Matrix Binomials
Type 1 Internal Assessment

Matrices are rectangular arrays of numbers that are arranged in rows and columns, however the regular rules of algebra do not apply.

Let X=$\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$ and Y=$\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}$ and calculate X^2, X^3, X^4 ; Y^2, Y^3, $Y^{4.}$

$$\rightarrow \quad X^2=\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}^2\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}\times\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}=\begin{bmatrix}2 & 2\\2 & 2\end{bmatrix}$$

$$\rightarrow \quad X^3=\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}^3\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}\times\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}\times\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}=\begin{bmatrix}4 & 4\\4 & 4\end{bmatrix}$$

$$\rightarrow \quad X^4=\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}^4\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}\times\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}\times\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}\times\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}=\begin{bmatrix}8 & 8\\8 & 8\end{bmatrix}$$

Because matrices do not follow the algebraic rules of exponents, one can not simply distribute the exponent for each matrix value. Instead the matrix must be multiplied by itself however many times the exponent says. So for example, for X^2, the matrix X must by multiplied with itself two times.

The pattern that has emerged is that with every increasing power the matrix value increases with an exponential power of 2.
2^1 is equal to 2 showed by the matrix $\begin{bmatrix}2 & 2\\2 & 2\end{bmatrix}$. 2^2 is equal to 4 showed by the matrix $\begin{bmatrix}4 & 4\\4 & 4\end{bmatrix}$ and similarly 2^3 is equal to 8 represented in the matrix $\begin{bmatrix}8 & 8\\8 & 8\end{bmatrix}$.

$$\rightarrow \quad Y^2=\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}^2\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}\times\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}=\begin{bmatrix}2 & -2\\-2 & 2\end{bmatrix}$$

$$\rightarrow \quad Y^3=\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}^3\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}\times\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}\times\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}=\begin{bmatrix}4 & -4\\-4 & 4\end{bmatrix}$$

$$\rightarrow \quad Y^4=\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}^4\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}\times\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}\times\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}\times\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}$$

$$=\begin{bmatrix}8 & -8\\-8 & 8\end{bmatrix}$$

The pattern is very similar to the one above except that all the negatives in the original matrix will also become negatives.

Based on the results above we can conclude that for each exponent value X^n the matrix value will result in X^{n-1} and the same goes for Y^n resulting in Y^{n-1}. Combining these terms would result in $(X+Y)^{n-1}$.
$(X+Y)^{n-1}$:

$$(X+Y)^{3-1} = \left(\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix} + \begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}\right)^{3-1}$$

$$(X+Y)^{2} = \left(\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix} + \begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}\right)^{2}$$

$$(X+Y)^{2} = \left(\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix} \times \begin{bmatrix}1 & 1\\1 & 1\end{bmatrix} + \begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix} \times \begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}\right)$$ X^2+Y^2

$$(X+Y)^{2} = \begin{bmatrix}4 & 0\\0 & 4\end{bmatrix}$$ *which is the same as* $X^{n-1}+Y^{n-1}$

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Now let **A**=*a*X and **B**=*b*Y where *a* and *b* are constants or scalars.

A=**-1**X

$$A^2 = -1\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}^2 \rightarrow -1\begin{bmatrix}2 & 2\\2 & 2\end{bmatrix} = \begin{bmatrix}-2 & -2\\-2 & -2\end{bmatrix}$$

$$A^3 = -1\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}^3 \rightarrow -1\begin{bmatrix}4 & 4\\4 & 4\end{bmatrix} = \begin{bmatrix}-4 & -4\\-4 & -4\end{bmatrix}$$

$$A^4 = -1\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}^4 \rightarrow -1\begin{bmatrix}8 & 8\\8 & 8\end{bmatrix} = \begin{bmatrix}-8 & -8\\-8 & -8\end{bmatrix}$$

When multiplied by -1, the matrix values become negative but remain the same.

A=**-½**X

$$A^2 = -\tfrac{1}{2}\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}^2 \rightarrow -\tfrac{1}{2}\begin{bmatrix}2 & 2\\2 & 2\end{bmatrix} = \begin{bmatrix}-1 & -1\\-1 & -1\end{bmatrix}$$

$$A^3 = -\tfrac{1}{2}\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}^3 \rightarrow -\tfrac{1}{2}\begin{bmatrix}4 & 4\\4 & 4\end{bmatrix} = \begin{bmatrix}-2 & -2\\-2 & -2\end{bmatrix}$$

$$A^4 = -\tfrac{1}{2}\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}^4 \rightarrow -\tfrac{1}{2}\begin{bmatrix}8 & 8\\8 & 8\end{bmatrix} = \begin{bmatrix}-4 & -4\\-4 & -4\end{bmatrix}$$

When multiplied by -½, the matrix values become negative. Also the values are divided by two (or halved).

A=½X

$$A^2 = \tfrac{1}{2}\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}^2 \rightarrow \tfrac{1}{2}\begin{bmatrix}2 & 2\\2 & 2\end{bmatrix} = \begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$$

$$A^3 = \tfrac{1}{2}\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}^3 \rightarrow \tfrac{1}{2}\begin{bmatrix}4 & 4\\4 & 4\end{bmatrix} = \begin{bmatrix}2 & 2\\2 & 2\end{bmatrix}$$

$$A^4 = \tfrac{1}{2}\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}^4 \rightarrow \tfrac{1}{2}\begin{bmatrix}8 & 8\\8 & 8\end{bmatrix} = \begin{bmatrix}4 & 4\\4 & 4\end{bmatrix}$$
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When multiplied by ½, the matrix values stay positive. Also the values are divided by two (or halved).

A=**1**X

$A^2 = 1\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}^2 \rightarrow 1\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

$A^3 = 1\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}^3 \rightarrow 1\begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix}$

$A^4 = 1\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}^4 \rightarrow 1\begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix} = \begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix}$

When multiplied by 1, the matrix values remain the same.

A=**3**X

$A^2 = 3\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}^2 \rightarrow 3\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 6 \\ 6 & 6 \end{bmatrix}$

$A^3 = 3\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}^3 \rightarrow 3\begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix} = \begin{bmatrix} 12 & 12 \\ 12 & 12 \end{bmatrix}$

$A^4 = 3\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}^4 \rightarrow 3\begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix} = \begin{bmatrix} 24 & 24 \\ 24 & 24 \end{bmatrix}$

The matrix values are all multiplied by 3.

A=**5**X

$A^2 = 5\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}^2 \rightarrow 5\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 10 & 10 \\ 10 & 10 \end{bmatrix}$

$A^3 = 5\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}^3 \rightarrow 5\begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix} = \begin{bmatrix} 20 & 20 \\ 20 & 20 \end{bmatrix}$

$A^4 = 5\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}^4 \rightarrow 5\begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix} = \begin{bmatrix} 40 & 40 \\ 40 & 40 \end{bmatrix}$

The matrix values are all multiplied by 5.

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B= **-1**Y

$B^2 = -1\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}^2 \rightarrow -1\begin{bmatrix} -2 & -2 \\ -2 & -2 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

$B^3 = -1\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}^3 \rightarrow -1\begin{bmatrix} -4 & -4 \\ -4 & -4 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix}$

$B^4 = -1\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}^4 \rightarrow -1\begin{bmatrix} -8 & -8 \\ -8 & -8 \end{bmatrix} = \begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix}$

When multiplied by -1, the matrix values become positive but remain the same. The reverse results of when matrix A was multiplied by -1.
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B= **-**½Y

$B^2 = -\frac{1}{2}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}^2 \rightarrow --\frac{1}{2}\begin{bmatrix}2 & -2\\ -2 & 2\end{bmatrix} = \begin{bmatrix}-1 & 1\\ 1 & -1\end{bmatrix}$

$B^3 = -\frac{1}{2}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}^3 \rightarrow -\frac{1}{2}\begin{bmatrix}4 & -4\\ -4 & 4\end{bmatrix} = \begin{bmatrix}-2 & 2\\ 2 & -2\end{bmatrix}$

$B^4 = -\frac{1}{2}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}^4 \rightarrow -\frac{1}{2}\begin{bmatrix}8 & -8\\ -8 & 8\end{bmatrix} = \begin{bmatrix}-4 & 4\\ 4 & -4\end{bmatrix}$

When multiplied by -1, the matrix values are divided by two (or halved) and all positive and negative the signs are switched in relation with the original matrix.

B= **½**Y

$B^2 = \frac{1}{2}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}^2 \rightarrow -\frac{1}{2}\begin{bmatrix}2 & -2\\ -2 & 2\end{bmatrix} = \begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$

$B^3 = \frac{1}{2}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}^3 \rightarrow \frac{1}{2}\begin{bmatrix}4 & -4\\ -4 & 4\end{bmatrix} = \begin{bmatrix}2 & -2\\ -2 & 2\end{bmatrix}$

$B^4 = \frac{1}{2}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}^4 \rightarrow \frac{1}{2}\begin{bmatrix}8 & -8\\ -8 & 8\end{bmatrix} = \begin{bmatrix}4 & -4\\ -4 & 4\end{bmatrix}$

When multiplied by -1, the matrix values are divided by two (or halved) and all positive and negative the signs remain the same as in the original matrix (key difference when divides by -½).

B= **1**Y

$B^2 = 1\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}^2 \rightarrow -1\begin{bmatrix}-2 & -2\\ -2 & -2\end{bmatrix} = \begin{bmatrix}-2 & -2\\ -2 & -2\end{bmatrix}$

$B^3 = 1\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}^3 \rightarrow 1\begin{bmatrix}-4 & -4\\ -4 & -4\end{bmatrix} = \begin{bmatrix}-4 & -4\\ -4 & -4\end{bmatrix}$

$B^4 = 1\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}^4 \rightarrow 1\begin{bmatrix}-8 & -8\\ -8 & -8\end{bmatrix} = \begin{bmatrix}-8 & -8\\ -8 & -8\end{bmatrix}$

When multiplied by 1, the matrix values remain negative. The reverse results of when matrix A was multiplied by 1.

B= **3**Y

$B^2 = 3\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}^2 \rightarrow -3\begin{bmatrix}-2 & -2\\ -2 & -2\end{bmatrix} = \begin{bmatrix}-6 & -6\\ -6 & -6\end{bmatrix}$

$B^3 = 3\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}^3 \rightarrow 3\begin{bmatrix}-4 & -4\\ -4 & -4\end{bmatrix} = \begin{bmatrix}-12 & -12\\ -12 & -12\end{bmatrix}$

$$B^4=3\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}^4 \rightarrow 3\begin{bmatrix}-8 & -8\\ -8 & -8\end{bmatrix}=\begin{bmatrix}-24 & -24\\ -24 & -24\end{bmatrix}$$

When multiplied by 3, the matrix values remain negative.

B= **3**Y

$$B^2= 3\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}^2 \rightarrow 3\begin{bmatrix}-2 & -2\\ -2 & -2\end{bmatrix}=\begin{bmatrix}-6 & -6\\ -6 & -6\end{bmatrix}$$

$$B^3= 3\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}^3 \rightarrow 3\begin{bmatrix}-4 & -4\\ -4 & -4\end{bmatrix}=\begin{bmatrix}-12 & -12\\ -12 & -12\end{bmatrix}$$

$$B^4=3\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}^4 \rightarrow 3\begin{bmatrix}-8 & -8\\ -8 & -8\end{bmatrix}=\begin{bmatrix}-24 & -24\\ -24 & -24\end{bmatrix}$$

The matrix values are all multiplied by 3.

From these results and using the scalars -1, - ½, ½, and 1 we can conclude that:

- The scalar -1 reverses all signs of matrix values
- The scalar 1 has all signs stay the same for matrix values
- The scalar – ½ halves and reverses all signs of matrix values
- The scalar ½ halves and has all signs remain the same for matrix values
- The scalars of 3 and 5 are simply multiplied by the matrix v alues and do not change the signs of the matrix values.

This information can guide us to find expressions for A^n, B^n and $(A+B)^n$.

$A^n = a \times X^n$ (whatever the exponent of A is, it will also be the exponent for X)

$B^n = b \times Y^n$ (whatever the exponent of B is, it will also be the exponent for Y)

$(A+B)^n = [(a\times X) + (b\times Y)]^n$

Test the validity of the statement:

a=1 & b=5 & n=2

$$(A+B)^2 = [\ (1\times\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}) + (5\times\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix})\ ]^2$$

$$(A+B)^2 = [\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix} + \begin{bmatrix}5 & -5\\ -5 & 5\end{bmatrix}]^2$$

$$(A+B)^2 = \begin{bmatrix}6 & -4\\ -4 & 6\end{bmatrix}^2$$

$$(A+B)^2 = \begin{bmatrix}52 & -48\\ -48 & 52\end{bmatrix}$$

a=2 & b=3 & n=4

$(A+B)^4 = [\ (2\times\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}) + (3\times\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix})\ ]^4$

$(A+B)^4 = [\begin{bmatrix}2 & 2\\ 2 & 2\end{bmatrix} + \begin{bmatrix}3 & -3\\ -3 & 3\end{bmatrix}]^4$

$(A+B)^4 = \begin{bmatrix}5 & -1\\ -1 & 5\end{bmatrix}^4$

$(A+B)^4 = \begin{bmatrix}776 & -520\\ -520 & 776\end{bmatrix}$

The pattern follows matrix $M=\begin{bmatrix}a+b & a-b\\ a-b & a+b\end{bmatrix}$

The top left and bottom right matrix values are positive values as the corresponding values in the matrices X and Y are also positive. Contrarily, the values in the top right and the bottom left are negative as the values in the some location on the Y matrix are also negative.

Now when the scalars are substituted for *a* and *b* in matrix M you receive the same results as when you substitute them in the statement for (A+B) without calculating the result with the exponent.

Scalars: a = 1 & b = 5

$M=\begin{bmatrix}1+5 & 1-5\\ 1-5 & 1+5\end{bmatrix}$

$M = \begin{bmatrix}6 & -4\\ -4 & 6\end{bmatrix}$ *This is the same result (A+B) when its scalars are 5 and 1*

Scalars: a = 2 & b = 3

$M = \begin{bmatrix}2+3 & 2-3\\ 2-3 & 2+3\end{bmatrix}$

$M = \begin{bmatrix}5 & -1\\ -1 & 5\end{bmatrix}$ *This is the same result (A+B) when its scalars are 2 and 3*

This allows us to show that M=A+B

M = A + B *remembering that A=a×X & B=b×Y*

$\begin{bmatrix}6 & -4\\ -4 & 6\end{bmatrix} = (1\times\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}) + (5\times\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix})$

$$\begin{bmatrix} 6 & -4 \\ -4 & 6 \end{bmatrix} = (\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 5 & -5 \\ -5 & 5 \end{bmatrix})$$

$$\begin{bmatrix} 6 & -4 \\ -4 & 6 \end{bmatrix} = \begin{bmatrix} 6 & -4 \\ -4 & 6 \end{bmatrix}$$

M = A + B *remembering that A=a×X & B=b×Y*

$$\begin{bmatrix} 5 & -1 \\ -1 & 5 \end{bmatrix} = (2\times\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}) + (3\times\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix})$$

$$\begin{bmatrix} 5 & -1 \\ -1 & 5 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 5 & -1 \\ -1 & 5 \end{bmatrix} = \begin{bmatrix} 5 & -1 \\ -1 & 5 \end{bmatrix}$$

Taking it a step further we can also show that $M^2=A^2+B^2$

M^2 = A^2 + B^2 *remembering that A=a×X & B=b×Y*

$$\begin{bmatrix} 6 & -4 \\ -4 & 6 \end{bmatrix}^2 = (1\times\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix})^2 + (5\times\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix})^2$$

$$\begin{bmatrix} 6 & -4 \\ -4 & 6 \end{bmatrix}^2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}^2 + \begin{bmatrix} 5 & -5 \\ -5 & 5 \end{bmatrix}^2$$

$$\begin{bmatrix} 52 & -48 \\ -48 & 52 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} 50 & -50 \\ -50 & 50 \end{bmatrix}$$

$$\begin{bmatrix} 52 & -48 \\ -48 & 52 \end{bmatrix} = \begin{bmatrix} 52 & -48 \\ -48 & 52 \end{bmatrix}$$

M^2 = A^2 + B^2 *remembering that A=a×X & B=b×Y*

$$\begin{bmatrix} 5 & -1 \\ -1 & 5 \end{bmatrix}^2 = (2\times\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix})^2 + (3\times\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix})^2$$

$$\begin{bmatrix} 5 & -1 \\ -1 & 5 \end{bmatrix}^2 = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}^2 + \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix}^2$$

$$\begin{bmatrix} 26 & -10 \\ -10 & 26 \end{bmatrix} = \begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix} + \begin{bmatrix} 18 & -18 \\ -18 & 18 \end{bmatrix}$$

$$\begin{bmatrix} 26 & -10 \\ -10 & 26 \end{bmatrix} = \begin{bmatrix} 26 & -10 \\ -10 & 26 \end{bmatrix}$$

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We can conclude from the investigation above that since:

$M^n=A^n+B^n$

$M^n=(a\times X)^n + (b\times Y)^n$
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Making $M^n= (aX)^n+(bY)^n$ the general statement.

To test the validity of this statement let n=5, a=6 and b=7

$$\begin{bmatrix}13 & -1\\ -1 & 13\end{bmatrix}^5 = (6\times\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix})^5 + (7\times\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix})^5$$

$$\begin{bmatrix}13 & -1\\ -1 & 13\end{bmatrix}^5 = \begin{bmatrix}6 & 6\\ 6 & 6\end{bmatrix}^5 + \begin{bmatrix}7 & -7\\ -7 & 7\end{bmatrix}^5$$

$$\begin{bmatrix}393328 & -144496\\ -144496 & 393328\end{bmatrix} = \begin{bmatrix}124416 & 124416\\ 124416 & 124416\end{bmatrix} + \begin{bmatrix}268912 & -268912\\ -268912 & 268912\end{bmatrix}$$

$$\begin{bmatrix}393328 & -144496\\ -144496 & 393328\end{bmatrix} = \begin{bmatrix}393328 & -144496\\ -144496 & 393328\end{bmatrix}$$

To further test the validity of the statement let n=4, a=7 and b=5

$$\begin{bmatrix}12 & 2\\ 2 & 12\end{bmatrix}^4 = (7\times\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix})^4 + (5\times\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix})^4$$

$$\begin{bmatrix}12 & 2\\ 2 & 12\end{bmatrix}^4 = \begin{bmatrix}7 & 7\\ 7 & 7\end{bmatrix}^4 + \begin{bmatrix}5 & -5\\ -5 & 5\end{bmatrix}^4$$

$$\begin{bmatrix}24208 & 14208\\ 14208 & 24208\end{bmatrix} = \begin{bmatrix}19208 & 19208\\ 19208 & 19208\end{bmatrix} + \begin{bmatrix}5000 & -5000\\ -5000 & 5000\end{bmatrix}$$

$$\begin{bmatrix}24208 & 14208\\ 14208 & 24208\end{bmatrix} = \begin{bmatrix}24208 & 14208\\ 14208 & 24208\end{bmatrix}$$

To further test the validity of the statement let n=3, a=9 and b=11

$$\begin{bmatrix}20 & -2\\ -2 & 20\end{bmatrix}^3 = (9\times\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix})^3 + (11\times\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix})^3$$

$$\begin{bmatrix}20 & -2\\ -2 & 20\end{bmatrix}^3 = \begin{bmatrix}9 & 9\\ 9 & 9\end{bmatrix}^3 + \begin{bmatrix}11 & -11\\ -11 & 11\end{bmatrix}^3$$

$$\begin{bmatrix}8240 & -2408\\ -2408 & 8240\end{bmatrix} = \begin{bmatrix}2916 & 2916\\ 2916 & 2916\end{bmatrix} + \begin{bmatrix}5324 & -5324\\ -5324 & 5324\end{bmatrix}$$

$$\begin{bmatrix}8240 & -2408\\ -2408 & 8240\end{bmatrix} = \begin{bmatrix}8240 & -2408\\ -2408 & 8240\end{bmatrix}$$

To further test the validity of the statement let n=3, a=-9 and b=11

$$\begin{bmatrix}2 & -20\\ -20 & 2\end{bmatrix}^3 = (-9\times\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix})^3 + (11\times\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix})^3$$

$$\begin{bmatrix}2 & -20\\ -20 & 2\end{bmatrix}^3 = \begin{bmatrix}-9 & -9\\ -9 & -9\end{bmatrix}^3 + \begin{bmatrix}11 & -11\\ -11 & 11\end{bmatrix}^3$$

$$\begin{bmatrix} 2408 & -8240 \\ -8240 & 2408 \end{bmatrix} = \begin{bmatrix} -2916 & -2916 \\ -2916 & -2916 \end{bmatrix} + \begin{bmatrix} 5324 & -5324 \\ -5324 & 5324 \end{bmatrix}$$

$$\begin{bmatrix} 2408 & -8240 \\ -8240 & 2408 \end{bmatrix} = \begin{bmatrix} 2408 & -8240 \\ -8240 & 2408 \end{bmatrix}$$

In all these case the general statement is correct and works. However, the scope of the general statement is limited to positive exponents as well as the matrices X and Y. We have only tested a very small sample of the different matrix values and its infinite combinations for the matrices X and Y.

If we were to use different matrix values for X and Y the general statement would not apply:

Let n=3, a=9 and b=11 but let $X = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$ and let $Y = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$

$$\begin{bmatrix} 20 & -2 \\ -2 & 20 \end{bmatrix}^3 = (9\times\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix})^3 + (11\times\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix})^3$$

$$\begin{bmatrix} 20 & -2 \\ -2 & 20 \end{bmatrix}^3 = \begin{bmatrix} 18 & 18 \\ 18 & 18 \end{bmatrix}^3 + \begin{bmatrix} 22 & -22 \\ -22 & 22 \end{bmatrix}^3$$

$$\begin{bmatrix} 8240 & -2408 \\ -2408 & 8240 \end{bmatrix} = \begin{bmatrix} 23328 & 23328 \\ 23328 & 23328 \end{bmatrix} + \begin{bmatrix} 42592 & -42592 \\ -42592 & 42592 \end{bmatrix}$$

$$\begin{bmatrix} 8240 & -2408 \\ -2408 & 8240 \end{bmatrix} \neq \begin{bmatrix} 65920 & -19364 \\ -19364 & 65920 \end{bmatrix}$$

As one can see, the general statement does not apply when the values for the matrices X and Y are changed and only works correctly when:

$X=\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ and $Y=\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$.

To reach my general statement I used my previous findings.
I found that $A^n = a\times X^n$ and that $B^n = b\times Y^n$. This in turn allowed me to come to the conclusion that $(A+B)^n = [(a\times X) + (b\times Y)]^n$.

Using the knowledge that $M=\begin{bmatrix} a+b & a-b \\ a-b & a+b \end{bmatrix}$ and that M=A+B and that $M^2=A^2+B^2$

I could reach the following conclusion:

$M^n=A^n+B^n$

$M^n=(a\times X)^n + (b\times Y)^n$ *substituting A for a×X and B for b×Y*

This is how I reached by general statement.