Internal Assessment - Matrix Binomials

The assignment of this internal assessment fundamentally consisted of nine goals. Those nine goals are finding X^n, Y^n, $(X+Y)^n$, A^n, B^n, $(A+B)^n$, proving M = A + B, proving $M^2 = A^2 + B^2$, and finding M^n. With the initial matrices of *X* and *Y*, the assignment was found using calculations based off of *X* and *Y*. The assignment was also found using the value of *A* and *B*. Values for *X, Y, A,* and *B* were then used to find values for *M*.

All calculations were made using the GDC

$$X = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$X^2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

$$X^3 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix}$$

$$X^4 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix}$$

If $X^2 = X^n$, then 2 = n.
Using the expression 2^{n-1}, we find that it equals 2 when multiplied to X which is why $X^2 = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$.

If the expression is $X^n = 2^{n-1}(X)$, then:

$$X^n = \begin{bmatrix} 2^{n-1} & 2^{n-1} \\ 2^{n-1} & 2^{n-1} \end{bmatrix}$$

Testing for n = 4 would mean: $X^4 = 2^{4-1}(X)$

$$X^4 = 2^{4-1} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = 8 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix}$$

Testing for n = 15

$X^{15} = (X^4)(X^4)(X^4)(X^3) = \begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix}\begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix}\begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix}\begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix} = \begin{bmatrix} 16384 & 16384 \\ 16384 & 16384 \end{bmatrix}$

Verify using $X^n = \begin{bmatrix} 2^{n-1} & 2^{n-1} \\ 2^{n-1} & 2^{n-1} \end{bmatrix}$

$X^{15} = \begin{bmatrix} 2^{15-1} & 2^{15-1} \\ 2^{15-1} & 2^{15-1} \end{bmatrix} = \begin{bmatrix} 2^{14} & 2^{14} \\ 2^{14} & 2^{14} \end{bmatrix} = \begin{bmatrix} 16384 & 16384 \\ 16384 & 16384 \end{bmatrix}$

Testing for n = 9

$X^9 = (X^4)(X^4)(X) = \begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix}\begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 256 & 256 \\ 256 & 256 \end{bmatrix}$

Verify using $X^n = \begin{bmatrix} 2^{n-1} & 2^{n-1} \\ 2^{n-1} & 2^{n-1} \end{bmatrix}$

$X^9 = \begin{bmatrix} 2^{9-1} & 2^{9-1} \\ 2^{9-1} & 2^{9-1} \end{bmatrix} = \begin{bmatrix} 2^8 & 2^8 \\ 2^8 & 2^8 \end{bmatrix} = \begin{bmatrix} 256 & 256 \\ 256 & 256 \end{bmatrix}$

Testing for n = 13

$X^{13} = (X^4)(X^4)(X^4)(X) = \begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix}\begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix}\begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 4096 & 4096 \\ 4096 & 4096 \end{bmatrix}$

Verify using $X^n = \begin{bmatrix} 2^{n-1} & 2^{n-1} \\ 2^{n-1} & 2^{n-1} \end{bmatrix}$

$X^{13} = \begin{bmatrix} 2^{13-1} & 2^{13-1} \\ 2^{13-1} & 2^{13-1} \end{bmatrix} = \begin{bmatrix} 2^{12} & 2^{12} \\ 2^{12} & 2^{12} \end{bmatrix} = \begin{bmatrix} 4096 & 4096 \\ 4096 & 4096 \end{bmatrix}$

Therefore, we can conclude that $X^n = 2^{n-1}(X)$ which, after multiplying $2^{n-1}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, equals

$X^n = \begin{bmatrix} 2^{n-1} & 2^{n-1} \\ 2^{n-1} & 2^{n-1} \end{bmatrix}$

$$Y = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$Y^2 = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$$

$$Y^3 = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix}$$

$$Y^4 = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}$$

If $Y^2 = Y^n$, then $2 = n$.
Using the expression 2^{n-1}, we find that it equals 2 when multiplied to Y which is why $Y^2 = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$.

If the expression is $Y^n = 2^{n-1}(Y)$, then:

$$Y^n = \begin{bmatrix} 2^{n-1} & -2^{n-1} \\ -2^{n-1} & 2^{n-1} \end{bmatrix}$$

Testing for n = 4 would result in $Y^4 = 2^{4-1}(Y)$

$$Y^4 = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}$$

Verifying using $Y^n = \begin{bmatrix} 2^{n-1} & -2^{n-1} \\ -2^{n-1} & 2^{n-1} \end{bmatrix}$

$$Y^4 = \begin{bmatrix} 2^{4-1} & -2^{4-1} \\ -2^{4-1} & 2^{4-1} \end{bmatrix} = \begin{bmatrix} 2^3 & -2^3 \\ -2^3 & 2^3 \end{bmatrix} = \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}$$

Testing for n = 15

$$Y^{15} = (Y^4)(Y^4)(Y^4)(Y^3) = \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}\begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}\begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}\begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix} = \begin{bmatrix} 16384 & -16384 \\ -16384 & 16384 \end{bmatrix}$$

Verify using $Y^n = \begin{bmatrix} 2^{n-1} & -2^{n-1} \\ -2^{n-1} & 2^{n-1} \end{bmatrix}$

$$Y^{15} = \begin{bmatrix} 2^{15-1} & -2^{15-1} \\ -2^{15-1} & 2^{15-1} \end{bmatrix} = \begin{bmatrix} 2^{14} & -2^{14} \\ -2^{14} & 2^{14} \end{bmatrix} = \begin{bmatrix} 16384 & -16384 \\ -16384 & 16384 \end{bmatrix}$$

Testing for n = 9

$$Y^9 = (Y^4)(Y^4)(Y) = \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}\begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 256 & -256 \\ -256 & 256 \end{bmatrix}$$

Verify using $Y^n = \begin{bmatrix} 2^{n-1} & -2^{n-1} \\ -2^{n-1} & 2^{n-1} \end{bmatrix}$

$$Y^9 = \begin{bmatrix} 2^{9-1} & -2^{9-1} \\ -2^{9-1} & 2^{9-1} \end{bmatrix} = \begin{bmatrix} 2^8 & -2^8 \\ -2^8 & 2^8 \end{bmatrix} = \begin{bmatrix} 256 & -256 \\ -256 & 256 \end{bmatrix}$$

Testing for n = 13

$$Y^{15} = (Y^9)(Y^4) = \begin{bmatrix} 256 & -256 \\ -256 & 256 \end{bmatrix}\begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix} = \begin{bmatrix} 16384 & -16384 \\ -16384 & 16384 \end{bmatrix}$$

Verify using $Y^n = \begin{bmatrix} 2^{n-1} & -2^{n-1} \\ -2^{n-1} & 2^{n-1} \end{bmatrix}$

$$Y^{15} = \begin{bmatrix} 2^{15-1} & -2^{15-1} \\ -2^{15-1} & 2^{15-1} \end{bmatrix} = \begin{bmatrix} 16384 & -16384 \\ -16384 & 16384 \end{bmatrix}$$

From this, we can conclude that $Y^n = 2^{n-1}(Y)$. When multiplied out, $2^{n-1}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$, this ultimately comes to $Y^n = \begin{bmatrix}2^{n-1} & -2^{n-1}\\ -2^{n-1} & 2^{n-1}\end{bmatrix}$.

Remember that the value of *X* and the value of *Y* are matrices. Essentially, *X* + *Y* is another matrix. Matrix *(X+Y)* will be called *Z*. Using various values we can find the values of Z^2, Z^3, Z^4, and Z^n. However, remember that Z is the value of the sum of *X* and *Y*.

$$Z = X + Y = \begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix} + \begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} = \begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}$$

$$Z^2 = \begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix} = \begin{bmatrix}4 & 0\\ 0 & 4\end{bmatrix}$$

$$Z^3 = \begin{bmatrix}4 & 0\\ 0 & 4\end{bmatrix}\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix} = \begin{bmatrix}8 & 0\\ 0 & 8\end{bmatrix}$$

$$Z^4 = \begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}\begin{bmatrix}8 & 0\\ 0 & 8\end{bmatrix} = \begin{bmatrix}16 & 0\\ 0 & 16\end{bmatrix}$$

Using information from above, we can test to see whether or not it is applicable when finding Z^n. Essentially, since $X^n = 2^{n-1}(X)$ and $Y^n = 2^{n-1}(Y)$, we'll test to see if $Z^n = 2^{n-1}(Z)$.

When n = 4

$$Z^4 = \begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix} = \begin{bmatrix}16 & 0\\ 0 & 16\end{bmatrix}$$

Verifying with $Z^n = 2^{n-1}(Z)$,

$$Z^4 = 2^{4-1}\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix} = 2^3\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix} = 8\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix} = \begin{bmatrix}16 & 0\\ 0 & 16\end{bmatrix}$$

Although it appears that $Z^n = 2^{n-1}(Z)$, we must check other values to determine that the equation does stay true.

When n = 15

$$Z^{15} = (Z^4)(Z^4)(Z^4)(Z^3) = \begin{bmatrix} 16 & 0 \\ 0 & 16 \end{bmatrix}\begin{bmatrix} 16 & 0 \\ 0 & 16 \end{bmatrix}\begin{bmatrix} 16 & 0 \\ 0 & 16 \end{bmatrix}\begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix} = \begin{bmatrix} 32768 & 32768 \\ 32768 & 32768 \end{bmatrix}$$

Verifying with $Z^n = 2^{n-1}(Z)$,

$$Z^{15} = 2^{15-1}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = 2^{14}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = 16384\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 32768 & 32768 \\ 32768 & 32768 \end{bmatrix}$$

When n = 9

$$Z^9 = (Z^4)(Z^4)(Z) = \begin{bmatrix} 16 & 0 \\ 0 & 16 \end{bmatrix}\begin{bmatrix} 16 & 0 \\ 0 & 16 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 512 & 0 \\ 0 & 512 \end{bmatrix}$$

Verifying with $Z^n = 2^{n-1}(Z)$,

$$Z^9 = 2^{9-1}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = 2^8\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = 256\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 512 & 0 \\ 0 & 512 \end{bmatrix}$$

Testing when n = 13

$$Z^{13} = (Z^4)(Z^4)(Z^4)(Z) = \begin{bmatrix} 16 & 0 \\ 0 & 16 \end{bmatrix}\begin{bmatrix} 16 & 0 \\ 0 & 16 \end{bmatrix}\begin{bmatrix} 16 & 0 \\ 0 & 16 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 8192 & 8192 \\ 8192 & 8192 \end{bmatrix}$$

With this evidence, we can come to the conclusion that $Z^n = 2^{n-1}(Z)$. However we must take into consideration that Z = (X + Y). This ultimately leads to:

$(X + Y)^n = 2^{n-1}(X + Y)$

Next, we'll consider *A*. *A* is a value and can be represented by $A = aX$. The value of *(a)* is always a constant, and we'll use the value of *X* from the previous work. Using this information, we'll find the values of A^2, A^3, A^4, and A^n.
To find a pattern within the values, we'll test using various values of *(a)*.

Remember that $X = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ and $A = aX$.

When $a = 2$,

$$A^2 = 2\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix} \bullet 2\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix} = \begin{bmatrix}2 & 2\\2 & 2\end{bmatrix} \bullet \begin{bmatrix}2 & 2\\2 & 2\end{bmatrix} = \begin{bmatrix}8 & 8\\8 & 8\end{bmatrix}$$

$$A^3 = \begin{bmatrix}2 & 2\\2 & 2\end{bmatrix}\begin{bmatrix}8 & 8\\8 & 8\end{bmatrix} = \begin{bmatrix}32 & 32\\32 & 32\end{bmatrix}$$

$$A^4 = \begin{bmatrix}2 & 2\\2 & 2\end{bmatrix}\begin{bmatrix}32 & 32\\32 & 32\end{bmatrix} = \begin{bmatrix}128 & 128\\128 & 128\end{bmatrix}$$

When $a = 4$

$$A^2 = 4\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix} \bullet 4\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix} = \begin{bmatrix}4 & 4\\4 & 4\end{bmatrix} \bullet \begin{bmatrix}4 & 4\\4 & 4\end{bmatrix} = \begin{bmatrix}32 & 32\\32 & 32\end{bmatrix}$$

$$A^3 = \begin{bmatrix}4 & 4\\4 & 4\end{bmatrix}\begin{bmatrix}32 & 32\\32 & 32\end{bmatrix} = \begin{bmatrix}256 & 256\\256 & 256\end{bmatrix}$$

$$A^4 = \begin{bmatrix}4 & 4\\4 & 4\end{bmatrix}\begin{bmatrix}256 & 256\\256 & 256\end{bmatrix} = \begin{bmatrix}2048 & 2048\\2048 & 2048\end{bmatrix}$$

When $a = 6$

$$A^2 = 6\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix} \bullet 6\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix} = \begin{bmatrix}6 & 6\\6 & 6\end{bmatrix} \bullet \begin{bmatrix}6 & 6\\6 & 6\end{bmatrix} = \begin{bmatrix}72 & 72\\72 & 72\end{bmatrix}$$

$$A^3 = \begin{bmatrix}6 & 6\\6 & 6\end{bmatrix}\begin{bmatrix}72 & 72\\72 & 72\end{bmatrix} = \begin{bmatrix}864 & 864\\864 & 864\end{bmatrix}$$

$$A^4 = \begin{bmatrix}6 & 6\\6 & 6\end{bmatrix}\begin{bmatrix}864 & 864\\864 & 864\end{bmatrix} = \begin{bmatrix}10368 & 10368\\10368 & 10368\end{bmatrix}$$

When $a = 8$

$$A^2 = 8\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix} \bullet 8\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix} = \begin{bmatrix}8 & 8\\8 & 8\end{bmatrix} \bullet \begin{bmatrix}8 & 8\\8 & 8\end{bmatrix} = \begin{bmatrix}128 & 128\\128 & 128\end{bmatrix}$$

$$A^3 = \begin{bmatrix}8 & 8\\8 & 8\end{bmatrix}\begin{bmatrix}128 & 128\\128 & 128\end{bmatrix} = \begin{bmatrix}2048 & 2048\\2048 & 2048\end{bmatrix}$$

$$A^4 = \begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix} \begin{bmatrix} 2048 & 2048 \\ 2048 & 2048 \end{bmatrix} = \begin{bmatrix} 32768 & 32768 \\ 32768 & 32768 \end{bmatrix}$$

When $a = 10$

$$A^2 = 10\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \bullet 10\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 10 & 10 \\ 10 & 10 \end{bmatrix} \bullet \begin{bmatrix} 10 & 10 \\ 10 & 10 \end{bmatrix} = \begin{bmatrix} 200 & 200 \\ 200 & 200 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 10 & 10 \\ 10 & 10 \end{bmatrix} \begin{bmatrix} 200 & 200 \\ 200 & 200 \end{bmatrix} = \begin{bmatrix} 4000 & 4000 \\ 4000 & 4000 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 10 & 10 \\ 10 & 10 \end{bmatrix} \begin{bmatrix} 4000 & 4000 \\ 4000 & 4000 \end{bmatrix} = \begin{bmatrix} 80000 & 80000 \\ 80000 & 80000 \end{bmatrix}$$

When $a = 5$

$$A^2 = 5\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \bullet 5\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 5 \\ 5 & 5 \end{bmatrix} \bullet \begin{bmatrix} 5 & 5 \\ 5 & 5 \end{bmatrix} = \begin{bmatrix} 50 & 50 \\ 50 & 50 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 5 & 5 \\ 5 & 5 \end{bmatrix} \begin{bmatrix} 50 & 50 \\ 50 & 50 \end{bmatrix} = \begin{bmatrix} 500 & 500 \\ 500 & 500 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 5 & 5 \\ 5 & 5 \end{bmatrix} \begin{bmatrix} 500 & 500 \\ 500 & 500 \end{bmatrix} = \begin{bmatrix} 5000 & 5000 \\ 5000 & 5000 \end{bmatrix}$$

When $a = 3$

$$A^2 = 3\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \bullet 3\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix} \bullet \begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 18 & 18 \\ 18 & 18 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} 18 & 18 \\ 18 & 18 \end{bmatrix} = \begin{bmatrix} 108 & 108 \\ 108 & 108 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} 108 & 108 \\ 108 & 108 \end{bmatrix} = \begin{bmatrix} 648 & 648 \\ 648 & 648 \end{bmatrix}$$

Because $A = aX$, we must remember that $X = 2^{n-1}(X)$. In finding A^n, we must find how the value of *(a)* changes. We can see this by looking at the results of A^n and how it corresponds to *(a)*.

Since A^n means $(aX)^n$, we can see that as $(a)^n \bullet (X)^n$. Since $X^n = X^n = 2^{n-1}(X)$, we can consequently combine the two equations which means that $A^n = 2^{n-1}(X)\ (a)^n$

We can verify this by testing the previous values with the equation.

When $a = 5$

$A^2 = \begin{bmatrix} 50 & 50 \\ 50 & 50 \end{bmatrix}$ $A^3 = \begin{bmatrix} 500 & 500 \\ 500 & 500 \end{bmatrix}$ $A^4 = \begin{bmatrix} 5000 & 5000 \\ 5000 & 5000 \end{bmatrix}$

Now using the equation $A^n = 2^{n-1}(X)\ (a)^n$

$$A^4 = \begin{bmatrix} 2^{4-1} & 2^{4-1} \\ 2^{4-1} & 2^{4-1} \end{bmatrix} 5^4 = \begin{bmatrix} 5000 & 5000 \\ 5000 & 5000 \end{bmatrix}$$

When $a = 10$

$A^2 = \begin{bmatrix} 200 & 200 \\ 200 & 200 \end{bmatrix}$ $A^3 = \begin{bmatrix} 4000 & 4000 \\ 4000 & 4000 \end{bmatrix}$ $A^4 = \begin{bmatrix} 80000 & 80000 \\ 80000 & 80000 \end{bmatrix}$

Now using the equation $A^n = 2^{n-1}(X)\ (a)^n$

$$A^4 = \begin{bmatrix} 2^{10-1} & 2^{10-1} \\ 2^{10-1} & 2^{10-1} \end{bmatrix} 10^4 = \begin{bmatrix} 80000 & 80000 \\ 80000 & 80000 \end{bmatrix}$$

When $a = 8$

$A^2 = \begin{bmatrix} 128 & 128 \\ 128 & 128 \end{bmatrix}$ $A^3 = \begin{bmatrix} 2048 & 2048 \\ 2048 & 2048 \end{bmatrix}$ $A^4 = \begin{bmatrix} 32768 & 32768 \\ 32768 & 32768 \end{bmatrix}$

Now using the equation $A^n = 2^{n-1}(X)\ (a)^n$

$$A^4 = \begin{bmatrix} 2^{8-1} & 2^{8-1} \\ 2^{8-1} & 2^{8-1} \end{bmatrix} 8^4 = \begin{bmatrix} 32768 & 32768 \\ 32768 & 32768 \end{bmatrix}$$

From this evidence, we can conclude that A^n equals $2^{n-1}(X)\ (a)^n$.

Next, we'll consider B. B is a value and can be represented by $B = aY$. The value of *(b)* is always a constant, and we'll use the value of X from the previous work. Using this information, we'll find the values of B^2, B^3, B^4, and B^n.
To find a pattern within the values, we'll test using various values of *(b)*.

Remember that $Y = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $B = bY$.

When $b = 2$,

$$B^2 = 2\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \bullet 2\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \bullet \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}$$

$$B^3 = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix} = \begin{bmatrix} 32 & -32 \\ -32 & 32 \end{bmatrix}$$

$$B^4 = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 32 & -32 \\ -32 & 32 \end{bmatrix} = \begin{bmatrix} 128 & -128 \\ -128 & 128 \end{bmatrix}$$

When $b = 4$

$$B^2 = 4\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \bullet 4\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix} \bullet \begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix} = \begin{bmatrix} 32 & -32 \\ -32 & 32 \end{bmatrix}$$

$$B^3 = \begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix}\begin{bmatrix} 32 & -32 \\ -32 & 32 \end{bmatrix} = \begin{bmatrix} 256 & -256 \\ -256 & 256 \end{bmatrix}$$

$$B^4 = \begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix}\begin{bmatrix} 256 & -256 \\ -256 & 256 \end{bmatrix} = \begin{bmatrix} 2048 & -2048 \\ -2048 & 2048 \end{bmatrix}$$

When $b = 6$

$$B^2 = 6\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \bullet 6\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 6 & -6 \\ -6 & 6 \end{bmatrix} \bullet \begin{bmatrix} 6 & -6 \\ -6 & 6 \end{bmatrix} = \begin{bmatrix} 72 & -72 \\ -72 & 72 \end{bmatrix}$$

$$B^3 = \begin{bmatrix} 6 & -6 \\ -6 & 6 \end{bmatrix}\begin{bmatrix} 72 & -72 \\ -72 & 72 \end{bmatrix} = \begin{bmatrix} 864 & -864 \\ -864 & 864 \end{bmatrix}$$

$$B^4 = \begin{bmatrix} 6 & -6 \\ -6 & 6 \end{bmatrix}\begin{bmatrix} 864 & -864 \\ -864 & 864 \end{bmatrix} = \begin{bmatrix} 10368 & -10368 \\ -10368 & 10368 \end{bmatrix}$$

When $b = 8$

$$B^2 = 8\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \bullet 8\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix} \bullet \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix} = \begin{bmatrix} 128 & -128 \\ -128 & 128 \end{bmatrix}$$

$$B^3 = \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}\begin{bmatrix} 128 & -128 \\ -128 & 128 \end{bmatrix} = \begin{bmatrix} 2048 & -2048 \\ -2048 & 2048 \end{bmatrix}$$

$$B^4 = \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}\begin{bmatrix} 2048 & -2048 \\ -2048 & 2048 \end{bmatrix} = \begin{bmatrix} 32768 & -32768 \\ -32768 & 32768 \end{bmatrix}$$

When $b = 10$

$$B^2 = 10\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \bullet 10\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 10 & -10 \\ -10 & 10 \end{bmatrix} \bullet \begin{bmatrix} 10 & -10 \\ -10 & 10 \end{bmatrix} = \begin{bmatrix} 200 & -200 \\ -200 & 200 \end{bmatrix}$$

$$B^3 = \begin{bmatrix} 10 & -10 \\ -10 & 10 \end{bmatrix}\begin{bmatrix} 200 & -200 \\ -200 & 200 \end{bmatrix} = \begin{bmatrix} 4000 & -4000 \\ -4000 & 4000 \end{bmatrix}$$

$$B^4 = \begin{bmatrix} 10 & -10 \\ -10 & 10 \end{bmatrix}\begin{bmatrix} 4000 & -4000 \\ -4000 & 4000 \end{bmatrix} = \begin{bmatrix} 80000 & -80000 \\ -80000 & 80000 \end{bmatrix}$$

When $b = 5$

$$B^2 = 5\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \bullet 5\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 5 & -5 \\ -5 & 5 \end{bmatrix} \bullet \begin{bmatrix} 5 & -5 \\ -5 & 5 \end{bmatrix} = \begin{bmatrix} 50 & -50 \\ -50 & 50 \end{bmatrix}$$

$$B^3 = \begin{bmatrix} 5 & -5 \\ -5 & 5 \end{bmatrix}\begin{bmatrix} 50 & -50 \\ -50 & 50 \end{bmatrix} = \begin{bmatrix} 500 & -500 \\ -500 & 500 \end{bmatrix}$$

$$B^4 = \begin{bmatrix} 5 & -5 \\ -5 & 5 \end{bmatrix}\begin{bmatrix} 500 & -500 \\ -500 & 500 \end{bmatrix} = \begin{bmatrix} 5000 & -5000 \\ -5000 & 5000 \end{bmatrix}$$

When $b = 3$

$$B^2 = 3\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \bullet 3\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix} \bullet \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix} = \begin{bmatrix} 18 & -18 \\ -18 & 18 \end{bmatrix}$$

$$B^3 = \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix}\begin{bmatrix} 18 & -18 \\ -18 & 18 \end{bmatrix} = \begin{bmatrix} 108 & -108 \\ -108 & 108 \end{bmatrix}$$

$$B^4 = \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix}\begin{bmatrix} 108 & -108 \\ -108 & 108 \end{bmatrix} = \begin{bmatrix} 648 & -648 \\ -648 & 648 \end{bmatrix}$$

Because B = bY, we must remember that Y = 2^{n-1}(Y). In finding B^n, we must find how the value of *(b)* changes. We can see this by looking at the results of B^n and how it corresponds to *(b)*.

Since B^n means $(bY)^n$, we can see that as $(b)^n \bullet (Y)^n$. Since $Y^n = 2^{n-1}(Y)$, we can consequently combine the two equations which means that $B^n = 2^{n-1}(Y)\,(b)^n$

We can verify this by testing the previous values with the equation.

When $b = 5$

$$B^2 = \begin{bmatrix} 50 & -50 \\ -50 & 50 \end{bmatrix} \quad B^3 = \begin{bmatrix} 500 & -500 \\ -500 & 500 \end{bmatrix} \quad B^4 = \begin{bmatrix} 5000 & -5000 \\ -5000 & 5000 \end{bmatrix}$$

Now using the equation $B^n = 2^{n-1}(Y)\,(b)^n$

$$B^4 = \begin{bmatrix} 2^{4-1} & -2^{4-1} \\ -2^{4-1} & 2^{4-1} \end{bmatrix} 5^4 = \begin{bmatrix} 5000 & -5000 \\ -5000 & 5000 \end{bmatrix}$$

When $b = 10$

$$B^2 = \begin{bmatrix} 200 & -200 \\ -200 & 200 \end{bmatrix} \quad B^3 = \begin{bmatrix} 4000 & -4000 \\ -4000 & 4000 \end{bmatrix} \quad B^4 = \begin{bmatrix} 80000 & -80000 \\ -80000 & 80000 \end{bmatrix}$$

Now using the equation $B^n = 2^{n-1}(Y)\,(b)^n$

$$B^4 = \begin{bmatrix} 2^{10-1} & -2^{10-1} \\ -2^{10-1} & 2^{10-1} \end{bmatrix} 10^4 = \begin{bmatrix} 80000 & -80000 \\ -80000 & 80000 \end{bmatrix}$$

When $b = 8$

$$B^2 = \begin{bmatrix} 128 & -128 \\ -128 & 128 \end{bmatrix} \quad B^3 = \begin{bmatrix} 2048 & -2048 \\ -2048 & 2048 \end{bmatrix} \quad B^4 = \begin{bmatrix} 32768 & -32768 \\ -32768 & 32768 \end{bmatrix}$$

Now using the equation $B^n = 2^{n-1}(Y)\,(b)^n$

$$B^4 = \begin{bmatrix} 2^{8-1} & -2^{8-1} \\ -2^{8-1} & 2^{8-1} \end{bmatrix} 8^4 = \begin{bmatrix} 32768 & -32768 \\ -32768 & 32768 \end{bmatrix}$$

From this evidence, we can conclude that B^n equals $2^{n-1}(Y)\,(b)^n$.

Next we'll take into consideration the equation $(A+B)^n$.

Looking at (A+B), we can see that it translates to (aX+bY) meaning that *(a)* and *(b)* are still constants. Since *(a)* and *(b)* are not necessarily always the same values, we must also take into concern and test for different values.

When $a = 2$ and $b = 2$

$$(A+B)^2 = \left(2\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}+2\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}\right)^2 = \left(\begin{bmatrix}2 & 2\\2 & 2\end{bmatrix}+\begin{bmatrix}2 & -2\\-2 & 2\end{bmatrix}\right)^2 = \begin{bmatrix}4 & 0\\0 & 4\end{bmatrix}^2 = \begin{bmatrix}16 & 0\\0 & 16\end{bmatrix}$$

$$(A+B)^3 = \begin{bmatrix}4 & 0\\0 & 4\end{bmatrix}\begin{bmatrix}16 & 0\\0 & 16\end{bmatrix} = \begin{bmatrix}64 & 0\\0 & 64\end{bmatrix}$$

$$(A+B)^4 = \begin{bmatrix}4 & 0\\0 & 4\end{bmatrix}\begin{bmatrix}64 & 0\\0 & 64\end{bmatrix} = \begin{bmatrix}256 & 0\\0 & 256\end{bmatrix}$$

When $a = 4$ and $b = 4$

$$(A+B)^2 = \left(4\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}+4\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}\right)^2 = \left(\begin{bmatrix}4 & 4\\4 & 4\end{bmatrix}+\begin{bmatrix}4 & -4\\-4 & 4\end{bmatrix}\right)^2 = \begin{bmatrix}8 & 0\\0 & 8\end{bmatrix}^2 = \begin{bmatrix}64 & 0\\0 & 64\end{bmatrix}$$

$$(A+B)^3 = \begin{bmatrix}8 & 0\\0 & 8\end{bmatrix}\begin{bmatrix}64 & 0\\0 & 64\end{bmatrix} = \begin{bmatrix}512 & 0\\0 & 512\end{bmatrix}$$

$$(A+B)^4 = \begin{bmatrix}8 & 0\\0 & 8\end{bmatrix}\begin{bmatrix}512 & 512\\512 & 512\end{bmatrix} = \begin{bmatrix}4096 & 0\\0 & 4096\end{bmatrix}$$

When $a = 6$ and $b = 6$

$$(A+B)^2 = \left(6\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}+6\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}\right)^2 = \left(\begin{bmatrix}6 & 6\\6 & 6\end{bmatrix}+\begin{bmatrix}6 & -6\\-6 & 6\end{bmatrix}\right)^2 = \begin{bmatrix}12 & 0\\0 & 12\end{bmatrix}^2 = \begin{bmatrix}144 & 0\\0 & 144\end{bmatrix}$$

$$(A+B)^3 = \begin{bmatrix}12 & 0\\0 & 12\end{bmatrix}\begin{bmatrix}144 & 0\\0 & 144\end{bmatrix} = \begin{bmatrix}1728 & 0\\0 & 1728\end{bmatrix}$$

$$(A+B)^4 = \begin{bmatrix}12 & 0\\0 & 12\end{bmatrix}\begin{bmatrix}1728 & 0\\0 & 1728\end{bmatrix} = \begin{bmatrix}20736 & 0\\0 & 20736\end{bmatrix}$$

When $a = 8$ and $b = 8$

$$(A+B)^2 = \left(8\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}+8\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}\right)^2 = \left(\begin{bmatrix}8 & 8\\8 & 8\end{bmatrix}+\begin{bmatrix}8 & -8\\-8 & 8\end{bmatrix}\right)^2 = \begin{bmatrix}16 & 0\\0 & 16\end{bmatrix}^2 = \begin{bmatrix}256 & 0\\0 & 256\end{bmatrix}$$

$$(A+B)^3 = \begin{bmatrix}16 & 0\\0 & 16\end{bmatrix}\begin{bmatrix}256 & 0\\0 & 256\end{bmatrix} = \begin{bmatrix}4096 & 0\\0 & 4096\end{bmatrix}$$

$$(A+B)^4 = \begin{bmatrix}16 & 0\\0 & 16\end{bmatrix}\begin{bmatrix}4096 & 0\\0 & 4096\end{bmatrix} = \begin{bmatrix}65536 & 0\\0 & 65536\end{bmatrix}$$

Now we'll take into consideration values for when *(a)* and *(b)* are different.

When $a = 2$ and $b = 4$

$$(A+B)^2 = \left(2\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}+4\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}\right)^2 = \left(\begin{bmatrix}2 & 2\\2 & 2\end{bmatrix}+\begin{bmatrix}4 & -4\\-4 & 4\end{bmatrix}\right)^2 = \begin{bmatrix}6 & -2\\-2 & 6\end{bmatrix}^2 = \begin{bmatrix}40 & -24\\-24 & 40\end{bmatrix}$$

$$(A+B)^3 = \begin{bmatrix}6 & -2\\-2 & 6\end{bmatrix}\begin{bmatrix}40 & -24\\-24 & 40\end{bmatrix} = \begin{bmatrix}288 & -224\\-224 & 228\end{bmatrix}$$

$$(A+B)^4 = \begin{bmatrix}6 & -2\\-2 & 6\end{bmatrix}\begin{bmatrix}288 & -224\\-224 & 228\end{bmatrix} = \begin{bmatrix}2176 & -1920\\-1920 & 2176\end{bmatrix}$$

When $a = 8$ and $b = 6$

$$(A+B)^2 = \left(8\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}+6\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}\right)^2 = \left(\begin{bmatrix}8 & 8\\8 & 8\end{bmatrix}+\begin{bmatrix}6 & -6\\-6 & 6\end{bmatrix}\right)^2 = \begin{bmatrix}14 & 2\\2 & 14\end{bmatrix}^2 = \begin{bmatrix}200 & 56\\56 & 200\end{bmatrix}$$

$$(A+B)^3 = \begin{bmatrix}14 & 2\\2 & 14\end{bmatrix}\begin{bmatrix}200 & 56\\56 & 200\end{bmatrix} = \begin{bmatrix}2912 & 1184\\1184 & 2912\end{bmatrix}$$

$$(A+B)^4 = \begin{bmatrix}14 & 2\\2 & 14\end{bmatrix}\begin{bmatrix}2912 & 1184\\1184 & 2912\end{bmatrix} = \begin{bmatrix}43136 & 22400\\22400 & 43136\end{bmatrix}$$

To find $(A+B)^n$, we'll look at the data above. The information above suggests that $A^n+B^n = (A+B)^n$. However, we must test this with data from above to verify and confirm if it's true. While testing, we must remember to test for values of *a* and *b* that are both the same and different.

When $a = 2$ and $b = 2$

$(A+B)^2 = \begin{bmatrix} 16 & 0 \\ 0 & 16 \end{bmatrix}$ $(A+B)^3 = \begin{bmatrix} 64 & 0 \\ 0 & 64 \end{bmatrix}$ $(A+B)^4 = \begin{bmatrix} 256 & 0 \\ 0 & 256 \end{bmatrix}$

Verifying using $(A+B)^n = A^n + B^n$

$(A+B)^4 = A^4 + B^4 = \begin{bmatrix} 128 & 128 \\ 128 & 128 \end{bmatrix} + \begin{bmatrix} 128 & -128 \\ -128 & 128 \end{bmatrix} = \begin{bmatrix} 256 & 0 \\ 0 & 256 \end{bmatrix}$

When $a = 4$ and $b = 4$

$(A+B)^2 = \begin{bmatrix} 64 & 0 \\ 0 & 64 \end{bmatrix}$ $(A+B)^3 = \begin{bmatrix} 512 & 0 \\ 0 & 512 \end{bmatrix}$ $(A+B)^4 = \begin{bmatrix} 4096 & 0 \\ 0 & 4096 \end{bmatrix}$

Verifying using $(A+B)^n = A^n + B^n$

$(A+B)^4 = A^4 + B^4 = \begin{bmatrix} 2048 & 2048 \\ 2048 & 2048 \end{bmatrix} + \begin{bmatrix} 2048 & -2048 \\ -2048 & 2048 \end{bmatrix} = \begin{bmatrix} 4096 & 0 \\ 0 & 4096 \end{bmatrix}$

When $a = 6$ and $b = 6$

$(A+B)^2 = \begin{bmatrix} 144 & 0 \\ 0 & 144 \end{bmatrix}$ $(A+B)^3 = \begin{bmatrix} 1728 & 0 \\ 0 & 1728 \end{bmatrix}$ $(A+B)^4 = \begin{bmatrix} 20736 & 0 \\ 0 & 20736 \end{bmatrix}$

Verifying using $(A+B)^n = A^n + B^n$

$(A+B)^4 = A^4 + B^4 = \begin{bmatrix} 10368 & 10368 \\ 10368 & 10368 \end{bmatrix} + \begin{bmatrix} 10368 & -10368 \\ -10368 & 10368 \end{bmatrix} = \begin{bmatrix} 20736 & 0 \\ 0 & 20736 \end{bmatrix}$

When $a = 8$ and $b = 8$

$(A+B)^2 = \begin{bmatrix} 256 & 0 \\ 0 & 256 \end{bmatrix}$ $(A+B)^3 = \begin{bmatrix} 4096 & 0 \\ 0 & 4096 \end{bmatrix}$ $(A+B)^4 = \begin{bmatrix} 65536 & 0 \\ 0 & 65536 \end{bmatrix}$

Verifying using $(A+B)^n = A^n + B^n$

$(A+B)^4 = A^4 + B^4 = \begin{bmatrix} 32768 & 32768 \\ 32768 & 32768 \end{bmatrix} + \begin{bmatrix} 32768 & -32768 \\ -32768 & 32768 \end{bmatrix} = \begin{bmatrix} 65536 & 0 \\ 0 & 65536 \end{bmatrix}$

When $a = 2$ and $b = 4$

(A+B)^2 = $\begin{bmatrix} 40 & -24 \\ -24 & 40 \end{bmatrix}$ (A+B)^3 = $\begin{bmatrix} 288 & -224 \\ -224 & 228 \end{bmatrix}$ (A+B)^4 = $\begin{bmatrix} 2176 & -1920 \\ -1920 & 2176 \end{bmatrix}$

Verifying using (A+B)^n = A^n + B^n

(A+B)^4 = A^4 + B^4 = $\begin{bmatrix} 128 & 128 \\ 128 & 128 \end{bmatrix} + \begin{bmatrix} 2048 & -2048 \\ -2048 & 2048 \end{bmatrix} = \begin{bmatrix} 2176 & -1920 \\ -1920 & 2176 \end{bmatrix}$

When a = 8 and b = 6

(A+B)^2 = $\begin{bmatrix} 200 & 56 \\ 56 & 200 \end{bmatrix}$ (A+B)^3 = $\begin{bmatrix} 2912 & 1184 \\ 1184 & 2912 \end{bmatrix}$ (A+B)^4 = $\begin{bmatrix} 43136 & 22400 \\ 22400 & 43136 \end{bmatrix}$

Verifying using (A+B)^n = A^n + B^n

(A+B)^4 = A^4 +B^4 = $\begin{bmatrix} 32768 & 32768 \\ 32768 & 32768 \end{bmatrix} + \begin{bmatrix} 10386 & -10368 \\ -10368 & 10368 \end{bmatrix} = \begin{bmatrix} 43136 & 22400 \\ 22400 & 43136 \end{bmatrix}$

Ultimately, from the information above and the following verification, we can conclude that the formula for (A+B)^n is *(A+B)^n = A^n + B^n*.

Finally, we'll consider *M*. The value for *M* is $\begin{bmatrix} a+b & a-b \\ a-b & a+b \end{bmatrix}$. Although it is given that M = A + B, we must come to this conclusion using data and calculated information. Hence, since A = aX and B = bY, we must remember that the values for a and b are constants and we must consider that can be both the same and different. For this reason, we'll test multiple values for a and b to come to the conclusion that M = $\begin{bmatrix} a+b & a-b \\ a-b & a+b \end{bmatrix}$.

When a = 2 and b = 2

Using the formula for M, this would translate to $\begin{bmatrix} 2+2 & 2-2 \\ 2-2 & 2+2 \end{bmatrix}$ which equals $\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$.

Using the equation M = A + B

$$M = 2\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + 2\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

When a = 4 and b = 4

Using the formula for M, this would translate to $\begin{bmatrix} 4+4 & 4-4 \\ 4-4 & 4+4 \end{bmatrix}$ which equals $\begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$.

Using the equation M = A + B

$$M = 4\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + 4\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix} + \begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$$

When $a = 2$ and b = 4

Using the formula for M, this would translate to $\begin{bmatrix} 2+4 & 2-4 \\ 2-4 & 2+4 \end{bmatrix}$ which equals $\begin{bmatrix} 6 & -2 \\ -2 & 6 \end{bmatrix}$

Using the equation M = A + B

$$M = 2\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + 4\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix} = \begin{bmatrix} 6 & -2 \\ -2 & 6 \end{bmatrix}$$

When $a = 8$ and $b = 6$

Using the formula for M, this would translate to $\begin{bmatrix} 8+6 & 8-6 \\ 8-6 & 8+6 \end{bmatrix}$ which equals $\begin{bmatrix} 14 & 2 \\ 2 & 14 \end{bmatrix}$

Using the equation M = A + B

$$M = 8\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + 6\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix} + \begin{bmatrix} 6 & -6 \\ -6 & 6 \end{bmatrix} = \begin{bmatrix} 14 & 2 \\ 2 & 14 \end{bmatrix}$$

From this, we can conclude that the equation M = A + B is true because of the values tested and confirmed above.

This leads us to consider M^2. Since M = $\begin{bmatrix} a+b & a-b \\ a-b & a+b \end{bmatrix}$, then $M^2 = \begin{bmatrix} a+b & a-b \\ a-b & a+b \end{bmatrix}^2$.

To find the simplified value of M^2, multiply it out. This translates to $\begin{bmatrix} a+b & a-b \\ a-b & a+b \end{bmatrix}\begin{bmatrix} a+b & a-b \\ a-b & a+b \end{bmatrix}$.

When multiplying matrices, one must add up the products of corresponding rows and columns. However, dimensions must match. If two matrices were represented by $\begin{bmatrix} R1 & R2 \\ R3 & R4 \end{bmatrix}$ and $\begin{bmatrix} C1 & C2 \\ C3 & C4 \end{bmatrix}$, then the multiplied matrix would equal $\begin{bmatrix} (R1)(C1)+(R2)(C3) & (R1)(C2)+(R1)(C4) \\ (R2)(C1)+(R2)(C3) & (R2)(C2)+(R2)(C4) \end{bmatrix}$.

When using this information regarding M^2, we see that

$$M^2 = \begin{bmatrix} a+b & a-b \\ a-b & a+b \end{bmatrix} \begin{bmatrix} a+b & a-b \\ a-b & a+b \end{bmatrix}.$$

Multiplying this out would result in:

$$\begin{bmatrix} (a+b)^2+(a-b)^2 & (a+b)(a-b)+(a-b)(a+b) \\ (a+b)(a-b)+(a-b)(a+b) & (a+b)^2+(a-b)^2 \end{bmatrix}$$

After multiplying this out, the result is the 2x2 matrix

$$\begin{bmatrix} 2a^2+2b^2 & 2a^2-2b^2 \\ 2a^2-2b^2 & 2a^2+2b^2 \end{bmatrix}$$

Now that we have the simplified version of M^2, we can use it to show that $M^2 = A^2 + B^2$. However, since A = aX and B = bY, we must remember that the values for *a* and *b* are constants and we must consider that can be both the same and different. For this reason, we'll test multiple values for *a* and *b* to come to the conclusion that

$$M^2 = \begin{bmatrix} 2a^2+2b^2 & 2a^2-2b^2 \\ 2a^2-2b^2 & 2a^2+2b^2 \end{bmatrix}$$

When $a = 2$ and $b = 2$

Using the formula for M, this would translate to

$$\begin{bmatrix} 2(2)^2+2(2)^2 & 2(2)^2-2(2)^2 \\ 2(2)^2-2(2)^2 & 2(2)^2+2(2)^2 \end{bmatrix} = \begin{bmatrix} 16 & 0 \\ 0 & 16 \end{bmatrix}$$

Verifying using $M^2 = A^2 + B^2$

$$M^2 = \begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix} + \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix} = \begin{bmatrix} 16 & 0 \\ 0 & 16 \end{bmatrix}$$

When $a = 8$ and $b = 8$

Using the formula for M, this would translate to

$$\begin{bmatrix} 2(8)^2 + 2(8)^2 & 2(8)^2 - 2(8)^2 \\ 2(8)^2 - 2(8)^2 & 2(8)^2 + 2(8)^2 \end{bmatrix} = \begin{bmatrix} 256 & 0 \\ 0 & 256 \end{bmatrix}$$

Verifying using $M^2 = A^2 + B^2$

$$M^2 = \begin{bmatrix} 128 & 128 \\ 128 & 128 \end{bmatrix} + \begin{bmatrix} 128 & -128 \\ -128 & 128 \end{bmatrix} = \begin{bmatrix} 256 & 0 \\ 0 & 256 \end{bmatrix}$$

When $a = 2$ and $b = 4$

Using the formula for M, this would translate to

$$\begin{bmatrix} 2(2)^2 + 2(8)^2 & 2(2)^2 - 2(8)^2 \\ 2(2)^2 - 2(8)^2 & 2(2)^2 + 2(8)^2 \end{bmatrix} = \begin{bmatrix} 136 & -120 \\ -120 & 136 \end{bmatrix}$$

Verifying using $M^2 = A^2 + B^2$

$$M^2 = \begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix} + \begin{bmatrix} 128 & -128 \\ -128 & 128 \end{bmatrix} = \begin{bmatrix} 136 & -120 \\ -120 & 136 \end{bmatrix}$$

When $a = 8$ and $b = 6$

Using the formula for M, this would translate to

$$\begin{bmatrix} 2(8)^2 + 2(6)^2 & 2(8)^2 - 2(6)^2 \\ 2(8)^2 - 2(6)^2 & 2(8)^2 + 2(6)^2 \end{bmatrix} = \begin{bmatrix} 200 & 56 \\ 56 & 200 \end{bmatrix}$$

Verifying using $M^2 = A^2 + B^2$

$$M^2 = \begin{bmatrix} 128 & 128 \\ 128 & 128 \end{bmatrix} + \begin{bmatrix} 72 & -72 \\ -72 & 72 \end{bmatrix} = \begin{bmatrix} 200 & 56 \\ 56 & 200 \end{bmatrix}$$

Thus, we can conclude that $M^2 = A^2 + B^2$ because all of the values above have been the same whether it was the variable version of M^2, $\begin{bmatrix} 2a^2+2b^2 & 2a^2-2b^2 \\ 2a^2-2b^2 & 2a^2+2b^2 \end{bmatrix}$, or plugging into the equation $M^2 = A^2 + B^2$

Finally, we'll look at the value of M^n. To find the equation for M^n, we can look at the equation of M^2. When comparing M^n to M^2 we can see that n=2. Using the connection established, the equation $M^2 = A^2 + B^2$ can be translated to $M^n = A^n + B^n$. However, since M is a matrix expressed in terms of values of *(a)* and *(b)*, matrix M^n must also be expressed in terms of *(a)* and *(b)*. Since there is a connection between M^2 and M^n, being that n=2, we can connect the two equations once again. Since the previous equation for $(A+B)^n$ came to be $A^n + B^n$, this reinforces that $M^n = A^n + B^n$. However, relating this back to the original matrix form of M, we must consider that the original formula for M was M $=\begin{bmatrix} a+b & a-b \\ a-b & a+b \end{bmatrix}$. Therefore, M^n must equal $\begin{bmatrix} a+b & a-b \\ a-b & a+b \end{bmatrix}^n$. However, this translates to equal the same as $A^n + B^n$ because it is the same as $M^n = (a^n)(2^{n-1})(X) + (b^n)(2^{n-1})(Y)$. Nonetheless we must still test this with values to verify and confirm, and once again, we must take into consideration values of *(a)* and *(b)* that are both the same and different, and we can use both equations to make sure.

When $a = 2$ and $b = 2$

Using the equation $M^n = (a^n)(2^{n-1})(X) + (b^n)(2^{n-1})(Y)$.

$$M^2 = (2^2)(2)\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + (2^2)(2)\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 16 & 0 \\ 0 & 16 \end{bmatrix}$$

Verify using the $M^n = \begin{bmatrix} a+b & a-b \\ a-b & a+b \end{bmatrix}^n$

$$M^2 = \begin{bmatrix} 2+2 & 2-2 \\ 2-2 & 2+2 \end{bmatrix}^2 = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 16 & 0 \\ 0 & 16 \end{bmatrix}$$

When $a = 8$ and $b = 8$

Using the equation $M^n = (a^n)(2^{n-1})(X) + (b^n)(2^{n-1})(Y)$.

$$M^2 = (8^2)(128)\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + (8^2)(128)\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 256 & 0 \\ 0 & 256 \end{bmatrix}$$

Verify using the $M^n = \begin{bmatrix} a+b & a-b \\ a-b & a+b \end{bmatrix}^n$

$$M^2 = \begin{bmatrix} 8+8 & 8-8 \\ 8-8 & 8+8 \end{bmatrix}^2 = \begin{bmatrix} 256 & 0 \\ 0 & 256 \end{bmatrix}$$

When $a = 8$ and $b = 6$

Using the equation $M^n = (a^n)(2^{n-1})(X) + (b^n)(2^{n-1})(Y)$.

$$M^2 = (8^2)(128)\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + (6^2)(32)\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 9344 & 7040 \\ 7040 & 9344 \end{bmatrix}$$

Verify using the $M^n = \begin{bmatrix} a+b & a-b \\ a-b & a+b \end{bmatrix}^n$

$$M^2 = \begin{bmatrix} 8+6 & 8-6 \\ 8-6 & 8+6 \end{bmatrix}^2 = \begin{bmatrix} 9344 & 7040 \\ 7040 & 9344 \end{bmatrix}$$

From this, we can consequently conclude that $M^n = \begin{bmatrix} a+b & a-b \\ a-b & a+b \end{bmatrix}^n$ because when plugged in for various values of *(a)* and various values of *(b)*. The equation confirms and verifies to be true because we verified it using two equations that both equal $M^n = A^n + B^n$.

Ultimately, the nine goals were reached based off calculations of the initial values of *X* and *Y* and all values and calculations were proven using various equations to verify and confirm