Mathematics Portfolio Type I

The beginnings of matrices and determinants go back to the 2nd century B.C. although matrices can be seen back to the 4th century B.C., however, it was not until near the end of the 17th century that the ideas reappeared and developed further. J.J. O'Connor and E.F. Robertson stated that the Babylonians studied problems which used matrices, as well as the Chinese using matrices early on. All throughout history, the use of matrices has helped mankind progress.

A matrix function such as X = $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ and Y = $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$ can be used to figure out expressions. By calculating X^2, X^3, X^4; Y^2, Y^3, and Y^4 the values of X and Y can be solved for. $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = X^2$. By following rules of multiplying matrices, this can be shown as $\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} (a_{11})(b_{11})+(a_{12})(b_{21}) & (a_{11})(b_{12})+(a_{12})(b_{22}) \\ (a_{21})(b_{11})+(a_{22})(b_{12}) & (a_{21})(b_{12})+(a_{22})(b_{22}) \end{pmatrix}$.

Using X^2 we can conclude that $X^2 = \begin{pmatrix} (1)(1)+(1)(1) & (1)(1)+(1)(1) \\ (1)(1)+(1)(1) & (1)(1)+(1)(1) \end{pmatrix}$ or $\begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}$. One can generalize a statement of a pattern that develops as the matrix goes on. The expression is as follows, $X^n = \begin{pmatrix} 2^{n-1} & 2^{n-1} \\ 2^{n-1} & 2^{n-1} \end{pmatrix}$. The number 2 in the matrix comes from when the product of X^n is solved for. The value of 2^{n-1} is twice the value of X^n. The variable *n* represents what power the matrix is to, such as *n = 2, 3, 4*. We can now solve for the rest of the values of X^n.

$$X^3 = \begin{pmatrix} 2^{3-1} & 2^{3-1} \\ 2^{3-1} & 2^{3-1} \end{pmatrix} = \begin{pmatrix} 2^2 & 2^2 \\ 2^2 & 2^2 \end{pmatrix} = \begin{pmatrix} 4 & 4 \\ 4 & 4 \end{pmatrix}$$

$$X^4 = \begin{pmatrix} 2^{4-1} & 2^{4-1} \\ 2^{4-1} & 2^{4-1} \end{pmatrix} = \begin{pmatrix} 2^3 & 2^3 \\ 2^3 & 2^3 \end{pmatrix} = \begin{pmatrix} 8 & 8 \\ 8 & 8 \end{pmatrix}$$

Further values of X^n can be proven by the expression, $X^n = \begin{pmatrix} 2^{n-1} & 2^{n-1} \\ 2^{n-1} & 2^{n-1} \end{pmatrix}$.

$$X^5 = \begin{pmatrix} 2^{5-1} & 2^{5-1} \\ 2^{5-1} & 2^{5-1} \end{pmatrix} = \begin{pmatrix} 2^4 & 2^4 \\ 2^4 & 2^4 \end{pmatrix} = \begin{pmatrix} 16 & 16 \\ 16 & 16 \end{pmatrix}$$

$$X^6 = \begin{pmatrix} 2^{6-1} & 2^{6-1} \\ 2^{6-1} & 2^{6-1} \end{pmatrix} = \begin{pmatrix} 2^5 & 2^5 \\ 2^5 & 2^5 \end{pmatrix} = \begin{pmatrix} 32 & 32 \\ 32 & 32 \end{pmatrix}$$

Y^2 can be proven with the same expression by slightly changed. Since a_{12} and a_{21} are negative, we much change the expression as $Y^n = \begin{pmatrix} 2^{n-1} & -2^{n-1} \\ -2^{n-1} & 2^{n-1} \end{pmatrix}$ to meet the demands of a_{12} and a_{21}.

$$Y^2 = \begin{pmatrix} 2^{2-1} & -2^{2-1} \\ -2^{2-1} & 2^{2-1} \end{pmatrix} = \begin{pmatrix} 2^1 & -2^1 \\ -2^1 & 2^1 \end{pmatrix} = \begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}$$

$$Y^3 = \begin{pmatrix} 2^{3-1} & -2^{3-1} \\ -2^{3-1} & 2^{3-1} \end{pmatrix} = \begin{pmatrix} 2^2 & -2^2 \\ -2^2 & 2^2 \end{pmatrix} = \begin{pmatrix} 4 & -4 \\ -4 & 4 \end{pmatrix}$$

$$Y^4 = \begin{pmatrix} 2^{4-1} & -2^{4-1} \\ -2^{4-1} & 2^{4-1} \end{pmatrix} = \begin{pmatrix} 2^3 & -2^3 \\ -2^3 & 2^3 \end{pmatrix} = \begin{pmatrix} 8 & -8 \\ -8 & 8 \end{pmatrix}$$

Just to be sure the expression works again, we can find higher values of Y^n.

$$Y^7 = \begin{pmatrix} 2^{7-1} & -2^{7-1} \\ -2^{7-1} & 2^{7-1} \end{pmatrix} = \begin{pmatrix} 2^6 & -2^6 \\ -2^6 & 2^6 \end{pmatrix} = \begin{pmatrix} 64 & -64 \\ -64 & 64 \end{pmatrix}$$

$$Y^8 = \begin{pmatrix} 2^{8-1} & -2^{8-1} \\ -2^{8-1} & 2^{8-1} \end{pmatrix} = \begin{pmatrix} 2^7 & -2^7 \\ -2^7 & 2^7 \end{pmatrix} = \begin{pmatrix} 128 & -128 \\ -128 & 128 \end{pmatrix}$$

By using GDC, the values of X^n and Y^n were double checked for accuracy.

By having the values of X^n and Y^n, they can be used in the expression $(X+Y)^n$ to further advance the knowledge of matrices. $(X+Y)^n$ is the same as the expression $(X^n)+(Y^n)$. Addings matrices would be just like adding $a_{11}+b_{11}$ and $a_{12}+b_{12}$, by adding the values of the columns and rows used, the value of the product, that is trying to be solved, will be found. An example of finding what values that could be put in would be;

$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}^n + \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}^n$. Any variable will do for *n*, so let us make the value of *n* = 2 to be simple. $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}^2 + \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}^2 = \begin{pmatrix} 2^1 & 2^1 \\ 2^1 & 2^1 \end{pmatrix} + \begin{pmatrix} 2^1 & -2^1 \\ -2^1 & 2^1 \end{pmatrix} = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$. $\begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} = 4\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, and $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is the identity matrix of a 2 column by 2 rows, or a 2x2 matrix, which $\begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$ is. Now knowing what the identity is, an expression can be formed. $(X+Y)^n = 2^n\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

Let the value of *n* = 3, 4 to further prove and show the expression $(X+Y)^n = 2^n\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}^3 + \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}^3 = \begin{pmatrix} 4 & 4 \\ 4 & 4 \end{pmatrix} + \begin{pmatrix} 4 & -4 \\ -4 & 4 \end{pmatrix} = \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix} = 2^3\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}^4 + \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}^4 = \begin{pmatrix} 8 & 8 \\ 8 & 8 \end{pmatrix} + \begin{pmatrix} 8 & -8 \\ -8 & 8 \end{pmatrix} = \begin{pmatrix} 16 & -16 \\ -16 & 16 \end{pmatrix} = 2^4\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

The expression of $(X+Y)^n = 2^n\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ fits and is proven by the examples above.

Let **A** = a**X** and **B** = b**Y** wherea and b are constants. Let us use different values of a and b to calculate the values of $\mathbf{A}^2$, $\mathbf{A}^3$, $\mathbf{A}^4$; $\mathbf{B}^2$, $\mathbf{B}^3$, $\mathbf{B}^4$.

a = 4 for $\mathbf{A}^2$

$$4\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}\cdot 4\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix} = \begin{pmatrix}4 & 4\\4 & 4\end{pmatrix}\begin{pmatrix}4 & 4\\4 & 4\end{pmatrix} = \begin{pmatrix}32 & 32\\32 & 32\end{pmatrix}$$

Solving the first example, we can create an expression that should work. The expression for the value can be written as a^n2^{n-1}X. The 2^{n-1} comes from multiplying it with X, which is shown earlier on, $X^n = \begin{pmatrix}2^{n-1} & 2^{n-1}\\2^{n-1} & 2^{n-1}\end{pmatrix}$. The a^n comes from the constant of a which we solve the value of $\mathbf{A}^n$ and raise it to the n power.

Continuing using a = 4, $\mathbf{A}^3$ will now be solved for, using the expression.

$\mathbf{A}^3$

$$4^3\cdot 2^{3-1}\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix} = 64\cdot 2^2\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix} = 256\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix} = \begin{pmatrix}256 & 256\\256 & 256\end{pmatrix}$$

$\mathbf{A}^4$

$$4^4\cdot 2^{4-1}\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix} = 256\cdot 2^3\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix} = 2048\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix} = \begin{pmatrix}2048 & 2048\\2048 & 2048\end{pmatrix}$$

The same expression can be used for when **B** = b**Y**, b^n2^{n-1}Y. The 2^{n-1} comes from multiplying it with Y as shown earlier, $Y^n = \begin{pmatrix}2^{n-1} & -2^{n-1}\\-2^{n-1} & 2^{n-1}\end{pmatrix}$. b^n comes from the constant of b we solve the value of $\mathbf{B}^n$ and raise it to the n power. Since a and b need to be different constants, the value of b in this example will equal 5.

b =5

$\mathbf{B}^2$

$$5^2\cdot 2^{2-1}\begin{pmatrix}1 & -1\\-1 & 1\end{pmatrix} = 25\cdot 2^1\begin{pmatrix}1 & -1\\-1 & 1\end{pmatrix} = 50\begin{pmatrix}1 & -1\\-1 & 1\end{pmatrix} = \begin{pmatrix}50 & -50\\-50 & 50\end{pmatrix}$$

B^3

$$5^3 \cdot 2^{3-1}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} = 125 \cdot 2^2\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} = 500\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 500 & -500 \\ -500 & 500 \end{pmatrix}$$

B^4

$$5^4 \cdot 2^{4-1}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} = 625 \cdot 2^3\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} = 5000\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 5000 & -5000 \\ -5000 & 5000 \end{pmatrix}$$

Keeping in mind what **A** and **B** are, we can now find the expression for $(\mathbf{A}+\mathbf{B})^n$. By already having **A** and **B** solved for, the conclusion of $(\mathbf{A}+\mathbf{B})^n$, which is $(a^n 2^{n-1}X) + (b^n 2^{n-1}Y)$. For example, $a = 4$, $b = 5$, $n = 2$.

$(a^n 2^{n-1}X) + (b^n 2^{n-1}Y)$

$$(4^2 \cdot 2^{2-1}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}) + (5^2 \cdot 2^{2-1}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}) = (16 \cdot 2\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}) + (25 \cdot 2\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}) =$$
$$\begin{pmatrix} 32 & 32 \\ 32 & 32 \end{pmatrix} + \begin{pmatrix} 50 & -50 \\ -50 & 50 \end{pmatrix} = \begin{pmatrix} 82 & -18 \\ -18 & 82 \end{pmatrix}$$

$(aX + bY)^2$

$$(4\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} + 5\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix})^2 = \begin{pmatrix} 4 & 4 \\ 4 & 4 \end{pmatrix} + \begin{pmatrix} 5 & -5 \\ -5 & 5 \end{pmatrix} = \begin{pmatrix} 9 & -1 \\ -1 & 9 \end{pmatrix}^2 = \begin{pmatrix} 82 & -18 \\ -18 & 82 \end{pmatrix}$$

By using GDC, the conclusion of the expression $(a^n 2^{n-1}X) + (b^n 2^{n-1}Y)$ can be brought about by double checking answers. Using the knowledge learned from above, another example can be used.

$a = 5$, $b = 6$, $n = 3$

$$(5^3 \cdot 2^{3-1}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}) + (6^3 \cdot 2^{3-1}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}) = (125 \cdot 2^2\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}) + (216 \cdot 2^2\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}) =$$
$$\begin{pmatrix} 500 & 500 \\ 500 & 500 \end{pmatrix} + \begin{pmatrix} 864 & -864 \\ -864 & 864 \end{pmatrix} = \begin{pmatrix} 1364 & -364 \\ -364 & 1364 \end{pmatrix}$$

Now, let us consider **M** = $\begin{pmatrix} a+b & a-b \\ a-b & a+b \end{pmatrix}$. Since *a* and *b* are constants but of different values, that must be taken into consideration for **M** = $\begin{pmatrix} a+b & a-b \\ a-b & a+b \end{pmatrix}$.

Let *a* = 2 and *b* = 3

$$\mathbf{M} = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix} + \begin{pmatrix} 3 & -3 \\ -3 & 3 \end{pmatrix} = \begin{pmatrix} 5 & -1 \\ -1 & 5 \end{pmatrix}$$

Let *a* = 8 and *b* = 4.

$$\mathbf{M} = \begin{pmatrix} 8+4 & 8-4 \\ 8-4 & 8+4 \end{pmatrix} = \begin{pmatrix} 12 & 4 \\ 4 & 12 \end{pmatrix}$$

M = **A** + **B** which is the same as **M** = aX + bY

$$\mathbf{M} = \begin{pmatrix} 8 & 8 \\ 8 & 8 \end{pmatrix} + \begin{pmatrix} 4 & -4 \\ -4 & 4 \end{pmatrix} = \begin{pmatrix} 12 & 4 \\ 4 & 12 \end{pmatrix}$$

M = **A** + **B** which is the same as **M** = aX + bY which is the same as **M** = $\begin{pmatrix} a+b & a-b \\ a-b & a+b \end{pmatrix}$.

Now since **M** = **A** + **B** the step further would be to see what was $\mathbf{M}^2 = \mathbf{A}^2 + \mathbf{B}^2$, and using the GDC, the answers found can be double checked.

Let *a* = 2 and *b* = 3.

$$\mathbf{M}^2 = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}^2 + \begin{pmatrix} 3 & -3 \\ -3 & 3 \end{pmatrix}^2 = \begin{pmatrix} 8 & 8 \\ 8 & 8 \end{pmatrix} + \begin{pmatrix} 18 & -18 \\ -18 & 18 \end{pmatrix} = \begin{pmatrix} 26 & -10 \\ -10 & 26 \end{pmatrix}$$

Let *a* = 4 and *b* = 7

$$\mathbf{M}^2 = \begin{pmatrix} 4 & 4 \\ 4 & 4 \end{pmatrix}^2 + \begin{pmatrix} 7 & -7 \\ -7 & 7 \end{pmatrix}^2 = \begin{pmatrix} 32 & 32 \\ 32 & 32 \end{pmatrix} + \begin{pmatrix} 98 & -98 \\ -98 & 98 \end{pmatrix} = \begin{pmatrix} 130 & -66 \\ -66 & 130 \end{pmatrix}$$

The general statement for the equations above is $\mathbf{M}^n = a^n 2^{n-1}X + b^n 2^{n-1}Y$. The $a^n 2^{n-1}X + b^n 2^{n-1}Y$ part comes from page 6 with the general equations of what *a*X and *b*Y are. To test the validity of the general statement, different values of all the variables need to be used.

Let $a = 4$, $b = 5$, $n = 3$

$$\mathbf{M}^3 = 4^3 \cdot 2^{3-1}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} + 5^3 \cdot 2^{3-1}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} = 64 \cdot 2^2\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} + 125 \cdot 2^2\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} = 256\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} + 500\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 256 & 256 \\ 256 & 256 \end{pmatrix} + \begin{pmatrix} 500 & -500 \\ -500 & 500 \end{pmatrix} = \begin{pmatrix} 756 & -244 \\ -244 & 756 \end{pmatrix}$$

Let $a = 2$, $b = 6$, $n = 4$

$$\mathbf{M}^4 = 2^4 \cdot 2^{4-1}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} + 6^4 \cdot 2^{4-1}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} = 16 \cdot 2^3\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} + 1296 \cdot 2^3\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} = 128\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} + 10368\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 128 & 128 \\ 128 & 128 \end{pmatrix} + \begin{pmatrix} 10368 & -10368 \\ -10368 & 10368 \end{pmatrix} = \begin{pmatrix} 10496 & -10240 \\ -10240 & 10496 \end{pmatrix}$$

Matrices can be used in many different ways, such as answering real life problems. Matrices can be used in simple tasks, such as grocery shopping. Let's say you need to buy bread, a dozen eggs and milk, which costs \$2.50, \$2.00, and \$3.50 respectively. How much of each item would you need to buy to fit the matrix equation?

Let b = bread, e = dozen eggs and m =milk.

$$\begin{pmatrix} 2.50 \\ 2.00 \\ 3.50 \end{pmatrix} \begin{pmatrix} b \\ e \\ m \end{pmatrix} = \begin{pmatrix} 10.00 \\ 6.00 \\ 21.00 \end{pmatrix}$$

One step approach to this problem would be to solve it linearly.

2.50·b = 4.00
(2.50/2.50) ·b = 10.00/2.50
b = 4

2.00·e = 6.00
(2.00/2.00) ·e = 6.00/2.200
e = 3

3.50·m = 21.00
(3.50/3.50) ·m = 21/3.50
m = 6

$$\begin{pmatrix} 2.50 \\ 2.00 \\ 3.50 \end{pmatrix} \begin{pmatrix} 4 \\ 3 \\ 6 \end{pmatrix} = \begin{pmatrix} 10.00 \\ 6.00 \\ 21.00 \end{pmatrix}$$

The matrix problem is now solved with each variable being account for in the matrix equation.

There is flaw to the general statements above. The matrices can only be multiplied exponentially by any whole integer number that is greater than 0. The matrices must meet specific demands. They must have matching dimensions to be multiplied. An example is $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$. The inner parts of the matrices must match, which the example follows since it's a 2x**2** · **2**x2. The outer portions of the matrices are the resulting dimensions when multiplied. For instances, you cannot multiply 2x**1** · **2**x3 since the inner numbers in bold do not match. The matrices used in all examples for finding general states have all been 2x2, which limits the general statements to only 2x2s. The general statements would not work for a 3x3 or any others besides a 2x2.

The general statement is $\mathbf{M}^n = a^n 2^{n-1}X + b^n 2^{n-1}Y$. One would get to this general statement algebraically when multiplying A or B exponentially. The 2 in the equation is twice as much as the square numbers and that is where the number 2 comes from in the general statement. Since 2 receives less than the power n and this is where the section of $n-1$ arrives from in the equation $\mathbf{M}^n = a^n 2^{n-1}X + b^n 2^{n-1}Y$. When $\mathbf{M} = \mathbf{A} + \mathbf{B}$, $\mathbf{A} = a\mathbf{X}$ and $\mathbf{B} = b\mathbf{Y}$ are given earlier on in the paper from their expressions that were found by solving various problems. An example would be;

$a = 1$ $b = 3$, $n = 2$

$\mathbf{M}^2 = 1^2{\cdot}2^{1\text{-}1}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} + 3^2{\cdot}2^{1\text{-}1}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} = 1{\cdot}1\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} + 9{\cdot}1\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} =$

$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} + \begin{pmatrix} 9 & -9 \\ -9 & 9 \end{pmatrix} = \begin{pmatrix} 10 & -8 \\ -8 & 10 \end{pmatrix}.$

This is the algebraic step and method for solving the general statement of $\mathbf{M}^n = a^n 2^{n-1}X + b^n 2^{n-1}Y$

Works Cited

O'Connor, J.J. and Robertson. "Matrices and determinants." Matrices and determinants Feb 1996. 13 Feb 2008 .