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IB Standard Level Maths: Portfolio Piece 1
Parallels and Parallelograms

Table of Results for 4 transversals:

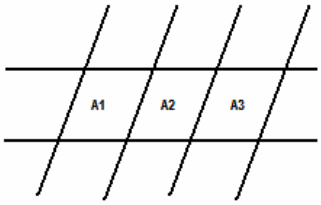
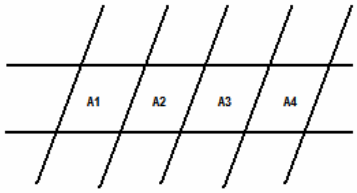
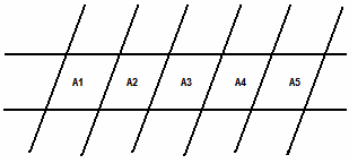
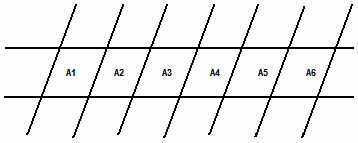
Transversals	Number of Parallelograms	Parallelograms	Diagram
4	6	A_1, A_2, A_3 $A_1 \cup A_2$, $A_2 \cup A_3$ $A_1 \cup A_2 \cup A_3$	

Table of Results for 5, 6 and 7 transversals:

5	10	A_1, A_2, A_3, A_4 $A_1 \cup A_2$, $A_2 \cup A_3$, $A_3 \cup A_4$ $A_1 \cup A_2 \cup A_3$, $A_2 \cup A_3 \cup A_4$ $A_1 \cup A_2 \cup A_3 \cup A_4$	
6	15	A_1, A_2, A_3, A_4, A_5 $A_1 \cup A_2$, $A_2 \cup A_3$, $A_3 \cup A_4$, $A_4 \cup A_5$ $A_1 \cup A_2 \cup A_3$, $A_2 \cup A_3 \cup A_4$, $A_3 \cup A_4 \cup A_5$ $A_1 \cup A_2 \cup A_3 \cup A_4$, $A_2 \cup A_3 \cup A_4 \cup A_5$ $A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5$	
7	21	A_1, A_2, A_3, A_4, A_5, A_6 $A_1 \cup A_2$, $A_2 \cup A_3$, $A_3 \cup A_4$, $A_4 \cup A_5$, $A_5 \cup A_6$ $A_1 \cup A_2 \cup A_3$, $A_2 \cup A_3 \cup A_4$, $A_3 \cup A_4 \cup A_5$, $A_4 \cup A_5 \cup A_6$ $A_1 \cup A_2 \cup A_3 \cup A_4$, $A_2 \cup A_3 \cup A_4 \cup A_5$, $A_3 \cup A_4 \cup A_5 \cup A_6$ $A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5$, $A_2 \cup A_3 \cup A_4 \cup A_5 \cup A_6$ $A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5 \cup A_6$	

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→ Let n = number of transversals and let p = number of parallelograms

Transversals (n)	**Parallelograms (p)**
2	1
3	3 (1 + 2)
4	6 (1 + 2 + 3)
5	10 (1 + 2 + 3 + 4)
6	15 (1 + 2 + 3 + 4 + 5)
7	21 (1 + 2 + 3 + 4 + 5 + 6)
n	1 + 2 + … + (n – 1)

Use of Technology:

Using the TI – 84 Plus, press *STAT* → 1: *Edit.*
Type in L_1, L_2: (2, 1)
(3, 3)
(4, 6)
…etc.

Using Quadreg, L_1, L_2, rule → $y = 0.5x^2 - 0.5x + 0$
$= \frac{1}{2}x^2 - \frac{1}{2}x$
$= (x^2 - x) \div 2$

→ If n is the number of transversals and p is the number of parallelograms, then the rule is:

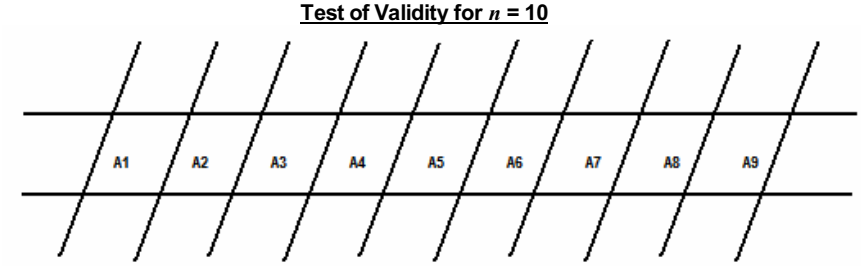
$$p = \tfrac{1}{2}n(n-1)$$

OR

$$p = (n^2 - n) \div 2$$

General statement:
If there are n transversals and two horizontal lines, then p = sum of all integers from 1 to (n – 1)

Test of Validity for n = 10



→ **e.g. 1)** 10 transversals, 2 horizontal lines

- A_1,
- A_2,
- A_3,
- A_4,
- A_5,
- A_6,
- A_7,
- A_8,
- A_9
  = 9

- $A_1 \cup A_2$,
- $A_2 \cup A_3$,
- $A_3 \cup A_4$,
- $A_4 \cup A_5$,
- $A_5 \cup A_6$,

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- $A_6 \cup A_7$,
- $A_7 \cup A_8$,
- $A_8 \cup A_9$
  = **8**

- $A_1 \cup A_2 \cup A_3$,
- $A_2 \cup A_3 \cup A_4$,
- $A_3 \cup A_4 \cup A_5$,
- $A_4 \cup A_5 \cup A_6$,
- $A_5 \cup A_6 \cup A_7$,
- $A_6 \cup A_7 \cup A_8$,
- $A_7 \cup A_8 \cup A_9$
  = **7**

- $A_1 \cup A_2 \cup A_3 \cup A_4$,
- $A_2 \cup A_3 \cup A_4 \cup A_5$,
- $A_3 \cup A_4 \cup A_5 \cup A_6$,
- $A_4 \cup A_5 \cup A_6 \cup A_7$,
- $A_5 \cup A_6 \cup A_7 \cup A_8$,
- $A_6 \cup A_7 \cup A_8 \cup A_9$
  = **6**

- $A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5$,
- $A_2 \cup A_3 \cup A_4 \cup A_5 \cup A_6$,
- $A_3 \cup A_4 \cup A_5 \cup A_6 \cup A_7$,
- $A_4 \cup A_5 \cup A_6 \cup A_7 \cup A_8$,
- $A_5 \cup A_6 \cup A_7 \cup A_8 \cup A_9$
  = **5**

- $A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5 \cup A_6$,
- $A_2 \cup A_3 \cup A_4 \cup A_5 \cup A_6 \cup A_7$,
- $A_3 \cup A_4 \cup A_5 \cup A_6 \cup A_7 \cup A_8$,
- $A_4 \cup A_5 \cup A_6 \cup A_7 \cup A_8 \cup A_9$
  = **4**

- $A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5 \cup A_6 \cup A_7$,
- $A_2 \cup A_3 \cup A_4 \cup A_5 \cup A_6 \cup A_7 \cup A_8$,
- $A_3 \cup A_4 \cup A_5 \cup A_6 \cup A_7 \cup A_8 \cup A_9$
  = **3**

- $A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5 \cup A_6 \cup A_7 \cup A_8$,
- $A_2 \cup A_3 \cup A_4 \cup A_5 \cup A_6 \cup A_7 \cup A_8 \cup A_9$
  = **2**

- $\cup A_2 \cup A_3 \cup A_4 \cup A_5 \cup A_6 \cup A_7 \cup A_8 \cup A_9$
  = **1**

➔ 45

p = sum of all integers from 1 to (10 – 1)
= sum of all integers from 1 to 9
= 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9
= 45

$p = (n^2 - n) \div 2$
$p = 10^2 - 10 \div 2$
$= 90 \div 2$
$= 45$

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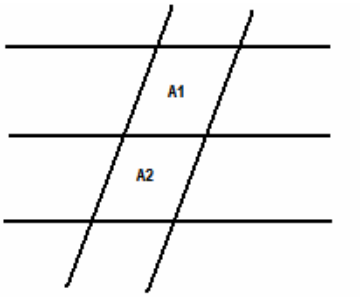
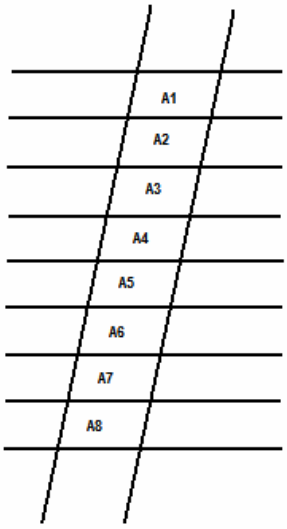
→ Let m = number of horizontal lines



If there are three horizontal lines, intersecting two transversals (m = 3, n = 2) then p = 3. Similarly, if there are three transversals, and two horizontal lines, (m = 2, n = 3), then we also obtain p = 3.

Conclusion:
Hence, m horizontal lines and n transversals produce the same amount of parallelograms as n horizontal lines and m transversals.

General Statement:
If there are m horizontal lines, and two transversals, then p = sum of all integers from 1 to (m – 1). Note that this rule is identical to the above investigation of *n* transversals and two horizontal lines.



Test of Validity for m = 10, n = 2
We will now prove that if m = 10, where n = 2, we will get the same p value of 45 as example 1 above where m = 2 and n = 10.

→ **e.g. 2)** 10 horizontal lines, 2 transversals

Manual method:

- A_1,
- A_2,
- A_3,
- A_4,
- A_5,
- A_6,
- A_7,
- A_8,
- A_9

= **9**

- $A_1 \cup A_2$,
- $A_2 \cup A_3$,
- $A_3 \cup A_4$,
- $A_4 \cup A_5$,
- $A_5 \cup A_6$,
- $A_6 \cup A_7$,
- $A_7 \cup A_8$,
- $A_8 \cup A_9$

= **8**

- $A_1 \cup A_2 \cup A_3$,
- $A_2 \cup A_3 \cup A_4$,
- $A_3 \cup A_4 \cup A_5$,
- $A_4 \cup A_5 \cup A_6$,
- $A_5 \cup A_6 \cup A_7$,
- $A_6 \cup A_7 \cup A_8$,
- $A_7 \cup A_8 \cup A_9$,

= **7**

- $A_1 \cup A_2 \cup A_3 \cup A_4$,
- $A_2 \cup A_3 \cup A_4 \cup A_5$,
- $A_3 \cup A_4 \cup A_5 \cup A_6$,
- $A_4 \cup A_5 \cup A_6 \cup A_7$,
- $A_5 \cup A_6 \cup A_7 \cup A_8$,
- $A_6 \cup A_7 \cup A_8 \cup A_9$

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= **6**

- $A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5$,
- $A_2 \cup A_3 \cup A_4 \cup A_5 \cup A_6$,
- $A_3 \cup A_4 \cup A_5 \cup A_6 \cup A_7$,
- $A_4 \cup A_5 \cup A_6 \cup A_7 \cup A_8$,
- $A_5 \cup A_6 \cup A_7 \cup A_8 \cup A_9$

= **5**

- $A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5 \cup A_6$,
- $A_2 \cup A_3 \cup A_4 \cup A_5 \cup A_6 \cup A_7$,
- $A_3 \cup A_4 \cup A_5 \cup A_6 \cup A_7 \cup A_8$,
- $A_4 \cup A_5 \cup A_6 \cup A_7 \cup A_8 \cup A_9$

= **4**

- $A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5 \cup A_6 \cup A_7$,
- $A_2 \cup A_3 \cup A_4 \cup A_5 \cup A_6 \cup A_7 \cup A_8$,
- $A_3 \cup A_4 \cup A_5 \cup A_6 \cup A_7 \cup A_8 \cup A_9$

= **3**

- $A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5 \cup A_6 \cup A_7 \cup A_8$,
- $A_2 \cup A_3 \cup A_4 \cup A_5 \cup A_6 \cup A_7 \cup A_8 \cup A_9$

= **2**

- $\cup A_2 \cup A_3 \cup A_4 \cup A_5 \cup A_6 \cup A_7 \cup A_8 \cup A_9$

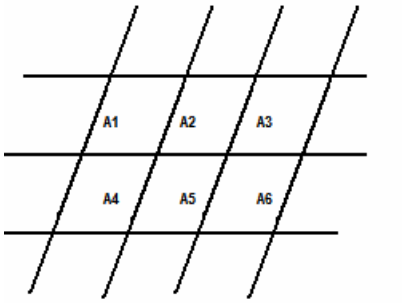
= **1**

➔ **sum of all parallelograms:** 45

<u>Using equation:</u>
p = sum of all integers from 1 to (10 – 1)
= sum of all integers from 1 to 9
= 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9
= 45

$p = (m^2 - m) \div 2$
$p = 10^2 - 10 \div 2$
= 90 ÷ 2
= 45

We now investigate p for m horizontal lines and n transversals where m, n > 2.
→ Consider the diagram below where m = 3 and n = 4.



- A_1,
- A_2,
- A_3,
- A_4,
- A_5,
- A_6

= **6**

- $A_1 \cup A_2$,
- $A_2 \cup A_3$,
- $A_4 \cup A_5$,
- $A_5 \cup A_6$,
- $A_1 \cup A_4$,
- $A_2 \cup A_5$,
- $A_3 \cup A_6$

= **7**

- $A_1 \cup A_2 \cup A_3$,
- $A_4 \cup A_5 \cup A_6$,

= **2**

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- $A_1 \cup A_2 \cup A_4 \cup A_5$,
- $A_2 \cup A_3 \cup A_5 \cup A_6$
  = 2

- $A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5 \cup A_6$
  = 1

→ p = 6 + 7 + 2 + 2 + 1
= 18

From the earlier conclusion, p will also equal 18 if m = 4 and n = 3.
Now refer to the table of p values below.

<u>Table of p values for different values of m and n</u>
m = parallel lines

→ The results below are processed data from Microsoft Excel.

n = transversals	**2**	**3**	**4**	**5**	**6**	**7**
2	1	3	6	10	15	21
3	3	9	18	30	45	63
4	6	18	36	60	90	126
5	10	30	60	100	150	210
6	15	45	90	150	225	
7	21	63	126	210		

etc.

From the table, if m = 2 and n = 5, then p = 10. Also, if m = 4 and n = 2, then p = 6. The table also shows that if m = 4 and n = 5, then p = 10 x 6. (= 60)

<u>General statement:</u>
It appears that for any diagram with m horizontal lines, and n transversals, the resultant value of p equals the product of p_1 and p_2, where;
p_1 = number of parallelograms for m horizontal lines and two transversals
p_2 = number of parallelograms for 2 horizontal lines and n transversals.

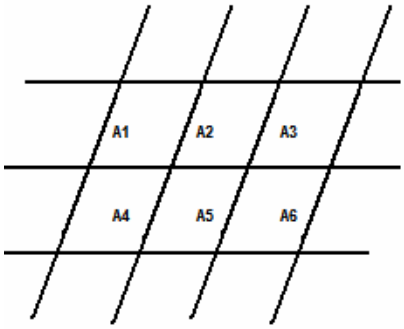
Hence, for any diagram with m horizontal lines and n transversals,

$$p = \tfrac{1}{2} m (m - 1) \times \tfrac{1}{2} n (n - 1)$$

OR

$$p = (m \times n) \div 4 (n - 1)(m - 1)$$

<u>Test of validity for m = 4, n = 3</u>



- A_1,
- A_2,
- A_3,
- A_4,
- A_5,
- A_6
  = 6

- $A_1 \cup A_2$,
- $A_2 \cup A_3$,
- $A_4 \cup A_5$,
- $A_5 \cup A_6$,
- $A_1 \cup A_4$,
- $A_2 \cup A_5$,

- $A_3 \cup A_6$
  = 7

- $A_1 \cup A_2 \cup A_3$,

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- $A_4 \cup A_5 \cup A_6$
  = 2

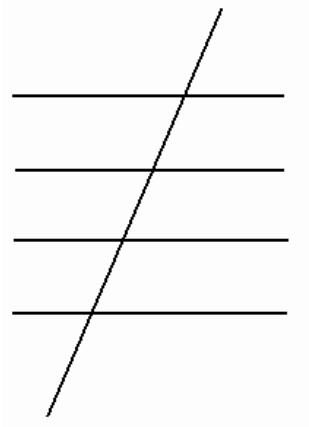
- $A_1 \cup A_2 \cup A_4 \cup A_5$,
- $A_2 \cup A_3 \cup A_5 \cup A_6$
  = 2

- $A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5 \cup A_6$
  = 1

→ p = 6 + 7 + 2 + 2 + 1
= 18

→ p = ½ m (m – 1) x ½ n (n – 1)
= ½ 4 (4 – 1) x ½ 3 (3 – 1)
= 6 x 3
= 18

Scope/limitations:
The formula will be valid for m, $n \geq 2$. If either value were to be 1 or 0, it would be impossible to create any parallelograms.



Explanation of generalisation:
A diagram with m horizontal lines and 2 transversals creates p_1 parallelograms.
A diagram with 2 horizontal lines and n transversals creates p_2 parallelograms.

It follows that if a diagram were created, with m horizontal lines, and n transversals, we would be able to fit p_1 parallelograms vertically and p_2 parallelograms horizontally, giving us a total of $p_1 \times p_2$ parallelograms.