IBO Internal Assessment

Mathematics SL Type 1

Matrix Binomials

$$x = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \quad y = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

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This assignment of my portfolio is to allow me to deal with matrix binomials and to investigate them through finding a series of statements.

Through my knowledge of algebra, matrices and sequences I will try to investigate these affirmations to find any relationships or patterns.

Through the use of both my TI 83 scientific calculator and the math type program.

Introduction

Matrix comes from a Latin word that means womb; and so where something is formed and produced. A matrix is a rectangular arrangement of numbers.[i]

The term "matrix" was thought up by some very famous mathematicians such as J. J. Sylvester. Cayley, Hamilton, Grassmann, Frobenius and von Neumann who helped with the development of matrices in 1848.

Since their first appearances long ago in ancient China (650 BC), they have remained very important mathematical tools. They are used for general arithmetic, in quantum mechanics, engineering, dance routines and many other areas which are surely unexpected.

$$x = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} y = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \qquad XY = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \qquad X + Y = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

Knowing the values of X and Y, I can then go on to calculate X^2, X^3, X^4 and Y^2, Y^3, Y^4.

In order to facilitate all my workings out, I will do this on my graphical display calculator.

X^2

$$X^2 = X * X$$

$$\therefore X^2 = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$X^2 = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}$$

Y^2

$$Y^2 = Y * Y$$

$$\therefore Y^2 = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

$$Y^2 = \begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}$$

X^3

$$X^3 = X^2 * X$$

$$\therefore X^3 = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$X^3 = \begin{pmatrix} 4 & 4 \\ 4 & 4 \end{pmatrix}$$

Y^3

$$Y^3 = Y^2 * Y$$

$$\therefore Y^3 = \begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

$$Y^3 = \begin{pmatrix} 4 & -4 \\ -4 & 4 \end{pmatrix}$$

X^4

$$X^4 = X^3 * X$$

$$\therefore X^3 = \begin{pmatrix} 4 & 4 \\ 4 & 4 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$X^3 = \begin{pmatrix} 8 & 8 \\ 8 & 8 \end{pmatrix}$$

Y^4

$$Y^4 = Y^3 * Y$$

$$\therefore Y^4 = \begin{pmatrix} 4 & -4 \\ -4 & 4 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

$$Y^4 = \begin{pmatrix} 8 & -8 \\ -8 & 8 \end{pmatrix}$$

From these calculations we can easily state that there is a pattern within the results of the variations of X and Y when there is an increase in the power of the matrix. I can clearly state that there is a relationship between the power of the matrix and the end product of the matrices.

$$X^2 / Y^2 \rightarrow 2$$

$$X^3 / Y^3 \rightarrow 4$$

For both the variables we get the same results except for the negative signs that represent inverse matrices depending on the initial matrix. But they always take upon the same position so it doesn't influence our answers. Through these results I see that a way to find a general statement for both X and Y I could use my knowledge of sequences and series along with that of matrices.

Considering integer powers of X and Y, we can find expressions for $X^n, Y^n, (X+Y)^n$.

Looking at both the matrices values, I realized that we were dealing with geometric sequences. Before following up with these I can simply move forward form the calculations on the previous page to continue towards finding these generalizations.

$$X^n$$

$$X^n = X^{n-1} * X$$

Taking into consideration $8 \div 4 = 2$ to give the constant value.

$$\therefore X^n = 2^{n-1} X$$

$$Y^n$$

$$Y^n = Y^{n-1} * Y$$

$$\therefore Y^n = 2^{n-1} Y$$

I must note that the value gives us only the progression for the scalar values and so when being multiplied to the matrix. It is necessary to find the final expressions that we multiply by 2^{n-1}.

In order to test the validity, we can try to put together a geometric sequence and try to put together a general statement like the one above.

$U^n =$ a term
$U^1 =$ first term
$r =$ common ratio
$n =$ the term related to sequence

$$x = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$X^n$$

$n = 1, 2, 3, 4$ corresponding 1,2,4,8

sequence is then $\rightarrow$ 1,2,4,8

$U^1 = 1$

$$Un = U_1 r^{n-1}$$

$$Un = 1 * 2^{n-1}$$

$$Un = 2^{n-1}$$

$$\therefore X^n = 2^{n-1} X$$

$$\rightarrow \begin{pmatrix} 2^{n-1} & 2^{n-1} \\ 2^{n-1} & 2^{n-1} \end{pmatrix}$$

$r = 8 \div 4 / 4 \div 2 \rightarrow r = 2$

Whilst for the Y matrix it would be slightly different as a cause to the negative signs of the inverse matrix.

$$\rightarrow \begin{pmatrix} 2^{n-1} & -2^{n-1} \\ -2^{n-1} & 2^{n-1} \end{pmatrix}$$

Through geometric sequences, I can see that what I had put together was actually correct. Now I am going to try and use different examples of integers to see if it really works most cases.

X^4

Using my Gdc, and from my calculations in page 3. Then using the general formula that I have put together we can see that this is correct.

$$X^4 = \begin{pmatrix} 8 & 8 \\ 8 & 8 \end{pmatrix}$$

$$X^n = 2^{n-1} X \longrightarrow X^4 = 2^{4-1}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$X^4 = 2^3\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$X^4 = 8\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\therefore X^4 \begin{pmatrix} 8 & 8 \\ 8 & 8 \end{pmatrix}$$

X^5

$$X^5 = \begin{pmatrix} 16 & 16 \\ 16 & 16 \end{pmatrix} \text{ Gdc}$$

$$X^n = 2^{n-1} X \longrightarrow X^5 = 2^{5-1}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$X^5 = 2^4\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$X^5 = 16\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\therefore X^5 = \begin{pmatrix} 16 & 16 \\ 16 & 16 \end{pmatrix}$$

As we already found the geometric sequence we can only assume that the values concerning Y also are correct. Here are just two examples to make sure we are getting our patterns correctly.

Y^4

$$Y^4 = \begin{pmatrix} 8 & -8 \\ -8 & 8 \end{pmatrix}$$

$$Y^4 = 2^{4-1}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

$$Y^4 = 2^3\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

$$Y^4 = 8\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

$$\therefore Y^4 \begin{pmatrix} 8 & -8 \end{pmatrix}$$

Y^5

$$Y^5 = \begin{pmatrix} 16 & -16 \\ -16 & 16 \end{pmatrix}$$

$$Y^5 = 2^{5-1}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

$$Y^5 = 2^4\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

$$Y^5 = 16\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

$$\therefore Y^5 = \begin{pmatrix} 16 & -16 \end{pmatrix}$$

Gdc

$Y^n = 2^{n-1}Y \longrightarrow$ $\qquad$ $Y^n = 2^{n-1}Y \longrightarrow$

Therefore, knowing $X^n = 2^{n-1}X$ **and** $Y^n = 2^{n-1}Y$ **, we can now find** $(X+Y)^n$ **.**

Assuming $(X+Y)^n = 2^{n-1}\left(X+Y\right)$

We can take as an example:

$n = 2$

$(X+Y)^2 = 2^{2-1}\left(X+Y\right)$

$(X+Y)^2 = 2\left(X+Y\right)$

$(X+Y)^2 = 2X+2Y$

$\left(\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}+\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}\right)^2 = 2\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}+2\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}$

$\begin{pmatrix}2 & 0\\ 0 & 2\end{pmatrix}^2 = \begin{pmatrix}2 & 2\\ 2 & 2\end{pmatrix}+\begin{pmatrix}2 & -2\\ -2 & 2\end{pmatrix}$

$\begin{pmatrix}4 & 0\\ 0 & 4\end{pmatrix} = \begin{pmatrix}4 & 0\\ 0 & 4\end{pmatrix} \rightarrow$ identity matrix of 2x2 (I)

$\therefore 4\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} = 2^n\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

$\therefore (X+Y)^2 = 2^1\left(2I\right)$

$\therefore$ since we know $X^n = 2^{n-1}X, Y^n = 2^{n-1}Y$

$(X+Y)^n = 2^{n-1}\left(2I\right)$

$n = 3$

$(X+Y)^3 = \begin{pmatrix}8 & 0\\ 0 & 8\end{pmatrix}$ Check on gdc.

Let's check our general statement for $(X+Y)^n$ with the following example.

$(X+Y)^n = 2^{n-1}\left(2I\right)$

$(X+Y)^3 = 2^{3-1}(2I)$

$(X+Y)^3 = 2^2\left(2I\right)$

$(X+Y)^3 = 4*2\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

$(X+Y)^3 = 8\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

$(X+Y)^3 = \begin{pmatrix}8 & 0\end{pmatrix}$

Letting $A = aX$ and $B = bY$ when $\begin{matrix} a \\ b \end{matrix}$ are constants: By different values of these constants we can find the following: $A^2, A^3, A^4, B^2, B^3, B^4$.

A

With this I am going to change the constants of a couple times to calculate the other components necessary. With the use of my gdc.

$A = aX$

$$x = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

<u>Constant -10</u>

A^2, A^3, A^4

$$A = -10X$$

$$A = -10\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \rightarrow$$

$$A = \begin{pmatrix} -10 & -10 \\ -10 & -10 \end{pmatrix}$$

$$A^2 = \left(-10X\right)^2$$

$$A^2 = -10\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}^2 \rightarrow$$

$$A^2 = \begin{pmatrix} -10 & -10 \\ -10 & -10 \end{pmatrix}^2$$

$$A^2 = \begin{pmatrix} 200 & 200 \\ 200 & 200 \end{pmatrix}$$

$$A^3 = \left(-10X\right)^3$$

$$A^3 = -10\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}^3$$

$$A^3 = \begin{pmatrix} -10 & -10 \\ -10 & -10 \end{pmatrix}^3 \rightarrow$$

$$A^3 = \begin{pmatrix} -4000 & -4000 \\ -4000 & -4000 \end{pmatrix}$$

$$A^4 = \left(-10X\right)^4$$

$$A^4 = -10\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}^4$$

$$A^4 = \begin{pmatrix} -10 & -10 \\ -10 & -10 \end{pmatrix}^4$$

$$A^4 = \begin{pmatrix} 80000 & 80000 \\ 80000 & 80000 \end{pmatrix}$$

<u>Constant 5</u>

A^2, A^3, A^4

$$A = 5X$$

$$A = 5\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \rightarrow$$

$$A = \begin{pmatrix} 5 & 5 \end{pmatrix}$$

$$A^2 = \left(5X\right)^2$$

$$A^2 = 5\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}^2$$

$$A^2 = \begin{pmatrix} 5 & 5 \\ 5 & 5 \end{pmatrix}^2 \rightarrow$$

$$A^2 = \begin{pmatrix} 50 & 50 \end{pmatrix}$$

$$A^3 = \left(5X\right)^3$$

$$A^3 = 5\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}^3$$

$$A^3 = \begin{pmatrix} 5 & 5 \\ 5 & 5 \end{pmatrix}^3 \rightarrow$$

$$A^3 = \begin{pmatrix} 500 & 500 \end{pmatrix}$$

$$A^4 = \left(5X\right)^4$$

$$A^4 = 5\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}^4$$

$$A^4 = \begin{pmatrix} 5 & 5 \\ 5 & 5 \end{pmatrix}^4$$

$$A^4 = \begin{pmatrix} 5000 & 5000 \end{pmatrix}$$

Constant 18

A^2, A^3, A^4

$A = 18X$

$A = 18\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

$A = \begin{pmatrix} 18 & 18 \\ 18 & 18 \end{pmatrix}$

$A^2 = |18X|^2$

$A^2 = 18\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}^2$

$A^2 = \begin{pmatrix} 18 & 18 \\ 18 & 18 \end{pmatrix}^2$

$A^2 = \begin{pmatrix} 648 & 648 \\ 648 & 648 \end{pmatrix}$

$A^3 = |18X|^3$

$A^3 = 18\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}^3$

$A^3 = \begin{pmatrix} 18 & 18 \\ 18 & 18 \end{pmatrix}^3$

$A^3 = \begin{pmatrix} 23328 & 23328 \\ 23328 & 23328 \end{pmatrix}$

$A^4 = |18X|^4$

$A^4 = 18\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}^4$

$A^4 = \begin{pmatrix} 18 & 18 \\ 18 & 18 \end{pmatrix}^4$

$A^4 = \begin{pmatrix} 839808 & 839808 \\ 839808 & 839808 \end{pmatrix}$

We substitute powers instead of $\rightarrow \begin{pmatrix} 2^{n-1} & 2^{n-1} \\ 2^{n-1} & 2^{n-1} \end{pmatrix}$ numbers.

$$X = 2^{n-1} X$$

$$a^n = 2^{n-1} X$$

$$\therefore a^n = a^n 2^{n-1} X$$

With this general statement made upon a I can see if with another example it works.

a Constant= 4

$$a^n = a^n 2^{n-1} X$$

$$a^4 = 4^4 2^{4-1} X$$

$$a^4 = 4^4 * 2^3 \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$a^4 = \begin{pmatrix} 2048 & 2048 \\ 2048 & 2048 \end{pmatrix}$$

$$B$$

$B = bY$

$b = \text{constant}$

$$y = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

Constant 2

$$B^2, B^3, B^4$$

$B = 2Y$

$$B = 2\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

$$B = \begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}$$

$$\rightarrow$$

$$B^2 = \left| 2Y \right|^2$$

$$B^2 = 2\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}^2$$

$$B^2 = \begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}^2$$

$$B^2 = \begin{pmatrix} 8 & -8 \\ -8 & 8 \end{pmatrix}$$

$$\rightarrow$$

$$B^3 = \left| 2Y \right|^3$$

$$B^3 = 2\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}^3$$

$$B^3 = \begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}^3$$

$$B^3 = \begin{pmatrix} 32 & -32 \\ -32 & 32 \end{pmatrix}$$

$$\rightarrow$$

$$B^4 = \left| 2Y \right|^4$$

$$B^4 = 2\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}^4$$

$$B^4 = \begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}^4$$

$$B^4 = \begin{pmatrix} 128 & -128 \\ -128 & 128 \end{pmatrix}$$

Constant 5

$$B^2, B^3, B^4$$

$B = 5Y$

$$B = 5\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

$$B = \begin{pmatrix} 5 & -5 \\ -5 & 5 \end{pmatrix}$$

$$\rightarrow$$

$$B^2 = \left| 5Y \right|^2$$

$$B^2 = 5\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}^2$$

$$B^2 = \begin{pmatrix} 5 & -5 \\ -5 & 5 \end{pmatrix}^2$$

$$B^2 = \begin{pmatrix} 50 & -50 \\ -50 & 50 \end{pmatrix}$$

$$\rightarrow$$

$$B^3 = \left| 5Y \right|^3$$

$$B^3 = 5\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}^3$$

$$B^3 = \begin{pmatrix} 5 & -5 \\ -5 & 5 \end{pmatrix}^3$$

$$B^3 = \begin{pmatrix} 500 & -500 \\ -500 & 500 \end{pmatrix}$$

$$\rightarrow$$

$$B^4 = \left| 5Y \right|^4$$

$$B^4 = 5\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}^4$$

$$B^4 = \begin{pmatrix} 5 & -5 \\ -5 & 5 \end{pmatrix}^4$$

$$B^4 = \begin{pmatrix} 5000 & -5000 \\ -5000 & 5000 \end{pmatrix}$$

Constant 10

$$B^2, B^3, B^4$$

$$B^2 = \left(10Y\right)^2$$

$$B = 10Y \quad B^2 = 10\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}^2$$

$$B = 10\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \quad B^2 = \begin{pmatrix} 10 & -10 \\ -10 & 10 \end{pmatrix}^2$$

$$B = \begin{pmatrix} 10 & -10 \\ -10 & 10 \end{pmatrix} \quad B^2 = \begin{pmatrix} 200 & -200 \\ -200 & 200 \end{pmatrix}$$

$$\rightarrow$$

$$B^3 = \left(10Y\right)^3$$

$$B^3 = 10\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}^3$$

$$B^3 = \begin{pmatrix} 10 & -10 \\ -10 & 10 \end{pmatrix}^3$$

$$B^3 = \begin{pmatrix} 4000 & -4000 \\ -4000 & 4000 \end{pmatrix}$$

$$\rightarrow$$

$$B^4 = \left(10Y\right)^4$$

$$B^4 = 10\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}^4$$

$$B^4 = \begin{pmatrix} 10 & -10 \\ -10 & 10 \end{pmatrix}^4$$

$$B^4 = \begin{pmatrix} 80000 & -80000 \\ -80000 & 80000 \end{pmatrix}$$

$$\rightarrow$$

$$Y^n = \begin{pmatrix} 2^{n-1} & 2^{n-1} \\ 2^{n-1} & 2^{n-1} \end{pmatrix}$$

$$\rightarrow Y = 2^{n-1} Y$$

$$\therefore B^n = 2^{n-1} Y$$

$$B = B^n 2^{n-1} Y$$

We can use as an example the constant value=4.

$$B = B^n 2^{n-1} Y$$

$$B^4 = 4^4 2^{4-1} X$$

$$B^4 = 4^4 * 2^3 \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

$$B^4 = \begin{pmatrix} 2048 & -2048 \\ -2048 & 2048 \end{pmatrix}$$

By considering Integer powers of A and B, we can find expressions for $A^n, B^n, \left(A+B\right)^n$.

Knowing the values of A^n, B^n, I can now deduce that of $\left(A+B\right)^n$.

$$A = a^n 2^{n-1} X$$

$$B = a^n 2^{n-1} Y$$

$$\left(A+B\right)^n \rightarrow \left(A+B\right)^2 = \left(A+B\right) + \left(A+B\right)$$

$$\left(A+B\right)^n \rightarrow \left(a^n 2^{n-1} X + a^n 2^{n-1} Y\right)$$

$$\left(A+B\right)^n = A^n + B^n \quad \text{(in terms of } X, Y\text{)}$$

$$\mapsto \left(A+B\right)^n = (AX)^n + (BY)^n$$

$$\left(A+B\right)^n = (AX+BY)^n$$

Giving an example with the following constants, I can prove my general statement.

$$a = 2, b = 5, n = 3$$

$$\left(A+B\right)^n = (aX+bY)^n$$

$$\left(A+B\right)^n = (2X+5Y)^3$$

$$\left(A+B\right)^n = (2\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} + 5\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix})^3$$

$$\left(A+B\right)^n = \left(\begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix} + \begin{pmatrix} 5 & -5 \\ -5 & 5 \end{pmatrix}\right)^3$$

$$\left(A+B\right)^n = \begin{pmatrix} 7 & -3 \\ -3 & 7 \end{pmatrix}^3$$

$$\left(A+B\right)^n = \begin{pmatrix} 532 & -468 \\ -468 & 532 \end{pmatrix}$$

In general:

$$\therefore \left| A+B \right|^{n} = (aX+bY)^{n}$$

Considering $M=\begin{pmatrix} a+b & a-b \\ a-b & a+b \end{pmatrix}$ which is an inverse matrix is clearly. I can show that $M=A+B$ and $M^2=A^2+B^2$. (A and B remain constants)

$$M = A+B$$

$$M=\begin{pmatrix} a+b & a-b \\ a-b & a+b \end{pmatrix}$$

$$\therefore A+B=\begin{pmatrix} a+b & a-b \\ a-b & a+b \end{pmatrix}$$

$$AX+BY=\begin{pmatrix} a+b & a-b \\ a-b & a+b \end{pmatrix}$$

$$A\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}+B\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}=\begin{pmatrix} a+b & a-b \\ a-b & a+b \end{pmatrix}$$

$$\begin{pmatrix} a & a \\ a & a \end{pmatrix}\begin{pmatrix} b & -b \\ -b & b \end{pmatrix}=\begin{pmatrix} a+b & a-b \\ a-b & a+b \end{pmatrix}$$

$$\therefore M = A+B$$

With this we can now give an example.

$$M = A+B$$

$$a=2, b=3$$

$$M=\begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}+\begin{pmatrix} 3 & -3 \\ -3 & 3 \end{pmatrix}=\begin{pmatrix} 5 & -1 \\ -1 & 5 \end{pmatrix} gdc$$

$$a=10, b=5$$

$$M=\begin{pmatrix} 10 & 10 \\ 10 & 10 \end{pmatrix}+\begin{pmatrix} 5 & -5 \\ -5 & 5 \end{pmatrix}=\begin{pmatrix} 15 & -5 \\ -5 & 15 \end{pmatrix} gdc$$

$$M = A + B \rightarrow M = aX + bY \rightarrow M = \begin{pmatrix} a+b & a-b \\ a-b & a+b \end{pmatrix}$$

Similarly $M^2 \rightarrow M^2 = A^2 + B^2$

$$\begin{pmatrix} a+b & a-b \\ a-b & a+b \end{pmatrix}^2 = A^2 + B^2$$

$$\begin{pmatrix} a+b & a-b \\ a-b & a+b \end{pmatrix}\begin{pmatrix} a+b & a-b \\ a-b & a+b \end{pmatrix} = (aX)^2 + (bY)^2$$

$$\begin{pmatrix} (a+b)(a+b)+(a-b)(a-b) & (a+b)(a-b)+(a-b)(a+b) \\ (a-b)(a+b)+(a+b)(a-b) & (a-b)(a-b)+(a+b)(a+b) \end{pmatrix} = \begin{pmatrix} a & a \\ a & a \end{pmatrix}\begin{pmatrix} a & a \\ a & a \end{pmatrix} + \begin{pmatrix} b & -b \\ -b & b \end{pmatrix}\begin{pmatrix} b & -b \\ -b & b \end{pmatrix}$$

$$\begin{pmatrix} a^2+2ab+b^2+a^2-2ab+b^2 & a^2-b^2+a^2-b^2 \\ a^2-b^2+a^2-b^2 & a^2-2ab+b^2+a^2+2ab+b^2 \end{pmatrix} = \begin{pmatrix} a(a)+a(a) & a(a)+a(a) \\ a(a)+a(a) & a(a)+a(a) \end{pmatrix} + \begin{pmatrix} b(b)+(-b)(-b) & (b)(-b)+(-b)(b) \\ (-b)(b)+(b)(-b) & (-b)(-b)+b(b) \end{pmatrix}$$

$$\begin{pmatrix} 2a^2+2b^2 & 2a^2-2b^2 \\ 2a^2-2b^2 & 2a^2+2b^2 \end{pmatrix} = \begin{pmatrix} 2a^2 & 2a^2 \\ 2a^2 & 2a^2 \end{pmatrix} + \begin{pmatrix} 2b^2 & -2b^2 \\ -2b^2 & 2b^2 \end{pmatrix}$$

$$\begin{pmatrix} 2a^2+2b^2 & 2a^2-2b^2 \\ 2a^2-2b^2 & 2a^2+2b^2 \end{pmatrix} = \begin{pmatrix} 2a^2+2b^2 & 2a^2-2b^2 \\ 2a^2-2b^2 & 2a^2+2b^2 \end{pmatrix}$$

$$M^2 = A^2 + B^2$$

The general statement for M^n in terms of aX, bY.

$$M = A + B \qquad M^2 = A^2 + B^2$$

$$M = \begin{pmatrix} a+b & a-b \\ a-b & a+b \end{pmatrix} \qquad M^2 = \begin{pmatrix} 2a^2+2b^2 & 2a^2-2b^2 \\ 2a^2-2b^2 & 2a^2+2b^2 \end{pmatrix}$$

$$M^n = A^n + B^n$$

$$M^n = (aX)^n + (bY)^n$$

$$M^n = A^n + B^n$$

$$M^n = a^n 2^{n-1} X + b^n 2^{n-1} Y$$

$$M^n = a^n 2^{n-1}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} + b^n 2^{n-1}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

Now, with a series of examples I am going to try and prove if this formula could be used in general for all types of integers and numbers or if there are any limitations in question.

$a=4, b=6, n=3$
$a=10, b=-5, n=2$
$a=-2, b=-8, n=4$
$a=-8, b=2, c=6$

$$M^n = a^n 2^{n-1}\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix} + b^n 2^{n-1}\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}$$

$a=4, b=6, n=3$
$a=-2, b=-8, n=4$

$$M^n = 4^3 2^{3-1}\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix} + 6^3 2^{3-1}\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}$$

$$M^n = 4^3 2^2\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix} + 6^3 2^2\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}$$

$$M^n = 256\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix} + 864\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}$$

$$M^n = \begin{pmatrix}256 & 256\\ 256 & 256\end{pmatrix} + \begin{pmatrix}864 & -864\\ -864 & 864\end{pmatrix}$$

$$M^n = \begin{pmatrix}1120 & -608\\ -608 & 1120\end{pmatrix}$$

$$M^n = -2^4 2^{4-1}\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix} + -8^4 2^{4-1}\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}$$

$$M^n = -2^4 2^3\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix} + -8^4 2^3\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}$$

$$M^n = 128\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix} + 32768\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}$$

$$M^n = \begin{pmatrix}128 & 128\\ 128 & 128\end{pmatrix} + \begin{pmatrix}32768 & -32768\\ -32768 & 32768\end{pmatrix}$$

$$M^n = \begin{pmatrix}32896 & -32640\\ -32640 & 32896\end{pmatrix}$$

$a=10, b=-5, n=2$
$a=-8, b=2, c=10$

$$M^n = 10^2 2^{2\ 1}\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix} + -5^2 2^{2\ 1}\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}$$

$$M^n = 10^2 2^1\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix} + -5^2 2^1\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}$$

$$M^n = 200\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix} + 50\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}$$

$$M^n = \begin{pmatrix}200 & 200\\ 200 & 200\end{pmatrix} + \begin{pmatrix}50 & -50\\ -50 & 50\end{pmatrix}$$

$$^{n} = \begin{pmatrix}250 & 150\\ 150 & 250\end{pmatrix}$$

$$M^n = -8^6 2^{6-1}\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix} + 2^6 2^{6-1}\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}$$

$$M^n = -8^6 2^5\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix} + 2^6 2^5\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}$$

$$M^n = -8388608\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix} + 2048\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}$$

$$M^n = \begin{pmatrix}-8388608 & -8388608\\ -8388608 & -8388608\end{pmatrix} + \begin{pmatrix}2048 & -2048\\ -2048 & 2048\end{pmatrix}$$

$$M^n = \begin{pmatrix}-8386560 & -8390656\\ -8390656 & -8386560\end{pmatrix}$$

Expressions

$$X^n = 2^{n-1}X$$

$$Y^n = 2^{n-1}Y$$

$$|X+Y|^n = 2^{n-1}(2I)$$

$$A^n = a^n 2^{n-1}X$$

$$B^n = b^n 2^{n-1}Y$$

$$(A+B)^n = (aX+bY)^n$$

$$M^n = a^n 2^{n-1}X + b^n 2^{n-1}Y$$

The a and b constants are rational and the n power is a natural number. After all my previous calculations I can clearly state that my generalizations only work upon these requirements. I tried to see if it worked with a variety of integers and they do, but they only work with integers, and one must make sure to use no negative powers.

From all the above made statements I realize that between them they all have a certain connection and we were able to realize how by using step by step algebra one can reach this. At times this could perhaps become a bit

complicated, but later on it became clearer.

Further examples and tasks that could be taken upon could be researching for other dimensions of matrices and making a more general statement that could justify in any situation. This would probably take up more time that I have left in a life time, but it is surely possible!

Powers have specific demands that one must notice when doing investigations such as the one above; matrices can only be multiplied by powers through whole integers numbers that are greater than 0. This is another limitation of our statements.

We must also understand than minimal knowledge on matrices is needed, such as remembering the rules of matrix multiplication and that the 2x2 2x2 have to have matching number to be able to multiply with each other.

As most students around the world probably asked themselves when they first came upon matrices, I asked myself what they were used for and why I needed to learn them if I would then never need them again. After having completed this assignment, I finally understood that they have several uses, and that they are actually used.

- Matrices can be used to encrypt and decrypt codes; that believe it or not are still in use. Have you ever noticed in hotels when they give you a safe in your closet? Well, if you ever happen to forget the code you put into it, they bring a machine up to your room that functions upon matrices and they decrypt your code.
- They are used in fields such as plumbing, engineering and traffic issues because they can be used to display networks allowing calculations to be worked out easily.
- When realizing you can solve simultaneous equations by matrices, didn't you feel a sudden relief?
- They are used in graphics, and in areas such as cartoon animation and computer/video games such as Nintendo and Wii.

Matrices are used overall in math and science but they surely do find a way to save people from a lot of calculations and saves them time.

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[i] Brown, William A. (1991), *Matrices and vector spaces*, New York: M. Dekker, chapter 1.1