The goal of this portfolio assessment is to find an expression for X^n, Y^n, $(X + Y)^n$, $[A^n, B^n, (A + B)^n]$ in both questions whilst expressing them: M^n in terms of aX and bY. The purpose of this assessment is to find out how we can interpret matrix binomials using different values and similarities to find the pattern occurring. We've been given a general statement to express M^n in terms of aX and bY, to do so we must substitute a into matrix X to get a new matrix 'A', and b into matrix Y to get the new matrix 'B'. The task given now is to see if the pattern really did work with other numbers, and to prove the general statement.

- ## Question 1

Let $X = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ and $Y = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$. Calculate X^2, X^3, X^4; Y^2, Y^3, Y^4.

By considering integer powers of X and Y, find expressions for X^n, Y^n, $(X + Y)^n$.

Alright now to calculate X^2, X^3, X^4; Y^2, Y^3, Y^4, I will firstly show how these matrices are multiplied, and then I shall use my graphics calculator to do the rest. As doing so I will also look for a pattern trend in which I can use to relate to fine the expression X^n, Y^n, $(X + Y)^n$. By doing so I will carefully look at how the matrix trend is created, therefore making it easier to find the expression.

I found that I can use a specific matrix property in order to find the expression as well as the arithmetic progression. This will ultimately determine how the expression is achieved and if it's feasible. I will continue using the rule throughout the first question, however, I may need to change the expression for the next question based upon my findings.

The first thing I am going to do is to demonstrate how a matrix is multiplied; a step by step method and then I shall continue using the graphics calculator. In the end I will also add a few examples to show if the expression found is truly valid.

*just a brief note before I begin the project, every red colored matrix means that this number has been added as an extra to find the pattern more clearly. Every yellow matrix indicates the question given from the worksheet.

The Original Matrix	X^n	Explanation of how the X^n was achieved
$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$	X^2	$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}^2 = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} x \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} (1x1)+(1x1) & (1x1)+(1x1) \\ (1x1)+(1x1) & (1x1)+(1x1) \end{pmatrix}$

		$= \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}$ To achieve this pattern you must multiply row by column.
$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$	X^3 $= X^2 \mathbf{x} X$	$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}^3 = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix} \times \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} (2x1)+(2x1) & (2x1)+(2x1) \\ (2x1)+(2x1) & (2x1)+(2x1) \end{pmatrix}$ $= \begin{pmatrix} 4 & 4 \\ 4 & 4 \end{pmatrix}$
$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$	X^4 $= X^3 \mathbf{x} X$	$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}^4 = \begin{pmatrix} 4 & 4 \\ 4 & 4 \end{pmatrix} \times \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} (4x1)+(4x1) & (4x1)+(4x1) \\ (4x1)+(4x1) & (4x1)+(4x1) \end{pmatrix}$ $= \begin{pmatrix} 8 & 8 \\ 8 & 8 \end{pmatrix}$
$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$	X^5 $= X^4 \mathbf{x} X$	$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}^5 = \begin{pmatrix} 8 & 8 \\ 8 & 8 \end{pmatrix} \times \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} (8x1)+(8x1) & (8x1)+(8x1) \\ (8x1)+(8x1) & (8x1)+(8x1) \end{pmatrix}$ $= \begin{pmatrix} 16 & 16 \\ 16 & 16 \end{pmatrix}$
$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$	X^6 $= X^5 \mathbf{x} X$	$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}^6 = \begin{pmatrix} 16 & 16 \\ 16 & 16 \end{pmatrix} \times \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} (16\ x1)+(16\ x1) & (16\ x1)+(16\ x1) \\ (16\ x1)+(16\ x1) & (16\ x1)+(16\ x1) \end{pmatrix}$ $= \begin{pmatrix} 32 & 32 \\ 32 & 32 \end{pmatrix}$

As you may notice, there are two differently colored matrices. This is because the last two matrices are extra help. I needed to add two more values to have a clearer view of the pattern occurring. As you can see I also showed the working out, however, I used a method of $X^n \mathbf{x} X$, meaning that every time I add a number I just multiply the answer by the same matrix again, with the power increasing as I go along. Now I'm going to use the same method for Matrix Y.

The Original Matrix	X^n	Explanation of how the Y^n was achieved

$\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	X^2	$\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}^2 = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} =$ $\begin{pmatrix} (1x1)+(-1x-1) & (-1x-1)+(-1x1) \\ (-1x1)+(-1x-1) & (-1x-1)+(1x1) \end{pmatrix}$ $= \begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}$ To achieve this pattern you must multiply row by column.
$\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	X^3 $= X^2 \mathbf{x}\, X$	$\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}^3 = \begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix} \times \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} =$ $\begin{pmatrix} (2x1)+(-2x-1) & (-2x-1)+(-2x1) \\ (-2x1)+(-2x-1) & (-2x-1)+(2x1) \end{pmatrix}$ $= \begin{pmatrix} 4 & -4 \\ -4 & 4 \end{pmatrix}$
$\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	X^4 $= X^3 \mathbf{x}\, X$	$\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}^4 = \begin{pmatrix} 4 & -4 \\ -4 & 4 \end{pmatrix} \times \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} =$ $\begin{pmatrix} (4x1)+(-4x-1) & (-4x-1)+(-4x1) \\ (-4x1)+(-4x-1) & (-4x-1)+(4x1) \end{pmatrix}$ $= \begin{pmatrix} 8 & -8 \\ -8 & 8 \end{pmatrix}$
$\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	X^5 $= X^4 \mathbf{x}\, X$	$\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}^5 = \begin{pmatrix} 8 & -8 \\ -8 & 8 \end{pmatrix} \times \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} =$ $\begin{pmatrix} (8x1)+(-8x-1) & (-8x-1)+(-8x1) \\ (-8x1)+(-8x-1) & (-8x-1)+(8x1) \end{pmatrix}$ $= \begin{pmatrix} 16 & -16 \\ -16 & 16 \end{pmatrix}$

$\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	X^6 $= X^5 \times X$	$\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}^6 = \begin{pmatrix} 16 & -16 \\ -16 & 16 \end{pmatrix} \times \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} =$ $\begin{pmatrix} (16 \times 1) + (-16 \times -1) & (-16 \times -1) + (-16 \times 1) \\ (-16 \times 1) + (-16 \times -1) & (-16 \times -1) + (16 \times 1) \end{pmatrix}$ $= \begin{pmatrix} 32 & -32 \\ -32 & 32 \end{pmatrix}$

As you might have noticed, this matrix has the same digits as Matrix A, however, the second and third numbers are negative, this and is due to the $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$ placement.

Now to find the expression of X^n, Y^n, $(X + Y)^n$, I found a clear pattern in the matrices developed. Each matrix was multiplied by 2. However, the power of X and Y also play a role which influence what's on the inside of the matrix, from that I started experimenting and I came up with solution. Since all the answers are multiples of 2 and gradually increase, one is taken away from the power of X and Y. I found that 1, 2, 4, 8, 16, 32 can also be determined as 2^0 2^1, 2^2, 2^3, 2^4, 2^5. Thus creating a pattern of 2^{n-1}.

<u>Example:</u>

- $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}^1 = \begin{pmatrix} 2^{n-1} & 2^{n-1} \\ 2^{n-1} & 2^{n-1} \end{pmatrix}$ in this case $n=1$ because $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}^1$ has a power of 1, therefore the answer to this would be $\begin{pmatrix} 2^{1-1} & 2^{1-1} \\ 2^{1-1} & 2^{1-1} \end{pmatrix} = \begin{pmatrix} 2^0 & 2^0 \\ 2^0 & 2^0 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$.
- $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}^2 = \begin{pmatrix} 2^{2-1} & 2^{2-1} \\ 2^{2-1} & 2^{2-1} \end{pmatrix} = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}$ n=2
- $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}^3 = \begin{pmatrix} 2^{3-1} & 2^{3-1} \\ 2^{3-1} & 2^{3-1} \end{pmatrix} = \begin{pmatrix} 4 & 4 \\ 4 & 4 \end{pmatrix}$ n=3

- $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}^4 = \begin{pmatrix} 2^{4-1} & 2^{4-1} \\ 2^{4-1} & 2^{4-1} \end{pmatrix} = \begin{pmatrix} 8 & 8 \\ 8 & 8 \end{pmatrix}$ n=4

Now that the pattern for matrix X has been found, you may come to realize that the highlighted green matrices are indeed multiples of two as well. So this pattern has been proved. $\begin{pmatrix} 2^{n-1} & 2^{n-1} \\ 2^{n-1} & 2^{n-1} \end{pmatrix}$

In order to find the pattern for matrix Y the same pattern will be used, however, negative signs will be drawn into the matrix so that it compliments $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$.

<u>Example:</u>

- $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}^1 = \begin{pmatrix} 2^{n-1} & -2^{n-1} \\ -2^{n-1} & 2^{n-1} \end{pmatrix}$ in this case $n=1$ because $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}^1$ has a power of 1, therefore the answer to this would be $\begin{pmatrix} 2^{1-1} & -2^{1-1} \\ -2^{1-1} & 2^{1-1} \end{pmatrix} = \begin{pmatrix} 2^0 & 2^0 \\ 2^0 & 2^0 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$.
- $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}^2 = \begin{pmatrix} 2^{2-1} & -2^{2-1} \\ -2^{2-1} & 2^{2-1} \end{pmatrix} = \begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}$ n=2
- $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}^3 = \begin{pmatrix} 2^{3-1} & -2^{3-1} \\ -2^{3-1} & 2^{3-1} \end{pmatrix} = \begin{pmatrix} 4 & -4 \\ -4 & 4 \end{pmatrix}$ n=3
- $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}^4 = \begin{pmatrix} 2^{4-1} & -2^{4-1} \\ -2^{4-1} & 2^{4-1} \end{pmatrix} = \begin{pmatrix} 8 & -8 \\ -8 & 8 \end{pmatrix}$ n=4

From my observation, the pattern found suits both matrices.

Now to prove that X^n, Y^n can also be equal to $(X + Y)^n$. I will demonstrate this by using two different numbers, which aren't given from the question in order to prove that the expression is correct and can work with other numbers given.

Matrix $X^n = \begin{pmatrix} 2^{n-1} & 2^{n-1} \\ 2^{n-1} & 2^{n-1} \end{pmatrix}$ and Matrix $Y^n = \begin{pmatrix} 2^{n-1} & -2^{n-1} \\ 2^{n-1} & 2^{n-1} \end{pmatrix}$

Then $(X+Y)^n = \left[\begin{pmatrix} 2^{n-1} & 2^{n-1} \\ 2^{n-1} & 2^{n-1} \end{pmatrix} + \begin{pmatrix} 2^{n-1} & -2^{n-1} \\ -2^{n-1} & 2^{n-1} \end{pmatrix}\right]^n$

Or $\begin{pmatrix} 2^n & (0)2^n \\ (0)2^n & 2^n \end{pmatrix} = 2^n \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}^n$

- $(X+Y)^5 = \left[\begin{pmatrix} 2^{5-1} & 2^{5-1} \\ 2^{5-1} & 2^{5-1} \end{pmatrix} + \begin{pmatrix} 2^{5-1} & -2^{5-1} \\ -2^{5-1} & 2^{5-1} \end{pmatrix}\right]^5$

$= \left[\begin{pmatrix} 16 & 16 \\ 16 & 16 \end{pmatrix} + \begin{pmatrix} 16 & -16 \\ -16 & 16 \end{pmatrix}\right]^5$

$= \left[\begin{pmatrix} 32 & 0 \\ 0 & 32 \end{pmatrix}\right]^5$

$= \begin{pmatrix} 33554432 & 0 \\ 0 & 33554432 \end{pmatrix}$

To prove this in terms of $2^n \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}^n$

$2^5 = \begin{pmatrix} 32 & 0 \\ 0 & 32 \end{pmatrix}^5$

$$= \begin{pmatrix} 33554432 & 0 \\ 0 & 33554432 \end{pmatrix}$$

- $(X + Y)^8 = \left[\begin{pmatrix} 2^{8-1} & 2^{8-1} \\ 2^{8-1} & 2^{8-1} \end{pmatrix} + \begin{pmatrix} 2^{8-1} & -2^{8-1} \\ -2^{8-1} & 2^{8-1} \end{pmatrix}\right]^8$

$$= \left[\begin{pmatrix} 128 & 128 \\ 128 & 128 \end{pmatrix} + \begin{pmatrix} 128 & -128 \\ -128 & 128 \end{pmatrix}\right]^8$$

$$= \left[\begin{pmatrix} 256 & 0 \\ 0 & 256 \end{pmatrix}\right]^8$$

$$= \begin{pmatrix} 1.844674407 & \in +19 & 0 & \\ & 0 & 1.844674407 & \in +19 \end{pmatrix}$$

Proving it in terms of $2^n \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}^n$

$$2^8 = \begin{pmatrix} 256 & 0 \\ 0 & 256 \end{pmatrix}^8$$

$$= \begin{pmatrix} 1.844674407 & \in +19 & 0 & \\ & 0 & 1.844674407 & \in +19 \end{pmatrix}$$

As you might have noticed there is a zero involved, which in this case makes this matrix an identity kind of matrix. Such as $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

Example 1:

$$IA = A \quad \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \left[\begin{pmatrix} 128 & 128 \\ 128 & 128 \end{pmatrix} + \begin{pmatrix} 128 & -128 \\ -128 & 128 \end{pmatrix} \right]^8 =$$

$$\begin{pmatrix} 1.844674407 & \in +19 & 0 & \\ & 0 & 1.844674407 & \in +19 \end{pmatrix}$$

- ## Question 2

Let A = aX and B = bY, where a and b are constants.
Use different values of a and b to calculate A^2, A^3, A^4; B^2, B^3, B^4.
By considering integer powers of A and B, find expressions for X^n, Y^n, $(X + Y)^n$.

In this part of the question I will use the GDC in order to calculate the answer of the matrix. I will choose to do so because in the previous question I have clearly shown how to multiply a matrix.

The Original Matrix	Value of a	A^n	Explanation of how the A^n was achieved

$a\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$	a = 2 $= 2 \times \begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$	$\begin{pmatrix}2 & 2\\2 & 2\end{pmatrix}^1$	Since this type of multiplication of a matrix is called a scalar matrix, the value of a is multiplied by the matrix given, which in this case is $\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$, thus becoming $\begin{pmatrix}2 & 2\\2 & 2\end{pmatrix}$ when multiplied by 2 (a). However the answer remains as 2 because the power of A is 1 (^{n}). + INTEGER
$a\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$	a = 2 $= 2 \times \begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$	$\begin{pmatrix}2 & 2\\2 & 2\end{pmatrix}^2$	$\begin{pmatrix}2 & 2\\2 & 2\end{pmatrix}\begin{pmatrix}2 & 2\\2 & 2\end{pmatrix} = \begin{pmatrix}8 & 8\\8 & 8\end{pmatrix}$
$a\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$	a = 2 $= 2 \times \begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$	$\begin{pmatrix}2 & 2\\2 & 2\end{pmatrix}^3$	$\begin{pmatrix}8 & 8\\8 & 8\end{pmatrix}\begin{pmatrix}2 & 2\\2 & 2\end{pmatrix} = \begin{pmatrix}32 & 32\\32 & 32\end{pmatrix}$
$a\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$	a = 2 $= 2 \times \begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$	$\begin{pmatrix}2 & 2\\2 & 2\end{pmatrix}^4$	$\begin{pmatrix}32 & 32\\32 & 32\end{pmatrix}\begin{pmatrix}2 & 2\\2 & 2\end{pmatrix} = \begin{pmatrix}128 & 128\\128 & 128\end{pmatrix}$
$a\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$	a = 2 $= 2 \times \begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$	$\begin{pmatrix}2 & 2\\2 & 2\end{pmatrix}^5$	$\begin{pmatrix}128 & 128\\128 & 128\end{pmatrix}\begin{pmatrix}2 & 2\\2 & 2\end{pmatrix}$ $= \begin{pmatrix}512 & 512\\512 & 512\end{pmatrix}$
$a\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$	a = 2 $= 2 \times \begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$	$\begin{pmatrix}2 & 2\\2 & 2\end{pmatrix}^6$	$\begin{pmatrix}512 & 512\\512 & 512\end{pmatrix}\begin{pmatrix}2 & 2\\2 & 2\end{pmatrix}$ $= \begin{pmatrix}2048 & 2048\\2048 & 2048\end{pmatrix}$
$a\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$	a = -2	$\begin{pmatrix}-2 & -2\\-2 & -2\end{pmatrix}^1$	The value of a (-2) is multiplied by the matrix

	$= -2 \times \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$		given, $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, which equals to $\begin{pmatrix} -2 & -2 \\ -2 & -2 \end{pmatrix}$ when multiplied by -2 (a). As the power increases, we'll soon come to realize that the matrix values change. - INTEGER
$a\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$	a = -2 $= -2 \times \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$	$\begin{pmatrix} -2 & -2 \\ -2 & -2 \end{pmatrix}^2$	$\begin{pmatrix} -2 & -2 \\ -2 & -2 \end{pmatrix}\begin{pmatrix} -2 & -2 \\ -2 & -2 \end{pmatrix}$ $= \begin{pmatrix} 8 & 8 \\ 8 & 8 \end{pmatrix}$
$a\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$	a = -2 $= -2 \times \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$	$\begin{pmatrix} -2 & -2 \\ -2 & -2 \end{pmatrix}^3$	$\begin{pmatrix} 8 & 8 \\ 8 & 8 \end{pmatrix}\begin{pmatrix} -2 & -2 \\ -2 & -2 \end{pmatrix}$ $= \begin{pmatrix} -32 & -32 \\ -32 & -32 \end{pmatrix}$
$a\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$	a = -2 $= -2 \times \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$	$\begin{pmatrix} -2 & -2 \\ -2 & -2 \end{pmatrix}^4$	$\begin{pmatrix} -32 & -32 \\ -32 & -32 \end{pmatrix}\begin{pmatrix} -2 & -2 \\ -2 & -2 \end{pmatrix}$ $= \begin{pmatrix} 128 & 128 \\ 128 & 128 \end{pmatrix}$
$a\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$	a = -2 $= -2 \times \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$	$\begin{pmatrix} -2 & -2 \\ -2 & -2 \end{pmatrix}^5$	$\begin{pmatrix} 128 & 128 \\ 128 & 128 \end{pmatrix}\begin{pmatrix} -2 & -2 \\ -2 & -2 \end{pmatrix}$ $= \begin{pmatrix} -512 & -512 \\ -512 & -512 \end{pmatrix}$
$a\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$	a = -2 $= -2 \times \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$	$\begin{pmatrix} -2 & -2 \\ -2 & -2 \end{pmatrix}^6$	$\begin{pmatrix} -512 & -512 \\ -512 & -512 \end{pmatrix}\begin{pmatrix} -2 & -2 \\ -2 & -2 \end{pmatrix}$

			$= \begin{pmatrix} 2048 & 2048 \\ 2048 & 2048 \end{pmatrix}$
The pattern found in this 'sequence' is that every new matrix is multiplied by 4. This is the pattern found the set of integers. I used the same number, by using it positively and negatively.			
$a\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$	$a = 0.25$ or $\frac{1}{4}$ $= \frac{1}{4} \times \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$	$\begin{pmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{pmatrix}^1$	The value of a (0.25) is multiplied by the matrix given, $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, which equals to $\begin{pmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ when multiplied by $\frac{1}{4}$ (a). In this matrix I used a fraction instead of a decimal, simply because it's easier to work with. + RATIONAL NUMBER
$a\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$	$a = 0.25$ or $\frac{1}{4}$ $= \frac{1}{4} \times \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$	$\begin{pmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{pmatrix}^2$	$\begin{pmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{pmatrix}\begin{pmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{pmatrix} = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} \end{pmatrix}$
$a\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$	$a = 0.25$ or $\frac{1}{4}$ $= \frac{1}{4} \times \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$	$\begin{pmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{pmatrix}^3$	$\begin{pmatrix} \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} \end{pmatrix}\begin{pmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{pmatrix} = \begin{pmatrix} \frac{1}{16} & \frac{1}{16} \\ \frac{1}{16} & \frac{1}{16} \end{pmatrix}$
$a\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$	$a = 0.25$ or $\frac{1}{4}$ $= \frac{1}{4} \times \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$	$\begin{pmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{pmatrix}^4$	$\begin{pmatrix} \frac{1}{16} & \frac{1}{16} \\ \frac{1}{16} & \frac{1}{16} \end{pmatrix}\begin{pmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{pmatrix} = \begin{pmatrix} \frac{1}{32} & \frac{1}{32} \\ \frac{1}{32} & \frac{1}{32} \end{pmatrix}$
$a\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$	$a = 0.25$ or $\frac{1}{4}$ $= \frac{1}{4} \times \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$	$\begin{pmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{pmatrix}^5$	$\begin{pmatrix} \frac{1}{32} & \frac{1}{32} \\ \frac{1}{32} & \frac{1}{32} \end{pmatrix}\begin{pmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ $= \begin{pmatrix} \frac{1}{64} & \frac{1}{64} \\ \frac{1}{64} & \frac{1}{64} \end{pmatrix}$

$a\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$	$a = 0.25$ or $\frac{1}{4}$ $= \frac{1}{4} \times \begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$	$\begin{pmatrix}\frac{1}{4} & \frac{1}{4}\\ \frac{1}{4} & \frac{1}{4}\end{pmatrix}^6$	$\begin{pmatrix}\frac{1}{64} & \frac{1}{64}\\ \frac{1}{64} & \frac{1}{64}\end{pmatrix}\begin{pmatrix}\frac{1}{4} & \frac{1}{4}\\ \frac{1}{4} & \frac{1}{4}\end{pmatrix}$ $= \begin{pmatrix}\frac{1}{128} & \frac{1}{128}\\ \frac{1}{128} & \frac{1}{128}\end{pmatrix}$
$a\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$	$a = -0.25$ or $-\frac{1}{4}$ $= -\frac{1}{4} \times \begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$	$\begin{pmatrix}-\frac{1}{4} & -\frac{1}{4}\\ -\frac{1}{4} & -\frac{1}{4}\end{pmatrix}^1$	The value of a (-0.25) is multiplied by the matrix given, $\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$, which equals to $\begin{pmatrix}-\frac{1}{4} & -\frac{1}{4}\\ -\frac{1}{4} & -\frac{1}{4}\end{pmatrix}$ when multiplied by $-\frac{1}{4}$ (a). In this matrix I used a fraction instead of a decimal, simply because it's easier to work with. - RATIONAL NUMBER
$a\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$	$a = -0.25$ or $-\frac{1}{4}$ $= -\frac{1}{4} \times \begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$	$\begin{pmatrix}-\frac{1}{4} & -\frac{1}{4}\\ -\frac{1}{4} & -\frac{1}{4}\end{pmatrix}^2$	$\begin{pmatrix}-\frac{1}{4} & -\frac{1}{4}\\ -\frac{1}{4} & -\frac{1}{4}\end{pmatrix}\begin{pmatrix}-\frac{1}{4} & -\frac{1}{4}\\ -\frac{1}{4} & -\frac{1}{4}\end{pmatrix}$ $= \begin{pmatrix}\frac{1}{8} & \frac{1}{8}\\ \frac{1}{8} & \frac{1}{8}\end{pmatrix}$
$a\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$	$a = 0.25$ or $\frac{1}{4}$ $= \frac{1}{4} \times \begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$	$\begin{pmatrix}-\frac{1}{4} & -\frac{1}{4}\\ -\frac{1}{4} & -\frac{1}{4}\end{pmatrix}^3$	$\begin{pmatrix}\frac{1}{8} & \frac{1}{8}\\ \frac{1}{8} & \frac{1}{8}\end{pmatrix}\begin{pmatrix}-\frac{1}{4} & -\frac{1}{4}\\ -\frac{1}{4} & -\frac{1}{4}\end{pmatrix}$

			$= \begin{pmatrix} -\frac{1}{16} & -\frac{1}{16} \\ -\frac{1}{16} & -\frac{1}{16} \end{pmatrix}$
$a\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$	$a = 0.25$ or $\frac{1}{4}$ $= \frac{1}{4} \times \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$	$\begin{pmatrix} -\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{4} \end{pmatrix}^4$	$\begin{pmatrix} -\frac{1}{16} & -\frac{1}{16} \\ -\frac{1}{16} & -\frac{1}{16} \end{pmatrix}\begin{pmatrix} -\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{4} \end{pmatrix}$ $= \begin{pmatrix} \frac{1}{32} & \frac{1}{32} \\ \frac{1}{32} & \frac{1}{32} \end{pmatrix}$
$a\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$	$a = 0.25$ or $\frac{1}{4}$ $= \frac{1}{4} \times \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$	$\begin{pmatrix} -\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{4} \end{pmatrix}^5$	$\begin{pmatrix} \frac{1}{32} & \frac{1}{32} \\ \frac{1}{32} & \frac{1}{32} \end{pmatrix}\begin{pmatrix} -\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{4} \end{pmatrix}$ $= \begin{pmatrix} -\frac{1}{64} & -\frac{1}{64} \\ -\frac{1}{64} & -\frac{1}{64} \end{pmatrix}$
$a\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$	$a = 0.25$ or $\frac{1}{4}$ $= \frac{1}{4} \times \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$	$\begin{pmatrix} -\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{4} \end{pmatrix}^6$	$\begin{pmatrix} -\frac{1}{64} & -\frac{1}{64} \\ -\frac{1}{64} & -\frac{1}{64} \end{pmatrix}\begin{pmatrix} -\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{4} \end{pmatrix}$ $= \begin{pmatrix} \frac{1}{128} & \frac{1}{128} \\ \frac{1}{128} & \frac{1}{128} \end{pmatrix}$
For this type of number, the rational number, I used both negative and positive numbers. However the pattern is different, at first the sequence has a multiple of 2 for the denominator.			

$a\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}$	$a = \sqrt{5}$ $= \sqrt{5} \times \begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}$	$\begin{pmatrix}\sqrt{5} & \sqrt{5}\\ \sqrt{5} & \sqrt{5}\end{pmatrix}^1$	The value of a ($\sqrt{5}$) is multiplied by the matrix given, $\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}$, which equals to $\begin{pmatrix}\sqrt{5} & \sqrt{5}\\ \sqrt{5} & \sqrt{5}\end{pmatrix}$ when multiplied by $\sqrt{5}$ (a). In this matrix I used a root 5 instead of a decimal, simply because it's easier to work with. + IRRATIONAL NUMBER
$a\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}$	$a = \sqrt{5}$ $= \sqrt{5} \times \begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}$	$\begin{pmatrix}\sqrt{5} & \sqrt{5}\\ \sqrt{5} & \sqrt{5}\end{pmatrix}^2$	$\begin{pmatrix}\sqrt{5} & \sqrt{5}\\ \sqrt{5} & \sqrt{5}\end{pmatrix}\begin{pmatrix}\sqrt{5} & \sqrt{5}\\ \sqrt{5} & \sqrt{5}\end{pmatrix}$ $= \begin{pmatrix}10 & 10\\ 10 & 10\end{pmatrix}$
$a\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}$	$a = \sqrt{5}$ $= \sqrt{5} \times \begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}$	$\begin{pmatrix}\sqrt{5} & \sqrt{5}\\ \sqrt{5} & \sqrt{5}\end{pmatrix}^3$	$\begin{pmatrix}10 & 10\\ 10 & 10\end{pmatrix}\begin{pmatrix}\sqrt{5} & \sqrt{5}\\ \sqrt{5} & \sqrt{5}\end{pmatrix}$ $= \begin{pmatrix}20\sqrt{5} & 20\sqrt{5}\\ 20\sqrt{5} & 20\sqrt{5}\end{pmatrix}$
$a\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}$	$a = \sqrt{5}$ $= \sqrt{5}x\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}$	$\begin{pmatrix}\sqrt{5} & \sqrt{5}\\ \sqrt{5} & \sqrt{5}\end{pmatrix}^4$	$\begin{pmatrix}20\sqrt{5} & 20\sqrt{5}\\ 20\sqrt{5} & 20\sqrt{5}\end{pmatrix}\begin{pmatrix}\sqrt{5} & \sqrt{5}\\ \sqrt{5} & \sqrt{5}\end{pmatrix}$ $\begin{pmatrix}200 & 200\\ 200 & 200\end{pmatrix}$
$a\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}$	$a = \sqrt{5}$ $= \sqrt{5}x\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}$	$\begin{pmatrix}\sqrt{5} & \sqrt{5}\\ \sqrt{5} & \sqrt{5}\end{pmatrix}^5$	$\begin{pmatrix}200 & 200\\ 200 & 200\end{pmatrix}\begin{pmatrix}\sqrt{5} & \sqrt{5}\\ \sqrt{5} & \sqrt{5}\end{pmatrix}$ $= \begin{pmatrix}400\sqrt{5} & 400\sqrt{5}\\ 400\sqrt{5} & 400\sqrt{5}\end{pmatrix}$

$a\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$	$a = \sqrt{5}$ $= \sqrt{5}x\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$	$\begin{pmatrix}\sqrt{5} & \sqrt{5}\\\sqrt{5} & \sqrt{5}\end{pmatrix}^6$	$\begin{pmatrix}400\sqrt{5} & 400\sqrt{5}\\400\sqrt{5} & 400\sqrt{5}\end{pmatrix}\begin{pmatrix}\sqrt{5} & \sqrt{5}\\\sqrt{5} & \sqrt{5}\end{pmatrix}$ $= \begin{pmatrix}4000 & 4000\\4000 & 4000\end{pmatrix}$
$a\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$	$a = -\sqrt{5}$ $=-\sqrt{5}x\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$	$\begin{pmatrix}-\sqrt{5} & -\sqrt{5}\\-\sqrt{5} & -\sqrt{5}\end{pmatrix}^1$	As you may have noticed, I am using a negative number now. - IRRATIONAL NUMBER
$a\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$	$a = -\sqrt{5}$ $= -\sqrt{5}x\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$	$\begin{pmatrix}-\sqrt{5} & -\sqrt{5}\\-\sqrt{5} & -\sqrt{5}\end{pmatrix}^2$	$\begin{pmatrix}-\sqrt{5} & -\sqrt{5}\\-\sqrt{5} & -\sqrt{5}\end{pmatrix}\begin{pmatrix}-\sqrt{5} & -\sqrt{5}\\-\sqrt{5} & -\sqrt{5}\end{pmatrix}$ $= \begin{pmatrix}10 & 10\\10 & 10\end{pmatrix}$
$a\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$	$a = -\sqrt{5}$ $= -\sqrt{5}x\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$	$\begin{pmatrix}-\sqrt{5} & -\sqrt{5}\\-\sqrt{5} & -\sqrt{5}\end{pmatrix}^3$	$\begin{pmatrix}10 & 10\\10 & 10\end{pmatrix}\begin{pmatrix}-\sqrt{5} & -\sqrt{5}\\-\sqrt{5} & -\sqrt{5}\end{pmatrix}$ $= \begin{pmatrix}-20\sqrt{5} & -20\sqrt{5}\\-20\sqrt{5} & -20\sqrt{5}\end{pmatrix}$
$a\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$	$a =-\sqrt{5}$ $= -\sqrt{5}x\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$	$\begin{pmatrix}-\sqrt{5} & -\sqrt{5}\\-\sqrt{5} & -\sqrt{5}\end{pmatrix}^4$	$\begin{pmatrix}-20\sqrt{5} & -20\sqrt{5}\\-20\sqrt{5} & -20\sqrt{5}\end{pmatrix}\begin{pmatrix}-\sqrt{5} & -\sqrt{5}\\-\sqrt{5} & -\sqrt{5}\end{pmatrix}$ $\begin{pmatrix}200 & 200\\200 & 200\end{pmatrix}$
$a\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$	$a = -\sqrt{5}$ $= -\sqrt{5}x\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$	$\begin{pmatrix}-\sqrt{5} & -\sqrt{5}\\-\sqrt{5} & -\sqrt{5}\end{pmatrix}^5$	$\begin{pmatrix}200 & 200\\200 & 200\end{pmatrix}\begin{pmatrix}-\sqrt{5} & -\sqrt{5}\\-\sqrt{5} & -\sqrt{5}\end{pmatrix}$ $= \begin{pmatrix}-400\sqrt{5} & -400\sqrt{5}\\-400\sqrt{5} & -400\sqrt{5}\end{pmatrix}$

$a\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$	$a = -\sqrt{5}$ $= -\sqrt{5}x\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$	$\begin{pmatrix}-\sqrt{5} & -\sqrt{5}\\-\sqrt{5} & -\sqrt{5}\end{pmatrix}^6$	$\begin{pmatrix}-400\sqrt{5} & -400\sqrt{5}\\-400\sqrt{5} & -400\sqrt{5}\end{pmatrix}\begin{pmatrix}-\sqrt{5} & -\sqrt{5}\\-\sqrt{5} & -\sqrt{5}\end{pmatrix}$ $= \begin{pmatrix}4000 & 4000\\4000 & 4000\end{pmatrix}$
Whilst calculating this sequence I came to notice that every second matrix is an integer. I noticed that this pattern went from multiples of two to 10. Every second matrix is multiplied by 10.			

However, knowing that the matrix A = aX a pattern conceived from here will determine the expression.

Example:

$$a\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix} = \begin{pmatrix}a & a\\a & a\end{pmatrix}$$

A^2, A^3, A^4, A^5, A^6

$$\begin{pmatrix}a & a\\a & a\end{pmatrix}^2 = \begin{pmatrix}a & a\\a & a\end{pmatrix}\begin{pmatrix}a & a\\a & a\end{pmatrix} = \begin{pmatrix}2a^2 & 2a^2\\2a^2 & 2a^2\end{pmatrix}$$

$$\begin{pmatrix}a & a\\a & a\end{pmatrix}^3 = \begin{pmatrix}a & a\\a & a\end{pmatrix}^2\begin{pmatrix}a & a\\a & a\end{pmatrix} = \begin{pmatrix}4a^3 & 4a^3\\4a^3 & 4a^3\end{pmatrix}$$

$$\begin{pmatrix}a & a\\a & a\end{pmatrix}^4 = \begin{pmatrix}a & a\\a & a\end{pmatrix}^3\begin{pmatrix}a & a\\a & a\end{pmatrix} = \begin{pmatrix}8a^4 & 8a^4\\8a^4 & 8a^4\end{pmatrix}$$

$$\begin{pmatrix}a & a\\a & a\end{pmatrix}^5 = \begin{pmatrix}a & a\\a & a\end{pmatrix}^4\begin{pmatrix}a & a\\a & a\end{pmatrix} = \begin{pmatrix}16a^5 & 16a^5\\16a^5 & 16a^5\end{pmatrix}$$

$$\begin{pmatrix}a & a\\a & a\end{pmatrix}^6 = \begin{pmatrix}a & a\\a & a\end{pmatrix}^5\begin{pmatrix}a & a\\a & a\end{pmatrix} = \begin{pmatrix}32a^6 & 32a^6\\32a^6 & 32a^6\end{pmatrix}$$

Now, using the previous matrix, a similar pattern has been derived from this matrix, however the only difference is the 'a'. the pattern is that 1, 2, 4, 8, 16, 32 can also be determined as 2^0 2^1, 2^2, 2^3, 2^4, 2^5. Thus creating a pattern of 2^{n-1}, therefore leaving 'a' as a^n. The new matrix evolved from this pattern will be $\begin{pmatrix}2^{n-1}a^n & 2^{n-1}a^n\\2^{n-1}a^n & 2^{n-1}a^n\end{pmatrix}$.

Example:

- Let a = 3 and n=5, hence

$$\begin{pmatrix} 2^{n-1}a^n & 2^{n-1}a^n \\ 2^{n-1}a^n & 2^{n-1}a^n \end{pmatrix} = \begin{pmatrix} 2^{5-1}3^5 & 2^{5-1}3^5 \\ 2^{5-1}3^5 & 2^{5-1}3^5 \end{pmatrix} = \begin{pmatrix} 2^4 3^5 & 2^4 3^5 \\ 2^4 3^5 & 2^4 3^5 \end{pmatrix} = \begin{pmatrix} 3888 & 3888 \\ 3888 & 3888 \end{pmatrix}$$

Or

$$3\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}^5 = \begin{pmatrix} 3 & 3 \\ 3 & 3 \end{pmatrix}^5 = \begin{pmatrix} 3888 & 3888 \\ 3888 & 3888 \end{pmatrix}$$

- Let a = -2.5 and n=3, hence

$$\begin{pmatrix} 2^{n-1}a^n & 2^{n-1}a^n \\ 2^{n-1}a^n & 2^{n-1}a^n \end{pmatrix} = \begin{pmatrix} 2^{3-1}\,-2.5^5 & 2^{3-1}\,-2.5^5 \\ 2^{3-1}\,-2.5^5 & 2^{3-1}\,-2.5^5 \end{pmatrix} = \begin{pmatrix} 2^2\,-2.5^3 & 2^2\,-2.5^3 \\ 2^2\,-2.5^3 & 2^2\,-2.5^3 \end{pmatrix} =$$

$$\begin{pmatrix} -62.5 & -62.5 \\ -62.5 & -62.5 \end{pmatrix}$$

Or

$$-2.5\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}^5 = \begin{pmatrix} -2.5 & -2.5 \\ -2.5 & -2.5 \end{pmatrix}^5 = \begin{pmatrix} -62.5 & -62.5 \\ -62.5 & -62.5 \end{pmatrix}$$

Now that this pattern has been proven using two types of numbers, I will not need to prove it any further.

Right now I will begin expanding on matrix B. I will use the same numbers as I did in matrix A just so that the matrices are even as well.

The Original Matrix	Value of b	B^n	Explanation of how the B^n was achieved
$b\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	a = 2 $= 2 \times \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	$\begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}^1$	Since this type of multiplication of a matrix is called a scalar matrix, the value of b is multiplied by the matrix given, which in this case is $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$, thus becoming $\begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}$ when multiplied by 2 (b). However the answer remains as 2 because the power of B is 1 (^{n}). + INTEGER
$b\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	a = 2 $= 2 \times \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	$\begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}^2$	$\begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}\begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}$ $= \begin{pmatrix} 8 & -8 \\ -8 & 8 \end{pmatrix}$
$b\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	a = 2 $= 2 \times \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	$\begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}^3$	$\begin{pmatrix} 8 & -8 \\ -8 & 8 \end{pmatrix}\begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}$ $= \begin{pmatrix} 32 & -32 \\ -32 & 32 \end{pmatrix}$
$b\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	a = 2 $= 2 \times \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	$\begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}^4$	$\begin{pmatrix} 32 & -32 \\ -32 & 32 \end{pmatrix}\begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}$ $= \begin{pmatrix} 128 & -128 \\ -128 & 128 \end{pmatrix}$
$b\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	a = 2 $= 2 \times \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	$\begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}^5$	$\begin{pmatrix} 128 & -128 \\ -128 & 128 \end{pmatrix}\begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}$ $= \begin{pmatrix} 512 & -512 \\ -512 & 512 \end{pmatrix}$

b $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	a = 2 $= 2 \times \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	$\begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}^6$	$\begin{pmatrix} 512 & -512 \\ -512 & 512 \end{pmatrix}\begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}$ $= \begin{pmatrix} 2048 & -2048 \\ -2048 & 2048 \end{pmatrix}$
b $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	a = -2 $= -2 \times \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	$\begin{pmatrix} -2 & 2 \\ 2 & -2 \end{pmatrix}^1$	The value of a (-2) is multiplied by the matrix given, $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$, which equals to $\begin{pmatrix} -2 & 2 \\ 2 & -2 \end{pmatrix}$ when multiplied by -2 (b). As the power increases, we'll soon come to realize that the matrix values change. - INTEGER
b $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	a = -2 $= -2 \times \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	$\begin{pmatrix} -2 & 2 \\ 2 & -2 \end{pmatrix}^2$	$\begin{pmatrix} -2 & 2 \\ 2 & -2 \end{pmatrix}\begin{pmatrix} -2 & 2 \\ 2 & -2 \end{pmatrix}$ $= \begin{pmatrix} 8 & -8 \\ -8 & 8 \end{pmatrix}$
b $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	a = -2 $= -2 \times \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	$\begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}^3$	$\begin{pmatrix} 8 & -8 \\ -8 & 8 \end{pmatrix}\begin{pmatrix} -2 & 2 \\ 2 & -2 \end{pmatrix}$ $= \begin{pmatrix} -32 & 32 \\ 32 & -32 \end{pmatrix}$
b $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	a = -2 $= -2 \times \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	$\begin{pmatrix} -2 & 2 \\ 2 & -2 \end{pmatrix}^4$	$\begin{pmatrix} -32 & 32 \\ 32 & -32 \end{pmatrix}\begin{pmatrix} -2 & 2 \\ 2 & -2 \end{pmatrix}$ $= \begin{pmatrix} 128 & -128 \\ -128 & 128 \end{pmatrix}$
b $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	a = -2 $= -2 \times \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	$\begin{pmatrix} -2 & 2 \\ 2 & -2 \end{pmatrix}^5$	$\begin{pmatrix} 128 & -128 \\ -128 & 128 \end{pmatrix}\begin{pmatrix} -2 & 2 \\ 2 & -2 \end{pmatrix}$

			$= \begin{pmatrix} -512 & 512 \\ 512 & -512 \end{pmatrix}$
b $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	a = 2 $= 2 \times \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	$\begin{pmatrix} -2 & 2 \\ 2 & -2 \end{pmatrix}^6$	$\begin{pmatrix} -512 & 512 \\ 512 & -512 \end{pmatrix} \begin{pmatrix} -2 & 2 \\ 2 & -2 \end{pmatrix}$ $= \begin{pmatrix} 2048 & -2048 \\ -2048 & 2048 \end{pmatrix}$
b $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	a = 0.25 or $\frac{1}{4}$ $= \frac{1}{4} \times \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	$\begin{pmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{pmatrix}^1$	The value of a (0.25) is multiplied by the matrix given, $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$, which equals to $\begin{pmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ when multiplied by $\frac{1}{4}$ (b). In this matrix I used a fraction instead of a decimal, simply because it's easier to work with. + RATIONAL NUMBER
b $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	a = 0.25 or $\frac{1}{4}$ $= \frac{1}{4} \times \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	$\begin{pmatrix} \frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4} \end{pmatrix}^2$	$\begin{pmatrix} \frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4} \end{pmatrix} \begin{pmatrix} \frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4} \end{pmatrix} = \begin{pmatrix} \frac{1}{8} & -\frac{1}{8} \\ -\frac{1}{8} & \frac{1}{8} \end{pmatrix}$
b $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	a = 0.25 or $\frac{1}{4}$ $= \frac{1}{4} \times \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	$\begin{pmatrix} \frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4} \end{pmatrix}^3$	$\begin{pmatrix} \frac{1}{8} & -\frac{1}{8} \\ -\frac{1}{8} & \frac{1}{8} \end{pmatrix} \begin{pmatrix} \frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4} \end{pmatrix} = \begin{pmatrix} \frac{1}{16} & -\frac{1}{16} \\ -\frac{1}{16} & \frac{1}{16} \end{pmatrix}$
b $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	a = 0.25 or $\frac{1}{4}$ $= \frac{1}{4} \times \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	$\begin{pmatrix} \frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4} \end{pmatrix}^4$	$\begin{pmatrix} \frac{1}{16} & -\frac{1}{16} \\ -\frac{1}{16} & \frac{1}{16} \end{pmatrix} \begin{pmatrix} \frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4} \end{pmatrix} = \begin{pmatrix} \frac{1}{32} & -\frac{1}{32} \\ -\frac{1}{32} & \frac{1}{32} \end{pmatrix}$

b $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	a = 0.25 or $\frac{1}{4}$ $= \frac{1}{4} \times \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	$\begin{pmatrix} \frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4} \end{pmatrix}^5$	$\begin{pmatrix} \frac{1}{32} & -\frac{1}{32} \\ -\frac{1}{32} & \frac{1}{32} \end{pmatrix} \begin{pmatrix} \frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4} \end{pmatrix} = \begin{pmatrix} \frac{1}{64} & -\frac{1}{64} \\ -\frac{1}{64} & \frac{1}{64} \end{pmatrix}$
b $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	a = 0.25 or $\frac{1}{4}$ $= \frac{1}{4} \times \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	$\begin{pmatrix} \frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4} \end{pmatrix}^6$	$\begin{pmatrix} \frac{1}{64} & -\frac{1}{64} \\ -\frac{1}{64} & \frac{1}{64} \end{pmatrix} \begin{pmatrix} \frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4} \end{pmatrix}$ $= \begin{pmatrix} \frac{1}{128} & -\frac{1}{128} \\ -\frac{1}{128} & \frac{1}{128} \end{pmatrix}$
b $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	a = -0.25 or $-\frac{1}{4}$ $= -\frac{1}{4} \times \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	$\begin{pmatrix} -\frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & -\frac{1}{4} \end{pmatrix}^1$	The value of a (-0.25) is multiplied by the matrix given, $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$, which equals to $\begin{pmatrix} -\frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & -\frac{1}{4} \end{pmatrix}$ when multiplied by $-\frac{1}{4}$ (b). In this matrix I used a fraction instead of a decimal, simply because it's easier to work with. - RATIONAL NUMBER
b $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	a = -0.25 or $-\frac{1}{4}$ $= -\frac{1}{4} \times \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	$\begin{pmatrix} -\frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & -\frac{1}{4} \end{pmatrix}^2$	$\begin{pmatrix} -\frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & -\frac{1}{4} \end{pmatrix} \begin{pmatrix} -\frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & -\frac{1}{4} \end{pmatrix} = \begin{pmatrix} \frac{1}{8} & -\frac{1}{8} \\ -\frac{1}{8} & \frac{1}{8} \end{pmatrix}$
b $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	a = -0.25 or $-\frac{1}{4}$ $= -\frac{1}{4} \times \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	$\begin{pmatrix} -\frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & -\frac{1}{4} \end{pmatrix}^3$	$\begin{pmatrix} \frac{1}{8} & -\frac{1}{8} \\ -\frac{1}{8} & \frac{1}{8} \end{pmatrix} \begin{pmatrix} -\frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & -\frac{1}{4} \end{pmatrix} = \begin{pmatrix} -\frac{1}{16} & \frac{1}{16} \\ \frac{1}{16} & -\frac{1}{16} \end{pmatrix}$

b $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	a = -0.25 or $-\frac{1}{4}$ $= -\frac{1}{4} \times \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	$\begin{pmatrix} -\frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & -\frac{1}{4} \end{pmatrix}^4$	$\begin{pmatrix} -\frac{1}{16} & \frac{1}{16} \\ \frac{1}{16} & -\frac{1}{16} \end{pmatrix}\begin{pmatrix} -\frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & -\frac{1}{4} \end{pmatrix} = \begin{pmatrix} \frac{1}{64} & -\frac{1}{64} \\ -\frac{1}{64} & \frac{1}{64} \end{pmatrix}$
b $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	a = -0.25 or $-\frac{1}{4}$ $= -\frac{1}{4} \times \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	$\begin{pmatrix} -\frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & -\frac{1}{4} \end{pmatrix}^5$	$\begin{pmatrix} \frac{1}{64} & -\frac{1}{64} \\ -\frac{1}{64} & \frac{1}{64} \end{pmatrix}\begin{pmatrix} -\frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & -\frac{1}{4} \end{pmatrix} = \begin{pmatrix} -\frac{1}{256} & \frac{1}{256} \\ \frac{1}{256} & -\frac{1}{256} \end{pmatrix}$
b $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	a = -0.25 or $-\frac{1}{4}$ $= -\frac{1}{4} \times \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	$\begin{pmatrix} -\frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & -\frac{1}{4} \end{pmatrix}^6$	$\begin{pmatrix} -\frac{1}{256} & \frac{1}{256} \\ \frac{1}{256} & -\frac{1}{256} \end{pmatrix}\begin{pmatrix} -\frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & -\frac{1}{4} \end{pmatrix}$ $= \begin{pmatrix} \frac{1}{1024} & -\frac{1}{1024} \\ -\frac{1}{1024} & \frac{1}{1024} \end{pmatrix}$
b $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	$a = \sqrt{5}$ $= \sqrt{5} \times \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	$\begin{pmatrix} \sqrt{5} & -\sqrt{5} \\ -\sqrt{5} & \sqrt{5} \end{pmatrix}^1$	The value of a ($\sqrt{5}$) is multiplied by the matrix given, $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$, which equals to $\begin{pmatrix} \sqrt{5} & -\sqrt{5} \\ -\sqrt{5} & \sqrt{5} \end{pmatrix}$ when multiplied by $\sqrt{5}$ (b). In this matrix I used a root 5 instead of a decimal, simply because it's easier to work with. + IRRATIONAL NUMBER
b $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	$a = \sqrt{5}$ $= \sqrt{5} \times \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	$\begin{pmatrix} \sqrt{5} & -\sqrt{5} \\ -\sqrt{5} & \sqrt{5} \end{pmatrix}^2$	$\begin{pmatrix} \sqrt{5} & -\sqrt{5} \\ -\sqrt{5} & \sqrt{5} \end{pmatrix}\begin{pmatrix} \sqrt{5} & -\sqrt{5} \\ -\sqrt{5} & \sqrt{5} \end{pmatrix}$ $= \begin{pmatrix} 10 & -10 \\ -10 & 10 \end{pmatrix}$
b $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	$a = \sqrt{5}$ $= \sqrt{5} \times \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	$\begin{pmatrix} \sqrt{5} & -\sqrt{5} \\ -\sqrt{5} & \sqrt{5} \end{pmatrix}^3$	$\begin{pmatrix} 10 & -10 \\ -10 & 10 \end{pmatrix}\begin{pmatrix} \sqrt{5} & -\sqrt{5} \\ -\sqrt{5} & \sqrt{5} \end{pmatrix}$

			$= \begin{pmatrix} 20\sqrt{5} & -20\sqrt{5} \\ -20\sqrt{5} & 20\sqrt{5} \end{pmatrix}$
$b\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	$a = \sqrt{5}$ $= \sqrt{5} \times \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	$\begin{pmatrix} \sqrt{5} & -\sqrt{5} \\ -\sqrt{5} & \sqrt{5} \end{pmatrix}^4$	$\begin{pmatrix} 20\sqrt{5} & -20\sqrt{5} \\ -20\sqrt{5} & 20\sqrt{5} \end{pmatrix}\begin{pmatrix} \sqrt{5} & -\sqrt{5} \\ -\sqrt{5} & \sqrt{5} \end{pmatrix}$ $\begin{pmatrix} 200 & -200 \\ -200 & 200 \end{pmatrix}$
$b\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	$a = \sqrt{5}$ $= \sqrt{5} \times \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	$\begin{pmatrix} \sqrt{5} & -\sqrt{5} \\ -\sqrt{5} & \sqrt{5} \end{pmatrix}^5$	$\begin{pmatrix} 200 & -200 \\ -200 & 200 \end{pmatrix}\begin{pmatrix} \sqrt{5} & -\sqrt{5} \\ -\sqrt{5} & \sqrt{5} \end{pmatrix}$ $= \begin{pmatrix} 400\sqrt{5} & -400\sqrt{5} \\ -400\sqrt{5} & 400\sqrt{5} \end{pmatrix}$
$b\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	$a = \sqrt{5}$ $= \sqrt{5} \times \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	$\begin{pmatrix} \sqrt{5} & -\sqrt{5} \\ -\sqrt{5} & \sqrt{5} \end{pmatrix}^6$	$\begin{pmatrix} 400\sqrt{5} & -400\sqrt{5} \\ -400\sqrt{5} & 400\sqrt{5} \end{pmatrix}\begin{pmatrix} \sqrt{5} & -\sqrt{5} \\ -\sqrt{5} & \sqrt{5} \end{pmatrix}$ $= \begin{pmatrix} 4000 & -4000 \\ -4000 & 4000 \end{pmatrix}$
$b\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	$a = -\sqrt{5}$ $= -\sqrt{5} x\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	$\begin{pmatrix} -\sqrt{5} & \sqrt{5} \\ \sqrt{5} & -\sqrt{5} \end{pmatrix}^1$	As you may have noticed, I am using a negative number now. - IRRATIONAL NUMBER
$b\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	$a = -\sqrt{5}$ $= -\sqrt{5} x\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	$\begin{pmatrix} -\sqrt{5} & \sqrt{5} \\ \sqrt{5} & -\sqrt{5} \end{pmatrix}^2$	$\begin{pmatrix} -\sqrt{5} & \sqrt{5} \\ \sqrt{5} & -\sqrt{5} \end{pmatrix}\begin{pmatrix} -\sqrt{5} & \sqrt{5} \\ \sqrt{5} & -\sqrt{5} \end{pmatrix}$ $= \begin{pmatrix} 10 & -10 \\ -10 & 10 \end{pmatrix}$

$b\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}$	$a=-\sqrt{5}$ $=-\sqrt{5}x\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}$	$\begin{pmatrix}-\sqrt{5} & \sqrt{5}\\ \sqrt{5} & -\sqrt{5}\end{pmatrix}^3$	$\begin{pmatrix}10 & -10\\ -10 & 10\end{pmatrix}\begin{pmatrix}-\sqrt{5} & \sqrt{5}\\ \sqrt{5} & -\sqrt{5}\end{pmatrix}$ $=\begin{pmatrix}-20\sqrt{5} & 20\sqrt{5}\\ 20\sqrt{5} & -20\sqrt{5}\end{pmatrix}$
$b\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}$	$a=-\sqrt{5}$ $=-\sqrt{5}x\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}$	$\begin{pmatrix}-\sqrt{5} & \sqrt{5}\\ \sqrt{5} & -\sqrt{5}\end{pmatrix}^4$	$\begin{pmatrix}-20\sqrt{5} & 20\sqrt{5}\\ 20\sqrt{5} & -20\sqrt{5}\end{pmatrix}\begin{pmatrix}-\sqrt{5} & \sqrt{5}\\ \sqrt{5} & -\sqrt{5}\end{pmatrix}$ $\begin{pmatrix}200 & -200\\ -200 & 200\end{pmatrix}$
$b\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}$	$a=\sqrt{5}$ $=-\sqrt{5}x\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}$	$\begin{pmatrix}-\sqrt{5} & \sqrt{5}\\ \sqrt{5} & -\sqrt{5}\end{pmatrix}^5$	$\begin{pmatrix}200 & -200\\ -200 & 200\end{pmatrix}\begin{pmatrix}-\sqrt{5} & \sqrt{5}\\ \sqrt{5} & -\sqrt{5}\end{pmatrix}$ $=\begin{pmatrix}-400\sqrt{5} & 400\sqrt{5}\\ 400\sqrt{5} & -400\sqrt{5}\end{pmatrix}$
$b\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}$	$a=-\sqrt{5}$ $=-\sqrt{5}x\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}$	$\begin{pmatrix}-\sqrt{5} & \sqrt{5}\\ \sqrt{5} & -\sqrt{5}\end{pmatrix}^6$	$\begin{pmatrix}-400\sqrt{5} & 400\sqrt{5}\\ 400\sqrt{5} & -400\sqrt{5}\end{pmatrix}\begin{pmatrix}-\sqrt{5} & \sqrt{5}\\ \sqrt{5} & -\sqrt{5}\end{pmatrix}$ $=\begin{pmatrix}4000 & -4000\\ -4000 & 4000\end{pmatrix}$

Now that the matrix B = bY, a pattern was conceived from here will determine the expression.

Example:

$$b\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}=\begin{pmatrix}b & -b\\ -b & b\end{pmatrix}$$

B^2, B^3, B^4, B^5, B^6

$$\begin{pmatrix}b & -b\\ -b & b\end{pmatrix}^2=\begin{pmatrix}b & -b\\ -b & b\end{pmatrix}\begin{pmatrix}b & -b\\ -b & b\end{pmatrix}=\begin{pmatrix}2b^2 & -2b^2\\ -2b^2 & 2b^2\end{pmatrix}$$

$$\begin{pmatrix} b & -b \\ -b & b \end{pmatrix}^3 = \begin{pmatrix} b & -b \\ -b & b \end{pmatrix}^2 \begin{pmatrix} b & -b \\ -b & b \end{pmatrix} = \begin{pmatrix} 4b^3 & -4b^3 \\ -4b^3 & 4b^3 \end{pmatrix}$$

$$\begin{pmatrix} b & -b \\ -b & b \end{pmatrix}^4 = \begin{pmatrix} b & -b \\ -b & b \end{pmatrix}^3 \begin{pmatrix} b & -b \\ -b & b \end{pmatrix} = \begin{pmatrix} 8b^4 & -8b^4 \\ -8b^4 & 8b^4 \end{pmatrix}$$

$$\begin{pmatrix} b & -b \\ -b & b \end{pmatrix}^5 = \begin{pmatrix} b & -b \\ -b & b \end{pmatrix}^4 \begin{pmatrix} b & -b \\ -b & b \end{pmatrix} = \begin{pmatrix} 16b^5 & -16b^5 \\ -16b^5 & 16b^5 \end{pmatrix}$$

$$\begin{pmatrix} b & -b \\ -b & b \end{pmatrix}^6 = \begin{pmatrix} b & -b \\ -b & b \end{pmatrix}^5 \begin{pmatrix} b & -b \\ -b & b \end{pmatrix} = \begin{pmatrix} 32b^6 & -32b^6 \\ -32b^6 & 32b^6 \end{pmatrix}$$

Now, using the previous matrix, a similar pattern has been derived from this matrix, however the only difference is the '*b*'. The pattern is that 1, 2, 4, 8, 16, 32 can also be determined as 2^0 2^1, 2^2, 2^3, 2^4, 2^5. Thus creating a pattern of 2^{n-1}, therefore leaving '*b*' as b^n. The new matrix evolved from this pattern will be $\begin{pmatrix} 2^{n-1}b^n & -2^{n-1}b^n \\ -2^{n-1}b^n & 2^{n-1}b^n \end{pmatrix}$.

<u>Example:</u>

- Let b = 7 and n=4, hence

$$\begin{pmatrix} 2^{n-1}b^n & -2^{n-1}b^n \\ -2^{n-1}b^n & 2^{n-1}b^n \end{pmatrix} = \begin{pmatrix} 2^{4-1}7^4 & -2^{4-1}7^4 \\ -2^{4-1}7^4 & 2^{4-1}7^4 \end{pmatrix} = \begin{pmatrix} 2^3 7^4 & -2^3 7^4 \\ -2^3 7^4 & 2^3 7^4 \end{pmatrix} =$$

$$\begin{pmatrix} 19208 & -19208 \\ -19208 & 19208 \end{pmatrix}$$

Or

$$7\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}^4 = \begin{pmatrix} 7 & -7 \\ -7 & 7 \end{pmatrix}^4 = \begin{pmatrix} 19208 & -19208 \\ -19208 & 19208 \end{pmatrix}$$

- Let b = -0.6 and n=3, hence

$$\begin{pmatrix} 2^{n-1}b^n & -2^{n-1}b^n \\ -2^{n-1}b^n & 2^{n-1}b^n \end{pmatrix} = \begin{pmatrix} 2^2 -0.6^3 & -2^2 -0.6^3 \\ -2^2 -0.6^3 & 2^2 -0.6^3 \end{pmatrix} = \begin{pmatrix} -0.864 & 0.864 \\ 0.864 & -0.864 \end{pmatrix}$$

Or

$$-0.6\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}^6 = \begin{pmatrix} -0.6 & 0.6 \\ 0.6 & -0.6 \end{pmatrix}^3 = \begin{pmatrix} -0.864 & 0.864 \\ 0.864 & -0.864 \end{pmatrix}$$

Now to express these terms, to find the expression of $(A + B)^n$

$$\left[\begin{pmatrix} 2^{n-1}a^n & 2^{n-1}a^n \\ 2^{n-1}a^n & 2^{n-1}a^n \end{pmatrix} + \begin{pmatrix} 2^{n-1}b^n & -2^{n-1}b^n \\ -2^{n-1}b^n & 2^{n-1}b^n \end{pmatrix}\right]^n$$

- Question 3

Now consider M = $\begin{pmatrix} a+b & a-b \\ a-b & a+b \end{pmatrix}$

Show that M= A + B, and that $M^2 = A^2 + B^2$.

Hence, find the general statement that expresses M^n in terms of *aX* and *bY*.

M = A + B

- M = aX + bY , $a\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} + b\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$
- $\begin{pmatrix} a & a \\ a & a \end{pmatrix} + \begin{pmatrix} b & -b \\ -b & b \end{pmatrix} = \begin{pmatrix} a+b & a-b \\ a-b & a+b \end{pmatrix}$

As you can see I used the previous matrix to show how matrix M is reached. Now to show that $M^2 = A^2 + B^2$, I will use the same variables to show how M^2 is attained.

$M^2 = A^2 + B^2$

- $\begin{pmatrix} a & a \\ a & a \end{pmatrix}\begin{pmatrix} a & a \\ a & a \end{pmatrix} + \begin{pmatrix} b & -b \\ -b & b \end{pmatrix}\begin{pmatrix} b & -b \\ -b & b \end{pmatrix}$

$$= \begin{pmatrix} 2a^2 & 2a^2 \\ 2a^2 & 2a^2 \end{pmatrix} + \begin{pmatrix} 2b^2 & -2b^2 \\ -2b^2 & 2b^2 \end{pmatrix} = \begin{pmatrix} 2a^2+2b^2 & 2a^2-2b^2 \\ 2a^2-2b^2 & 2a^2+2b^2 \end{pmatrix}$$

which can also be written as $\begin{pmatrix} a+b & a-b \\ a-b & a+b \end{pmatrix}^2$.

To find the general statement which expresses M^n in terms of *aX* and *bY*.

The general statement would be $\begin{pmatrix} a+b & a-b \\ a-b & a+b \end{pmatrix}^n$.

Now I am going to test the validity of the general statement found ($M^n = A^n + B^n$)

- a = 3, b = 4, n = 2

$$\begin{pmatrix} 2(3)^2+2(4)^2 & 2(3)^2-2(4)^2 \\ 2(3)^2-2(4)^2 & 2(3)^2+2(4)^2 \end{pmatrix} = \begin{pmatrix} 18+32 & 18-32 \\ 18-32 & 18+32 \end{pmatrix} = \begin{pmatrix} 50 & -14 \\ -14 & 50 \end{pmatrix}$$

Or

$$\begin{pmatrix} 3+4 & 3-4 \\ 3-4 & 3+4 \end{pmatrix}^2 = \begin{pmatrix} 7 & -1 \\ -1 & 7 \end{pmatrix}^2 = \begin{pmatrix} 50 & -14 \\ -14 & 50 \end{pmatrix}$$

- a = -3, b = -4, n=2

$$\begin{pmatrix} 2(-3)^2+2(-4)^2 & 2(-3)^2-2(-4)^2 \\ 2(-3)^2-2(-4)^2 & 2(-3)^2+2(-4)^2 \end{pmatrix} = \begin{pmatrix} 18+32 & 18-32 \\ 18-32 & 18+32 \end{pmatrix} = \begin{pmatrix} 50 & -14 \\ -14 & 50 \end{pmatrix}$$

Or

$$\begin{pmatrix} -3+(-4) & -3-(-4) \\ -3-(-4) & -3+(-4) \end{pmatrix}^2 = \begin{pmatrix} -7 & 1 \\ 1 & -7 \end{pmatrix}^2 = \begin{pmatrix} 50 & -14 \\ -14 & 50 \end{pmatrix}$$

- a = 0.5, b = -5, n=4

$$\begin{pmatrix} 8(0.5)^4+8(-5)^4 & 8(0.5)^4-8(-5)^4 \\ 8(0.5)^4-8(-5)^4 & 8(0.5)^4+8(-5)^4 \end{pmatrix} = \begin{pmatrix} 0.5+5000 & 0.5-5000 \\ 0.5-5000 & 0.5+5000 \end{pmatrix} =$$

$$\begin{pmatrix} 5000.5 & -4999.5 \\ -4999.5 & 5000.5 \end{pmatrix}$$

$$\begin{pmatrix} 0.5+(-5) & 0.5-(-5) \\ 0.5-(-5) & 0.5-(-5) \end{pmatrix}^4 = \begin{pmatrix} -4.5 & 5.5 \\ 5.5 & -4.5 \end{pmatrix}^4 = \begin{pmatrix} 5000.5 & -4999.5 \\ -4999.5 & 5000.5 \end{pmatrix}$$

Now I shall discuss the scopes and limitations I encountered during this portfolio. I realized that I can not have the power (^{n}) as a negative or as a fraction or even as a root. Hence proving my point that the power can only be natural numbers. I will prove this right now giving 3 examples.

NEGATIVE

- a = 3, b = 2, n = -2

$\begin{pmatrix} 3+2 & 3-2 \\ 3-2 & 3-2 \end{pmatrix}^{-2} = \begin{pmatrix} 5 & 1 \\ 1 & 5 \end{pmatrix}^{-2}$ = this is not possible. Syntax error.

SQUARE ROOT

- a = 3, b = 2, n = √2

$\begin{pmatrix} 3+2 & 3-2 \\ 3-2 & 3-2 \end{pmatrix}^{\sqrt{2}} = \begin{pmatrix} 5 & 1 \\ 1 & 5 \end{pmatrix}^{\sqrt{2}}$ = this is not possible. Syntax error.

FRACTION

- a = 3, b = 2, n = $\frac{1}{3}$

$\begin{pmatrix} 3+2 & 3-2 \\ 3-2 & 3-2 \end{pmatrix}^{\frac{1}{3}} = \begin{pmatrix} 5 & 1 \\ 1 & 5 \end{pmatrix}^{\frac{1}{3}}$ = this is not possible. Syntax error.

As you may have noticed I used the same numbers for the matrix because, no matter what number is inputted into the matrix you will get a syntax error, because the powers do not exist.

As I conclude this project I have shown all the working out, and I shown how the general statement is processed and I have also shown the different ways of which a matrix can be expressed whilst getting the same answer. The general statement is basically another way of showing how $M^n = (A + B)^n$ can be shown. This project was to strengthen out knowledge about matrix binomials and how they can be used in just simple sequences.