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Maths Portfolio

SL Type 1

Matrix Binomials

In this mathematics portfolio we are instructed to investigate matrix binomials and algebraically find a general statement that combines perfectly with our matrices and equations given.

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MATRIX BINOMIALS

Given that:

$X=\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}$ and $Y=\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}$

We calculated $X^2, X^3, X^4;\ Y^2, Y^3, Y^4$

Therefore:

$$X=\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}$$

$$X^2=\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}=\begin{pmatrix}2 & 2\\ 2 & 2\end{pmatrix}$$

$$X^3=\begin{pmatrix}2 & 2\\ 2 & 2\end{pmatrix}\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}=\begin{pmatrix}4 & 4\\ 4 & 4\end{pmatrix}$$

$$X^4=\begin{pmatrix}4 & 4\\ 4 & 4\end{pmatrix}\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}=\begin{pmatrix}8 & 8\\ 8 & 8\end{pmatrix}$$

And

$$Y=\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}$$

$$Y^2=\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}=\begin{pmatrix}2 & -2\\ -2 & 2\end{pmatrix}$$

$$Y^3=\begin{pmatrix}2 & -2\\ -2 & 2\end{pmatrix}\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}=\begin{pmatrix}4 & -4\\ -4 & 4\end{pmatrix}$$

$$Y^4=\begin{pmatrix}4 & -4\\ -4 & 4\end{pmatrix}\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}=\begin{pmatrix}8 & -8\\ -8 & 8\end{pmatrix}$$

We were then requested to find expressions for $X^n,\ Y^n$ and $(X+Y)^n$ by considering the integer powers of *X* and *Y*:

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$$X^n = 2^{n-1}X$$
$$Y^n = 2^{n-1}Y$$
$$(X+Y)^n = 2^{n-1}(X+Y)$$

These expressions were found by observing that the result of $X^n,\ Y^n \text{ and } (X+Y)^n$ was always the matrix to the power of *n* multiplied by 2 to the power of *n-1*. The sequence of results gives us: 1, 2, 4 and 8, reaffirming our expressions are correct because $1 = 2^0,\ 2 = 2^1,\ 4 = 2^2,\ 8 = 2^3$.

Given that:

$A = aX$ and $B = bY$ where *a* and *b* are constants

We were asked to find $A^2, A^3, A^4; B^2, B^3, B^4$ using different values of *a* and *b* and then find the expressions for $A^n,\ B^n \text{ and } (A+B)^n$.

Therefore:

$$A = a\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$
$$A = \begin{pmatrix} a & a \\ a & a \end{pmatrix}$$

And:

$$B = b\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$
$$B = \begin{pmatrix} b & -b \\ -b & b \end{pmatrix}$$

Assuming that *a* = 2

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$$A = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} 8 & 8 \\ 8 & 8 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} 8 & 8 \\ 8 & 8 \end{pmatrix}\begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} 32 & 32 \\ 32 & 32 \end{pmatrix}$$

$$A^4 = \begin{pmatrix} 32 & 32 \\ 32 & 32 \end{pmatrix}\begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} 128 & 128 \\ 128 & 128 \end{pmatrix}$$

Assuming that a = 3

$$A = \begin{pmatrix} 3 & 3 \\ 3 & 3 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 3 & 3 \\ 3 & 3 \end{pmatrix}\begin{pmatrix} 3 & 3 \\ 3 & 3 \end{pmatrix} = \begin{pmatrix} 18 & 18 \\ 18 & 18 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} 18 & 18 \\ 18 & 18 \end{pmatrix}\begin{pmatrix} 3 & 3 \\ 3 & 3 \end{pmatrix} = \begin{pmatrix} 108 & 108 \\ 108 & 108 \end{pmatrix}$$

$$A^4 = \begin{pmatrix} 108 & 108 \\ 108 & 108 \end{pmatrix}\begin{pmatrix} 3 & 3 \\ 3 & 3 \end{pmatrix} = \begin{pmatrix} 648 & 648 \\ 648 & 648 \end{pmatrix}$$

Assuming that a = 6

$$A = \begin{pmatrix} 6 & 6 \\ 6 & 6 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 6 & 6 \\ 6 & 6 \end{pmatrix}\begin{pmatrix} 6 & 6 \\ 6 & 6 \end{pmatrix} = \begin{pmatrix} 72 & 72 \\ 72 & 72 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} 72 & 72 \\ 72 & 72 \end{pmatrix}\begin{pmatrix} 6 & 6 \\ 6 & 6 \end{pmatrix} = \begin{pmatrix} 864 & 864 \\ 864 & 864 \end{pmatrix}$$

$$A^4 = \begin{pmatrix} 864 & 864 \\ 864 & 864 \end{pmatrix}\begin{pmatrix} 6 & 6 \\ 6 & 6 \end{pmatrix} = \begin{pmatrix} 10368 & 10368 \\ 10368 & 10368 \end{pmatrix}$$

Assuming that a = -4

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$$A=\begin{pmatrix}-4 & -4\\ -4 & -4\end{pmatrix}$$

$$A^2=\begin{pmatrix}-4 & -4\\ -4 & -4\end{pmatrix}\begin{pmatrix}-4 & -4\\ -4 & -4\end{pmatrix}=\begin{pmatrix}32 & 32\\ 32 & 32\end{pmatrix}$$

$$A^3=\begin{pmatrix}32 & 32\\ 32 & 32\end{pmatrix}\begin{pmatrix}-4 & -4\\ -4 & -4\end{pmatrix}=\begin{pmatrix}-256 & -256\\ -256 & -256\end{pmatrix}$$

$$A^4=\begin{pmatrix}-256 & -256\\ -256 & -256\end{pmatrix}\begin{pmatrix}-4 & -4\\ -4 & -4\end{pmatrix}=\begin{pmatrix}2048 & 2048\\ 3048 & 2048\end{pmatrix}$$

Assuming that *b* = -1

$$B=\begin{pmatrix}-1 & 1\\ 1 & -1\end{pmatrix}$$

$$B^2=\begin{pmatrix}-1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}-1 & 1\\ 1 & -1\end{pmatrix}=\begin{pmatrix}2 & -2\\ -2 & 2\end{pmatrix}$$

$$B^3=\begin{pmatrix}2 & -2\\ -2 & 2\end{pmatrix}\begin{pmatrix}-1 & 1\\ 1 & -1\end{pmatrix}=\begin{pmatrix}-4 & 4\\ 4 & -4\end{pmatrix}$$

$$B^4=\begin{pmatrix}-4 & 4\\ 4 & -4\end{pmatrix}\begin{pmatrix}-1 & 1\\ 1 & -1\end{pmatrix}=\begin{pmatrix}8 & -8\\ -8 & 8\end{pmatrix}$$

Assuming that *b* = 4

$$B=\begin{pmatrix}4 & -4\\ -4 & 4\end{pmatrix}$$

$$B^2=\begin{pmatrix}4 & -4\\ -4 & 4\end{pmatrix}\begin{pmatrix}4 & -4\\ -4 & 4\end{pmatrix}=\begin{pmatrix}32 & -32\\ -32 & 32\end{pmatrix}$$

$$B^3=\begin{pmatrix}32 & -32\\ -32 & 32\end{pmatrix}\begin{pmatrix}4 & -4\\ -4 & 4\end{pmatrix}=\begin{pmatrix}256 & -256\\ -256 & 256\end{pmatrix}$$

$$B^4=\begin{pmatrix}256 & -256\\ -256 & 256\end{pmatrix}\begin{pmatrix}4 & -4\\ -4 & 4\end{pmatrix}=\begin{pmatrix}2048 & -2048\\ -2048 & 2048\end{pmatrix}$$

Assuming that *b* = 5

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$$B=\begin{pmatrix}5 & -5\\ -5 & 5\end{pmatrix}$$

$$B^2=\begin{pmatrix}5 & -5\\ -5 & 5\end{pmatrix}\begin{pmatrix}5 & -5\\ -5 & 5\end{pmatrix}=\begin{pmatrix}50 & -50\\ -50 & 50\end{pmatrix}$$

$$B^3=\begin{pmatrix}50 & -50\\ -50 & 50\end{pmatrix}\begin{pmatrix}5 & -5\\ -5 & 5\end{pmatrix}=\begin{pmatrix}500 & -500\\ -500 & 500\end{pmatrix}$$

$$B^4=\begin{pmatrix}500 & -500\\ -500 & 500\end{pmatrix}\begin{pmatrix}5 & -5\\ -5 & 5\end{pmatrix}=\begin{pmatrix}5000 & -5000\\ -5000 & 5000\end{pmatrix}$$

Assuming that *b* = 7

$$B=\begin{pmatrix}7 & -7\\ -7 & 7\end{pmatrix}$$

$$B^2=\begin{pmatrix}7 & -7\\ -7 & 7\end{pmatrix}\begin{pmatrix}7 & -7\\ -7 & 7\end{pmatrix}=\begin{pmatrix}98 & -98\\ -98 & 98\end{pmatrix}$$

$$B^3=\begin{pmatrix}98 & -98\\ -98 & 98\end{pmatrix}\begin{pmatrix}7 & -7\\ -7 & 7\end{pmatrix}=\begin{pmatrix}1372 & -1372\\ -1372 & 1372\end{pmatrix}$$

$$B^4=\begin{pmatrix}1372 & -1372\\ -1372 & 1372\end{pmatrix}\begin{pmatrix}7 & -7\\ -7 & 7\end{pmatrix}=\begin{pmatrix}19208 & -19208\\ -19208 & 19208\end{pmatrix}$$

To find the expression for A^n I observed that the final results achieved were always the value chosen for *a* multiplied by the matrix *X* and the product to the power of *n*. For example (assuming *a* = 2):

$$A^n=2\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}^n$$

$$A^n=\begin{pmatrix}2 & 2\\ 2 & 2\end{pmatrix}^n$$

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As a result we can see that:

$$A^n = (aX)^n$$

$$A^n = a^n X^n$$

And since we know $X^n = 2^{n-1} X$

Hence: $A^n = a^n 2^{n-1} X$ or $A^n = a^n 2^{n-1}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

To find the expression for B^n I did the same as for A^n changing the value *a* to *b* and *X* to *Y.*
For example (assuming *b* = -1):

$$B^n = -1\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}^n$$

$$B^n = \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}^n$$

As a result we can see that:

$$B^n = (bY)^n$$

$$B^n = b^n Y^n$$

And since we know $Y^n = 2^{n-1} Y$

Hence: $B^n = b^n 2^{n-1} Y$ or $B^n = b^n 2^{n-1}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$

To find $(A+B)^n$ I used the binomial theorem

$$(A+B) = A + B$$

$$(A+B)^2 = A^2 + 2AB + B^2$$

$$(A+B)^3 = A^3 + 3A^2B + 3AB^2 + B^3$$

$$(A+B)^4 = A^4 + 4A^3B^2 + 4A^2B^2 + 4AB^3 + B^4$$

From the binomial theorem we can see that the values of *A* and *B* multiply by each other on every term except the first and the last, where we find A^n and B^n.

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However if we multiply matrix *A* by *B* we will see that the product will be a zero matrix.

$$\begin{pmatrix} a & a \\ a & a \end{pmatrix}\begin{pmatrix} b & -b \\ -b & b \end{pmatrix}=\begin{pmatrix} ab-ab & ab-ab \\ ab-ab & ab-ab \end{pmatrix}=\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

This allows us to cancel every term in which *A* multiplies *B* or vice-versa, as the result will be zero.

Therefore if we cancel these terms we will be only left with $A^n + B^n$.

This means that $(A+B)^n = A^n + B^n$

Now we were given:

$$M = \begin{pmatrix} a+b & a-b \\ a-b & a+b \end{pmatrix}$$

And asked to prove that $M = A+B$ and $M^2 = A^2 + B^2$
So:

$$\underset{A}{\begin{pmatrix} a & a \\ a & a \end{pmatrix}}+\underset{B}{\begin{pmatrix} b & -b \\ -b & b \end{pmatrix}}=\underset{M}{\begin{pmatrix} a+b & a-b \\ a-b & a+b \end{pmatrix}}$$

To find $M^2 = A^2 + B^2$, first we need to calculate *A²*, *B²* and *M².*
So:

$$A^2 = \begin{pmatrix} a & a \\ a & a \end{pmatrix}\begin{pmatrix} a & a \\ a & a \end{pmatrix}=\begin{pmatrix} 2a^2 & 2a^2 \\ 2a^2 & 2a^2 \end{pmatrix}$$

$$B^2 = \begin{pmatrix} b & -b \\ -b & b \end{pmatrix}\begin{pmatrix} b & -b \\ -b & b \end{pmatrix}=\begin{pmatrix} 2b^2 & -2b^2 \\ -2b^2 & 2b^2 \end{pmatrix}$$

$$M^2 = \begin{pmatrix} a+b & a-b \\ a-b & a+b \end{pmatrix}\begin{pmatrix} a+b & a-b \\ a-b & a+b \end{pmatrix}=\begin{pmatrix} 2a^2+2b^2 & 2a^2-2b^2 \\ 2a^2-2b^2 & 2a^2+2b^2 \end{pmatrix}$$

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Therefore:

$$\begin{pmatrix} 2a^2 & 2a^2 \\ 2a^2 & 2a^2 \end{pmatrix}\begin{pmatrix} 2b^2 & -2b^2 \\ -2b^2 & 2b^2 \end{pmatrix}=\begin{pmatrix} 2a^2+2b^2 & 2a^2-2b^2 \\ 2a^2-2b^2 & 2a^2+2b^2 \end{pmatrix}$$

To find the general statement that expresses M^n in terms of *aX* and *bY* we first need to continue the sequence and find M^3 and M^4.

So:

$$M^3 = M^2M$$

$$M^3 = \begin{pmatrix} 2a^2+2b^2 & 2a^2-2b^2 \\ 2a^2-2b^2 & 2a^2+2b^2 \end{pmatrix}\begin{pmatrix} a+b & a-b \\ a-b & a+b \end{pmatrix}=\begin{pmatrix} 4a^3+4b^3 & 4a^3-4b^3 \\ 4a^3-4b^3 & 4a^3+4b^3 \end{pmatrix}$$

$$M^4 = M^3M$$

$$M^4 = \begin{pmatrix} 4a^3+4b^3 & 4a^3-4b^3 \\ 4a^3-4b^3 & 4a^3+4b^3 \end{pmatrix}\begin{pmatrix} a+b & a-b \\ a-b & a+b \end{pmatrix}=\begin{pmatrix} 8a^4+8b^4 & 8a^4-8b^4 \\ 8a^4-8b^4 & 8a^4+8b^4 \end{pmatrix}$$

By analyzing all 4 values for M^n we can see that the result can be put into 2^n multiplied by *2a+2b* in a sequence.
Example:

$$a+b = 2^0(2a+2b)$$

$$2a^2+2b^2 = 2^1(2a+2b)$$

$$4a^3+4b^3 = 2^2(2a+2b)$$

$$8a^4+8b^4 = 2^3(2a+2b)$$

Hence we can say that our general statement is:

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$$M^n = 2^{n-1}(a+b)^n$$

$$M^n = 2^{n-1}(a^n X^n + b^n Y^n)$$

$$M^n = 2^{n-1}(a^n \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}^n + b^n \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}^n)$$

$$M^n = \begin{pmatrix} 2^{n-1}a^n & 2^{n-1}a^n \\ 2^{n-1}a^n & 2^{n-1}a^n \end{pmatrix} + \begin{pmatrix} 2^{n-1}b^n & -\left| 2^{n-1}b^n \right| \\ -\left| 2^{n-1}b^n \right| & 2^{n-1}b^n \end{pmatrix}$$

$$M^n = 2^{n-1}a^n X^n + 2^{n-1}b^n Y^n$$

$$M^n = a^n X^n + b^n Y^n$$

Testing the validity of my general statement, to do this we had to get the same results for both expressions:

$$M^n = a^n X^n + b^n Y^n$$

$$M^n = (A+B)^n$$

Assuming: *a* = 2, *b* = 4, *n* = 1

$$M^n = (A+B)^n$$

$$M^1 = \left(\begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix} + \begin{pmatrix} 4 & -4 \\ -4 & 4 \end{pmatrix} \right)^1$$

$$M^1 = \begin{pmatrix} 6 & -2 \\ -2 & 6 \end{pmatrix}^1$$

$$M = \begin{pmatrix} 6 & -2 \\ -2 & 6 \end{pmatrix}$$

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$$M^n = a^n X^n + b^n Y^n$$

$$M^1 = 2^1 \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}^1 + 4^1 \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}^1$$

$$M^n = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix} + \begin{pmatrix} 4 & -4 \\ -4 & 4 \end{pmatrix}$$

$$M^n = \begin{pmatrix} 6 & -2 \\ -2 & 6 \end{pmatrix}$$

Assuming: *a* = -1, *b* = -3, *n* = 1

$$M^n = (A + B)^n$$

$$M^1 = \left(\begin{pmatrix} -1 & -1 \\ -1 & -1 \end{pmatrix} + \begin{pmatrix} -3 & 3 \\ 3 & -3 \end{pmatrix} \right)^1$$

$$M^1 = \begin{pmatrix} -4 & 2 \\ 2 & -4 \end{pmatrix}^1$$

$$M = \begin{pmatrix} -4 & 2 \\ 2 & -4 \end{pmatrix}$$

$$M^n = a^n X^n + b^n Y^n$$

$$M^1 = -1^1 \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}^1 + (-3)^1 \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}^1$$

$$M^n = \begin{pmatrix} -1 & -1 \\ -1 & -1 \end{pmatrix} + \begin{pmatrix} -3 & 3 \\ 3 & -3 \end{pmatrix}$$

$$M^n = \begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix}$$

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Assuming: *a* = -5, *b* = 2, *n* = 1

$$M^n = (A+B)^n$$

$$M^1 = \left(\begin{pmatrix} -5 & -5 \\ -5 & -5 \end{pmatrix} + \begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix} \right)^1$$

$$M^1 = \begin{pmatrix} -3 & -7 \\ -7 & -3 \end{pmatrix}^1$$

$$M = \begin{pmatrix} -3 & -7 \\ -7 & -3 \end{pmatrix}$$

$$M^n = a^n X^n + b^n Y^n$$

$$M^1 = -5^1 \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}^1 + 2^1 \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}^1$$

$$M = \begin{pmatrix} -5 & -5 \\ -5 & -5 \end{pmatrix} + \begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}$$

$$M = \begin{pmatrix} -3 & -7 \\ -7 & -3 \end{pmatrix}$$

Assuming: *a* = 1, *b* = 3, *n* = 2

$$M^n = (A+B)^n$$

$$M^2 = \left(\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} + \begin{pmatrix} 3 & -3 \\ -3 & 3 \end{pmatrix} \right)^2$$

$$M^2 = \begin{pmatrix} 4 & -2 \\ -2 & 4 \end{pmatrix}^2$$

$$M^2 = \begin{pmatrix} 20 & -16 \\ -16 & 20 \end{pmatrix}$$

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$$M^n = a^n X^n + b^n Y^n$$

$$M^2 = 1^2 \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}^2 + 3^2 \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}^2$$

$$M^2 = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix} + \begin{pmatrix} 18 & -18 \\ -18 & 18 \end{pmatrix}$$

$$M^2 = \begin{pmatrix} 20 & -16 \\ -16 & 20 \end{pmatrix}$$

Assuming: *a* = 1/2, *b* = 2, *n* = 1

$$M^n = (A + B)^n$$

$$M^1 = \left(\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} + \begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix} \right)^1$$

$$M^1 = \begin{pmatrix} 2\frac{1}{2} & -1\frac{1}{2} \\ -1\frac{1}{2} & 2\frac{1}{2} \end{pmatrix}^1$$

$$M = \begin{pmatrix} 2\frac{1}{2} & -1\frac{1}{2} \\ -1\frac{1}{2} & 2\frac{1}{2} \end{pmatrix}$$

$$M^n = a^n X^n + b^n Y^n$$

$$M^1 = \frac{1}{2}^1 \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}^1 + 2^1 \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}^1$$

$$M^1 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} + \begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}$$

$$M = \begin{pmatrix} 2\frac{1}{2} & -1\frac{1}{2} \\ -1\frac{1}{2} & 2\frac{1}{2} \end{pmatrix}$$

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Assuming: *a* = 2, *b* = 2, *n* = -2

$$M^n = (A+B)^n$$

$$M^{-2} = \left(\begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix} + \begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}\right)^{-2}$$

$$M^{-2} = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}^{-2}$$

Not Possible

Assuming: *a* = 2, *b* = 2, *n* = 0

$$M^n = (A+B)^n$$

$$M^0 = \left(\begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix} + \begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}\right)^0$$

$$M^0 = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}^0$$

$$M^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$M^n = a^n X^n + b^n Y^n$$

$$M^0 = 2^0 \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}^0 + 4^0 \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}^0$$

$$M^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$M^0 = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

Not Compatible

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After testing the validity of my general statement we can see that the results for both formulas were mostly compatible for all numbers of *a*, *b* and positive *n*, proving the validity of our statement. However when *n* is zero or a negative integer we find some problems with it. As we can't power a matrix to a negative number *n* can't be a negative number and when 0 we find out all matrices to the power of 0 form identity matrices that when added in the formula $M^n = | A+B |^n$ it differs from our other result which is a identity matrix as well.

To get to this formula algebraically I did:

$$M = A + B$$

$$M^n = | A + B |^n$$

$$M^n = A^n + B^n$$

$$M^n = a^n X^n + b^n Y^n$$

$$A = aX$$

$$B = bY$$

This can be concluded by:

$$M^2 = (A+B)^2 = A^2 + 2AB + B^2$$

$$M^3 = (A+B)^3 = A^3 + 3A^2B + 3AB^2 + B^3$$

$$M^4 = (A+B)^4 = A^4 + 4A^3B^2 + 4A^2B^2 + 4AB^3 + B^4$$

By knowing that *AB =0*:

$$M^n = | A+B |^n$$

$$M^n = A^n + B^n$$

And finally I found the general expression:

$$M^n = a^n X^n + b^n Y^n$$