repeat the above steps for "n" values of 3, 4 and 5.

The answers should be as follows:

$$P^3 = \begin{pmatrix} 36 & 28 \\ 28 & 36 \end{pmatrix} \qquad P^4 = \begin{pmatrix} 136 & 120 \\ 120 & 136 \end{pmatrix} \qquad P^5 = \begin{pmatrix} 528 & 496 \\ 496 & 528 \end{pmatrix}$$

By using the graphing calculator to check our work, we see that by using the second method of finding the powers of a matrix to be incorrect to an extent. The second method of finding the powers of a matrix will only prove to be correct if used for a matrix like $A^n = \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}^n$, however is the matrix is $B^n = \begin{pmatrix} y & x \\ x & y \end{pmatrix}^n$, then the method no longer works. Therefore there is only two ways of solving for a power of a matrix and that is by long hand multiplying a matrix by "n" times, and a new method by using a graphing calculator.

Besides the matrix $P = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$ that was given to us, we are also given the matrix $S = \begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix}$. Thus we shall solve for "n" values of 3, 4 and 5.

Using our graphing calculators as before the answers for "n" values of 3, 4 and 5, should be as follows:

$$S^3 = \begin{pmatrix} 112 & 104 \\ 104 & 112 \end{pmatrix} \qquad S^4 = \begin{pmatrix} 656 & 640 \\ 640 & 656 \end{pmatrix} \qquad S^5 = \begin{pmatrix} 3904 & 3872 \\ 3872 & 3904 \end{pmatrix}$$

Therefore based on the above answers and those of the matrices [P] and [M] there must be a general formula to solving powers of matrices. This is also because it is deemed unreasonable and tedious work to find the power of a matrix, by multiplying a matrix by "n" times, especially when n = 100.

Another interesting fact given to us is:

$$P^2 = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}^2 = \begin{pmatrix} 10 & 6 \\ 6 & 10 \end{pmatrix} = 2\begin{pmatrix} 5 & 3 \\ 3 & 5 \end{pmatrix}; \quad P^2 = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}^2 = \begin{pmatrix} 10 & 6 \\ 6 & 10 \end{pmatrix} = 2\begin{pmatrix} 5 & 3 \\ 3 & 5 \end{pmatrix};$$

$$S^2 = \begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix}^2 = \begin{pmatrix} 20 & 16 \\ 16 & 20 \end{pmatrix} = 2\begin{pmatrix} 10 & 8 \\ 8 & 10 \end{pmatrix} S^2 = \begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix}^2 = \begin{pmatrix} 20 & 16 \\ 16 & 20 \end{pmatrix} = 2\begin{pmatrix} 10 & 8 \\ 8 & 10 \end{pmatrix}$$

Based on the above given matrices, it can be assumed that the power of a matrix can be divided by the power value "n." however after researching into the matter above, it would have seems that this is only available to even powers of "n" such as the multiples of 2. If the power of the matrix was three, then when the matrix that is powered is divided by 3, then we will receive a decimal number. Even though we receive a decimal number if the power was odd, the theory is correct and can be applied to other matrices, however the only drawback is the length of the decimal numbers as opposed to a well-rounded whole number.

The formula for the theory is as follows:

$$C = \begin{pmatrix} k+1 & k-1 \\ k-1 & k+1 \end{pmatrix}$$

$$C^2 = \begin{pmatrix} k+1 & k-1 \\ k-1 & k+1 \end{pmatrix}^2 = 2\left[\begin{pmatrix} \frac{k+1}{2} & \frac{k-1}{2} \\ \frac{k-1}{2} & \frac{k+1}{2} \end{pmatrix}^2\right]$$

All of the above matrices that was given above are all in the form of $\begin{pmatrix} k+1 & k-1 \\ k-1 & k+1 \end{pmatrix}$ where k=1, 2, 3.

We shall investigate the possibilities of any patterns and formulas that deals with matrices in the forms of $\begin{pmatrix} k+1 & k-1 \\ k-1 & k+1 \end{pmatrix}$, for k values of 4, 5, 6, and powers of 2 and 3 using the Ti-83 Graphing Calculator.

For the k value of 4, the matrix shall look like the following:

$$D = \begin{pmatrix} 5 & 3 \\ 3 & 5 \end{pmatrix}$$

And then using the graphing calculator and solving for powers of 2 and 3, we should receive the following answers:

$$D^2 = \begin{pmatrix} 5 & 3 \\ 3 & 5 \end{pmatrix}^2 = \begin{pmatrix} 34 & 30 \\ 30 & 34 \end{pmatrix} \qquad D^3 = \begin{pmatrix} 5 & 3 \\ 3 & 5 \end{pmatrix}^3 = \begin{pmatrix} 260 & 252 \\ 252 & 260 \end{pmatrix}$$

For the k value of 5, the matrix shall look like the following:

$$E = \begin{pmatrix} 6 & 4 \\ 4 & 6 \end{pmatrix}$$

And then using the graphing calculator and solving for powers of 2 and 3, we should receive the following answers:

$$E^2 = \begin{pmatrix} 6 & 4 \\ 4 & 6 \end{pmatrix}^2 = \begin{pmatrix} 52 & 48 \\ 48 & 52 \end{pmatrix} \qquad E^3 = \begin{pmatrix} 6 & 4 \\ 4 & 6 \end{pmatrix}^3 = \begin{pmatrix} 504 & 496 \\ 496 & 504 \end{pmatrix}$$

For the k value of 6, the matrix shall look like the following:

$$F = \begin{pmatrix} 7 & 5 \\ 5 & 7 \end{pmatrix}$$

And then using the graphing calculator and solving for powers of 2 and 3, we should receive the following answers:

$$F^2 = \begin{pmatrix} 7 & 5 \\ 5 & 7 \end{pmatrix}^2 = \begin{pmatrix} 74 & 70 \\ 70 & 74 \end{pmatrix} \qquad F^3 = \begin{pmatrix} 7 & 5 \\ 5 & 7 \end{pmatrix}^3 = \begin{pmatrix} 868 & 860 \\ 860 & 868 \end{pmatrix}$$

Assuming from the above results from calculating the powers of the matrices, we can safely conclude that their not just only a pattern of an identity matrix but also perhaps a formula that can be used to determine the result of a power of a matrix.

Determining the formula for a power of a matrix in the form of $\begin{pmatrix} k+1 & k-1 \\ k-1 & k+1 \end{pmatrix}$:

$$G = \begin{pmatrix} k+1 & k-1 \\ k-1 & k+1 \end{pmatrix}$$

$$G^2 = \begin{pmatrix} [(k+1)(k+1) + (k+1)(k-1)] & [(k-1)(k-1) + (k+1)(k-1)] \\ [(k-1)(k-1) + (k+1)(k-1)] & [(k+1)(k+1) + (k+1)(k-1)] \end{pmatrix}$$

$$G^2 = \begin{pmatrix} [(k^2+2k+1) + (k^2-1)] & [(k^2-2k+1) + (k^2-1)] \\ [(k^2-2k+1) + (k^2-1)] & [(k^2+2k+1) + (k^2-1)] \end{pmatrix}$$

$$G^2 = \begin{pmatrix} 2k^2+2k & 2k^2-2k \\ 2k^2-2k & 2k^2+2k \end{pmatrix}$$

$$G^2 = \begin{pmatrix} 2k(k+1) & 2k(k-1) \\ 2k(k-1) & 2k(k+1) \end{pmatrix}$$

$$G^2 = 2k\begin{pmatrix} (k+1) & (k-1) \\ (k-1) & (k+1) \end{pmatrix}$$

To determine if our new formula works and is proven correct we substitute 2 for k.

$$H = \begin{pmatrix} (2+1) & (2-1) \\ (2-1) & (2+1) \end{pmatrix}$$
$$H^2 = 2(2)\begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$$
$$H^2 = 4\begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix} \rightarrow \begin{pmatrix} 4(3) & 4(1) \\ 4(1) & 4(3) \end{pmatrix}$$
$$H^2 = \begin{pmatrix} 12 & 8 \\ 8 & 12 \end{pmatrix}$$

The final result when compared to the answer given on the graphing calculator is proven to be simply wrong. Therefore the formula which exists is not the correct formula as the one that was solved by us. Therefore, for the powers of matrices we are only given the only two options of solving the powers of matrices and that is either by hand by multiplying the matrices by "n" times (the value of the power) or to calculate the power by using a graphing calculator. Even though we are not able to determine a formula, we are able to however to discover a relationship between the "k" and "n" values when the equation is calculated.

When we take $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}^2 = \begin{pmatrix} 2^2 & 0^2 \\ 0^2 & 2^2 \end{pmatrix} = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$ we see that the difference between the values of 4 and 0 in the result of the power is 4 or in this case 2^2. When we take another example say $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}^5 = \begin{pmatrix} 2^5 & 0^5 \\ 0^5 & 2^5 \end{pmatrix} = \begin{pmatrix} 32 & 0 \\ 0 & 32 \end{pmatrix}$, we see that the difference between the values of 32 and 0 is 32 or in this case 2^5. Thus we can assume that the power of the matrix "n" plays a role in the value difference between the two values in the end result of the final answer. Therefore the difference between the two values of the end result of a power of a matrix is the outcome of $2^{(\text{power of the matrix})}$.

Using technology, in this case our graphing calculators, we investigated what will happen with the more extreme values of "k" and "n". We determined that the exact scope and limitations for "k" and "n" is quite far ahead of the capabilities of the calculators. We found out that as the end results of the matrices powers increased then so did the difference between the two values found in the end matrices, with relation to "k" and "n". Therefore the range of the of "k" and "n" is limitless and as for the limitations on "k" and "n" does not exist for now.

Therefore in conclusion, powers of matrices can be determined by hand by repeatedly multiplying the matrix by itself, "n" times, with "n" being the value of the power that the matrix is powered by. Another way is to use a graphing calculator to determine the new matrix. There also exists a relation between the value of "n" and the difference between the two values of the new matrix. The relation is that the difference between the two values is 2 to the power of "n". However, the student was not able to find a pattern or relationship that will exist between the values of "k" and "n". Also we also found that the end result of the new matrix forms an identity matrix. The above statements will perhaps be limited only to matrices in the form of $\begin{pmatrix} k+1 & k-1 \\ k-1 & k+1 \end{pmatrix}$, for other different types of matrices it is not known for it was not investigated by the student. Another conclusion that can be drawn up is that the power of a matrix can be divided by the power value of "n." Even though we receive a decimal number if the power was an odd value, the theory is correct and can be applied to other matrices, however the only drawback is the length of the decimal numbers as opposed to a well-rounded whole number.

The Matrix theory is a branch of mathematics that focuses on the study of matrices. A matrix is a rectangular table of elements, or entries, which may be numbers or, any abstract quantities that can be added and multiplied. A matrix may be squared or even raised to an integer power. However, for a matrix to be able to be raised to a power the matrix must first be a square matrix. Different values of a power whether it be an integer or a whole number, can create and manipulate the original matrix into a different form or branch of matrices. The portfolio not only investigates the powers of matrix but it also investigates the patterns and the meaning and value of everything beneath the surface. Matrices are created by humankind to further their technological advances and create a better society and increase the level of living standards in the world. The importance of understanding matrices and their operations will help students and even those who no longer participate in school, will help them create a better should they advance forward into the technological and engineering fields of careers. The purpose of this portfolio was to develop a skill necessary in that area of career, a skill of being able to decipher patterns and know how to manipulate them in order to apply them to the real world.