# Internal Assessment- Matrix Powers

The internal assessment will focused on observing patterns of matrix powers which will be the main key to find the general expression of matrix powers.

**1)**

$$M^2 = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

$$M^2 = \begin{pmatrix} 2*2+0*0 & 2*0+2*0 \\ 2*0+2*0 & 2*2+0*0 \end{pmatrix}$$

$$M^2 = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$$

$$M^3 = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

$$M^3 = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

$$M^3 = \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix}$$

**Explanation:** When matrix M is powered by 2 it gives a result of $M^2 = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$ from GDC, when M is powered by 3 it gives a result of $M^3 = \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix}$ from GDC and when M is powered by 4 it gives a result of $M^4 = \begin{pmatrix} 16 & 0 \\ 0 & 16 \end{pmatrix}$ from GDC. The pattern shown is that every time M is powered by a number after its preceding number it is multiply by 2. For instance, 4 shown in matrix $M^2$, 8 shown in matrix $M^3$, and 16 shown in matrix $M^4$. Thus, these results make a general expression that $M^n = 2^{n-1}\begin{pmatrix} 1^n+1 & 1^n-1 \\ 1^n-1 & 1^n+1 \end{pmatrix}$. This formula is proven correct because when $M^5 = 2^{5-1}\begin{pmatrix} 1^5+1 & 1^5-1 \\ 1^5-1 & 1^5+1 \end{pmatrix} = \begin{pmatrix} 32 & 0 \\ 0 & 32 \end{pmatrix}$ from GDC and this formula also works when $M^{10} = 2^{10-1}\begin{pmatrix} 1^{10}+1 & 1^{10}-1 \\ 1^{10}-1 & 1^{10}+1 \end{pmatrix} = \begin{pmatrix} 1024 & 0 \\ 0 & 1024 \end{pmatrix}$ from GDC.

**2)**

$$P^3 = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$$

$$P^3 = \begin{pmatrix} 10 & 6 \\ 6 & 10 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$$

$$P^3 = \begin{pmatrix} 36 & 28 \\ 28 & 36 \end{pmatrix} = 4\begin{pmatrix} 9 & 7 \\ 7 & 9 \end{pmatrix}$$

$$P^4 = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$$

$$P^4 = \begin{pmatrix} 10 & 6 \\ 6 & 10 \end{pmatrix}\begin{pmatrix} 10 & 6 \\ 6 & 10 \end{pmatrix}$$

$$P^4 = \begin{pmatrix} 136 & 120 \\ 120 & 136 \end{pmatrix} = 8\begin{pmatrix} 17 & 15 \\ 15 & 17 \end{pmatrix}$$

**Explanation:** When matrix P is powered by 3 it gives a result of $4\begin{pmatrix} 9 & 7 \\ 7 & 9 \end{pmatrix}$, when matrix P is powered by 4 it gives a result of $8\begin{pmatrix} 17 & 15 \\ 15 & 17 \end{pmatrix}$, and when matrix P is powered by 5 it gives a result of $16\begin{pmatrix} 33 & 31 \\ 31 & 33 \end{pmatrix}$. The pattern shown is that results have a common factor of $2^n$ such as 4 shown in matrix $P^3$, 8 shown in matrix $P^4$, and then16 shown in matrix $P^5$. Once the resulting matrix is factor out, the left over numbers inside the matrix has a general pattern of either $2^n + 1$ such as 9 when n = 3, 17 when n = 4 and then 33 when n = 5 or $2^n - 1$ such as 7 when n = 3, 9 when n = 4 and then 31 when n = 5. Thus, these results make a general expression that $P^n = 2^{n-1}\begin{pmatrix} 2^n + 1 & 2^n - 1 \\ 2^n - 1 & 2^n + 1 \end{pmatrix}$. This formula is proven correct because when $P^5 = 2^{5-1}\begin{pmatrix} 2^5 + 1 & 2^5 - 1 \\ 2^5 - 1 & 2^5 + 1 \end{pmatrix} = \begin{pmatrix} 528 & 496 \\ 496 & 528 \end{pmatrix}$ from GDC and this formula also works when $P^6 = 2^{6-1}\begin{pmatrix} 2^6 + 1 & 1^6 - 1 \\ 2^6 - 1 & 1^6 + 1 \end{pmatrix} = \begin{pmatrix} 2080 & 2016 \\ 2016 & 2080 \end{pmatrix}$ from GDC.

$$S^3 = \begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix}$$

$$S^3 = \begin{pmatrix} 20 & 16 \\ 16 & 20 \end{pmatrix}\begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix}$$

$$S^3 = \begin{pmatrix} 112 & 104 \\ 104 & 112 \end{pmatrix} = 4\begin{pmatrix} 28 & 26 \\ 26 & 28 \end{pmatrix}$$

$$S^4 = \begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix}$$

$$S^4 = \begin{pmatrix} 20 & 16 \\ 16 & 20 \end{pmatrix}\begin{pmatrix} 20 & 16 \\ 16 & 20 \end{pmatrix}$$

$$S^4 = \begin{pmatrix} 656 & 640 \\ 640 & 656 \end{pmatrix} = 8\begin{pmatrix} 82 & 80 \\ 80 & 82 \end{pmatrix}$$

**Explanation:** When matrix S is powered by 3 it gives a result of $4\begin{pmatrix} 28 & 26 \\ 26 & 28 \end{pmatrix}$ from GDC, when matrix S is powered by 4 it gives a result of $8\begin{pmatrix} 82 & 80 \\ 80 & 82 \end{pmatrix}$ from GDC, and when matrix S is powered by 5 it gives a result of $16\begin{pmatrix} 244 & 242 \\ 242 & 244 \end{pmatrix}$ from GDC. The pattern shown is that results have a common factor of $2^n$ such as 4 shown in matrix $S^3$, 8 shown in matrix $S^4$, and then16 shown in matrix $S^5$. Once the resulting matrix is factor out, the left over numbers inside the matrix has a general pattern of either $3^n + 1$ such as 28 when n = 3, 82 when $n = 4$, and then 244 when n = 5 or $3^n - 1$ such as 26 when n = 3, 80 when n = 4, and then 242 when n = 5. Thus, these results make a general expression that $S^n = 2^{n-1}\begin{pmatrix} 3^n+1 & 3^n-1 \\ 3^n-1 & 3^n+1 \end{pmatrix}$. This formula is proven correct because

when $S^5 = 2^{5-1}\begin{pmatrix} 3^5+1 & 3^5-1 \\ 3^5-1 & 3^5+1 \end{pmatrix} = \begin{pmatrix} 3904 & 3872 \\ 3872 & 3904 \end{pmatrix}$ from GDC and this formula also works

when $S^6 = 2^{6-1}\begin{pmatrix} 3^6+1 & 3^6-1 \\ 3^6-1 & 3^6+1 \end{pmatrix} = \begin{pmatrix} 23360 & 23296 \\ 23296 & 23360 \end{pmatrix}$ from GDC.

**3)** When $k=1, \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$, $k=2, \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$, $k=3, \begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix}$, $k=4, \begin{pmatrix} 5 & 5 \\ 3 & 3 \end{pmatrix}$, $k=5, \begin{pmatrix} 6 & 4 \\ 4 & 6 \end{pmatrix}$

The pattern shows that the results of $k=1$ is equivalent to result matrix $M$, where as the results of $k=2$ is equivalent to result matrix $P$ and the results of $k=3$ is equivalent to result matrix $S^n$. The pattern also shows that when $k=1$ the matrix formula is $M^n = 2^{n-1}\begin{pmatrix} 1^n+1 & 1^n-1 \\ 1^n-1 & 1^n+1 \end{pmatrix}$, when $k=2$ the matrix formula is $M^n = 2^{n-1}\begin{pmatrix} 1^n+1 & 1^n-1 \\ 1^n-1 & 1^n+1 \end{pmatrix}$,

and when $k=3$ the matrix formula is $S^n = 2^{n-1}\begin{pmatrix} 3^n+1 & 3^n-1 \\ 3^n-1 & 3^n+1 \end{pmatrix}$. This shows that any

number representing $k$ corresponds to $k^n+1$ and $k^n-1$. Thus, these results give a general expression that $\begin{pmatrix} k+1 & k-1 \\ k-1 & k+1 \end{pmatrix}^n = 2^{n-1}\begin{pmatrix} k^n+1 & k^n-1 \\ k^n-1 & k^n+1 \end{pmatrix}$**.** For instance, if $k=4$ and $n=5$

then $\begin{pmatrix} 4+1 & 4-1 \\ 4-1 & 4+1 \end{pmatrix}^5 = 2^{5-1}\begin{pmatrix} 4^5+1 & 4^5-1 \\ 4^5-1 & 4^5+1 \end{pmatrix} =$

$\begin{pmatrix} 5 & 3 \\ 3 & 5 \end{pmatrix}^5 = 16\begin{pmatrix} 1025 & 1023 \\ 1023 & 1025 \end{pmatrix} = \begin{pmatrix} 16400 & 16368 \\ 16368 & 16400 \end{pmatrix}$ from GDC.

**4)**

**Explanation:** After a further investigation with further values of k and n, the results shows that the limitation for n is that $n \neq$ all negative numbers and non-integers value because when a matrix is powered by negative numbers or non-integers value the calculator gives a math error text. **In addition there's no limitation for k.**

**5)**

In conclusion,