Crows Dropping Nuts

SL Type 2

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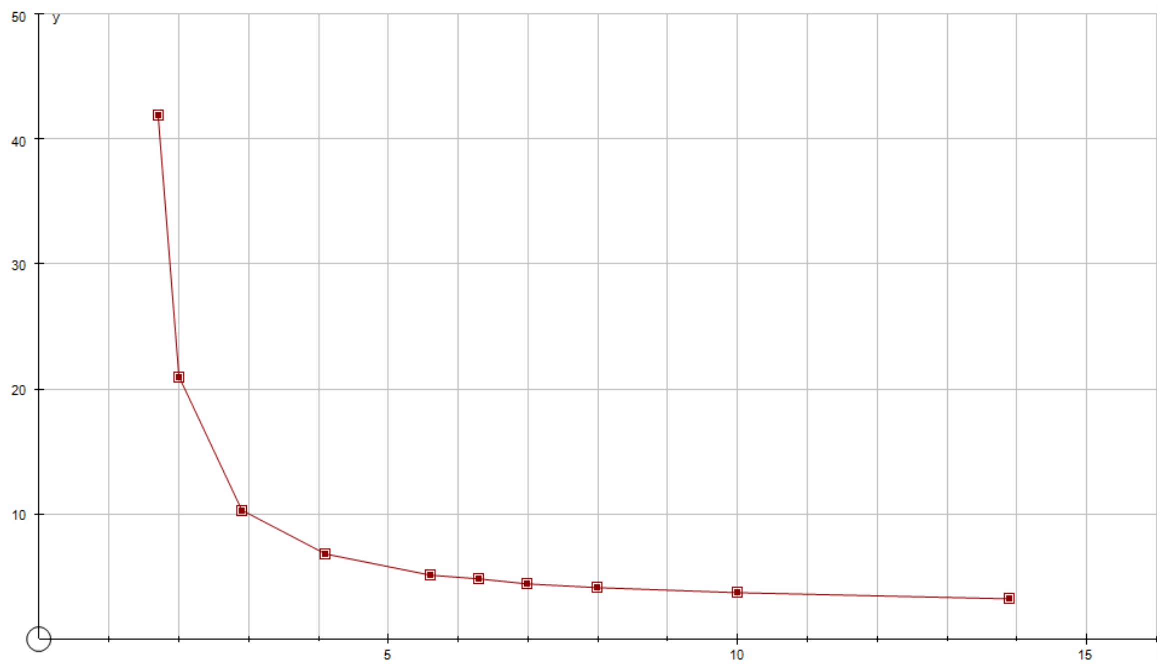
## Crows dropping nuts:

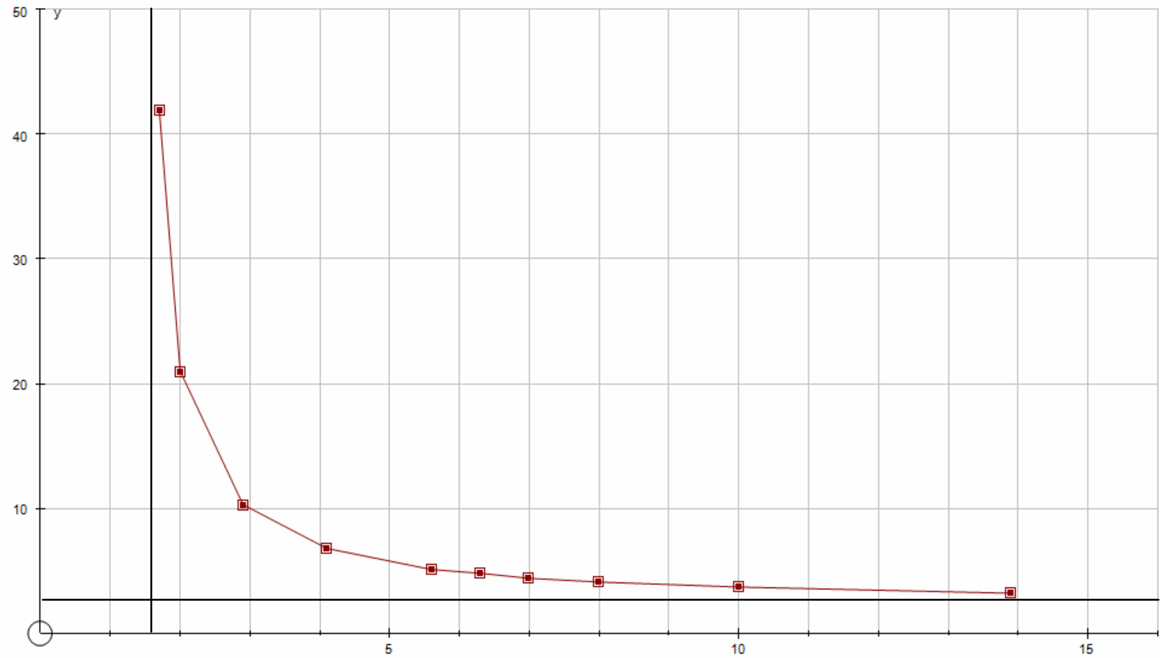
The table that is provided shows us the average of height and the number of times it takes to break the large nuts from that height.

| **Height of drop $(x)$** | 1.7 | 2.0 | 2.9 | 4.1 | 5.6 | 6.3 | 7.0 | 8.0 | 10.0 | 13.9 |
|---|---|---|---|---|---|---|---|---|---|---|
| **Number of drops $(y)$** | 42.0 | 21.0 | 10.3 | 6.8 | 5.1 | 4.8 | 4.4 | 4.1 | 3.7 | 3.2 |

Line graph depicting the table above, showing the frequency of drops by the height of the drop for a **large nut.**

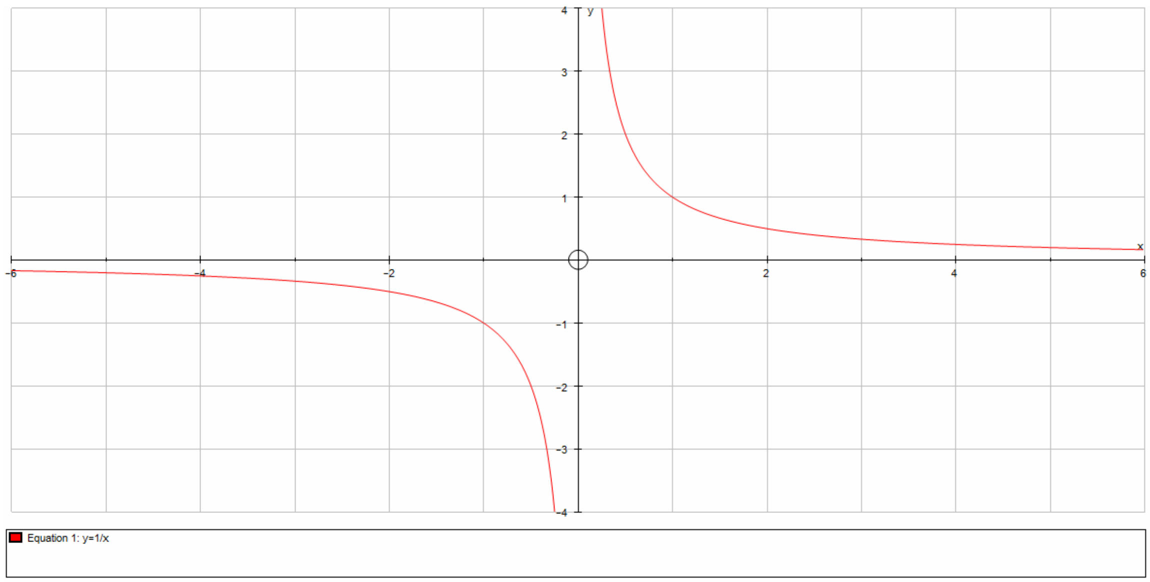



There numerous variables used for this graph. One such is the height at which the nut should be dropped affected the frequency, and this variable is put into an average. Another variable is the frequency of drops is also an average, where is it impossible to have 6.8 times of drops to open a nut. This has been converted into an average because it provides much clearer data, which could be put into one graph and distinguish the equation for it. Another variable is the size of the nut, where "large" is not very scientific and can vary in size and shape, which will consequently alter the frequency of drops it takes for it to crack open. Hence by creating a size range for the "large" nut will help to identify and shape the model better.

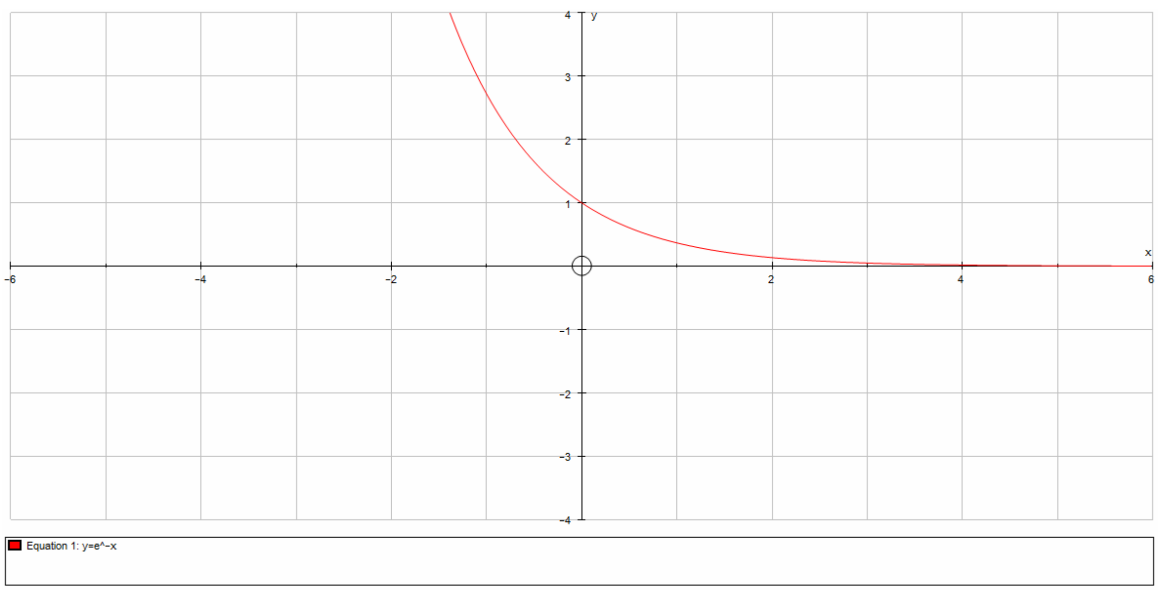



One such parameter that could be seen from the graph is the asymptotes, one on the y-axis and the other on the x-axis. This clearly suggests that this is not a precise graph, where both axis have infinite possibilities. For example when the height of the drop is too low, the frequency of drops is too high, therefore, in the real world this would not be possible, therefore suggesting parameters for the graph would have to be done. Yet the graph can also not touch the axis, as the model will not work as well, as at 0m a y amount of times the nut has to be dropped. Therefore this creates a paradox.

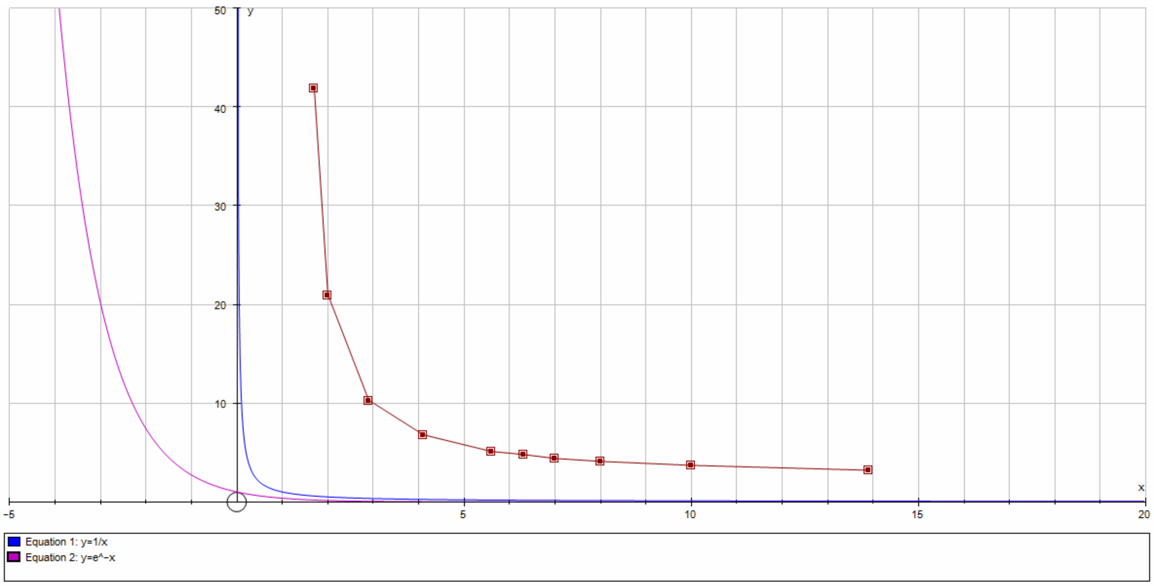
One function that could gimmick the original is $y = \frac{1}{x}$




Or the other function that could model this graph is the inverse exponential, where $y = e^{-x}$.




This function applies better to the "large nut" graph, as this shows only one graph unlike the inverse of x, this also shows the two isotopes that the "large nut" graph has, therefore showing a clear relevance between each other.




And this is clearly seen when comparing the original "large nut" graph (red line), to the inverse of x (blue line) and the inverse exponential (purple line). Therefore I will find the equation for the inverse exponential graph through simultaneous equations, whilst for the $y = \frac{1}{x}$ I plan to use autograph for trial and error in-putting these parameters:

$y = \frac{a}{x+b} + c$

## Creating the first model

Creating a model for the graph using inverse exponential. One can do this through the use of simultaneous equations, where by creating two constants and using the information the from the table we are able to find the equation. From using $\boldsymbol{y = ae^{-x} + b}$ and finding out the two constants, a and b we will be able to find a rough equation.

$\boldsymbol{y = ae^{-x} + b}$

First set of data: (x,y) (1.7,42)

First equation: $\boldsymbol{42 = ae^{-1.7} + b}$

Second set of data: (x,y) (13.9,3.2)

Second equation: $\boldsymbol{3.2 = ae^{-13.9} + b}$

Equating the two to find **a** and **b**:

$42 = ae^{-1.7} + b$

(Minus)

$3.2 = ae^{-13.9} + b$

(Equals)

$$38.8 = a0.1826826051$$

$a = 212.39$

(Substituting a into the equation)

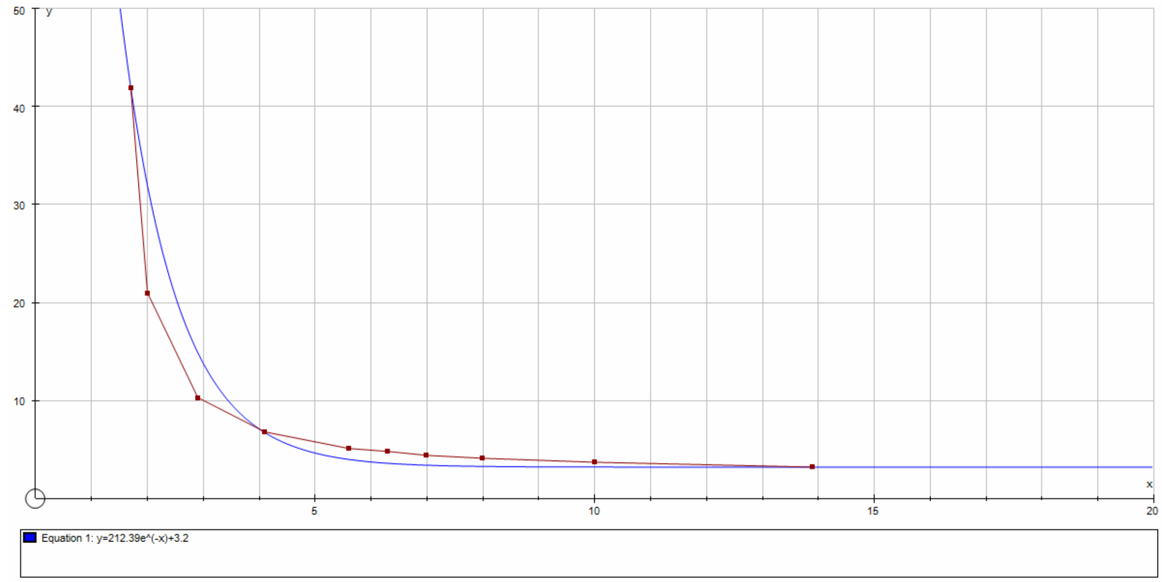
$$42 = 212.39e^{-1.7} + b$$

$$b = 3.2$$

(Therefore)

$$y = 212.39e^{-x} + 3.2 \quad 1.7 \geq x \leq 13.9$$

My model $y = 212.39e^{-x} + 3.2$ against the current graph

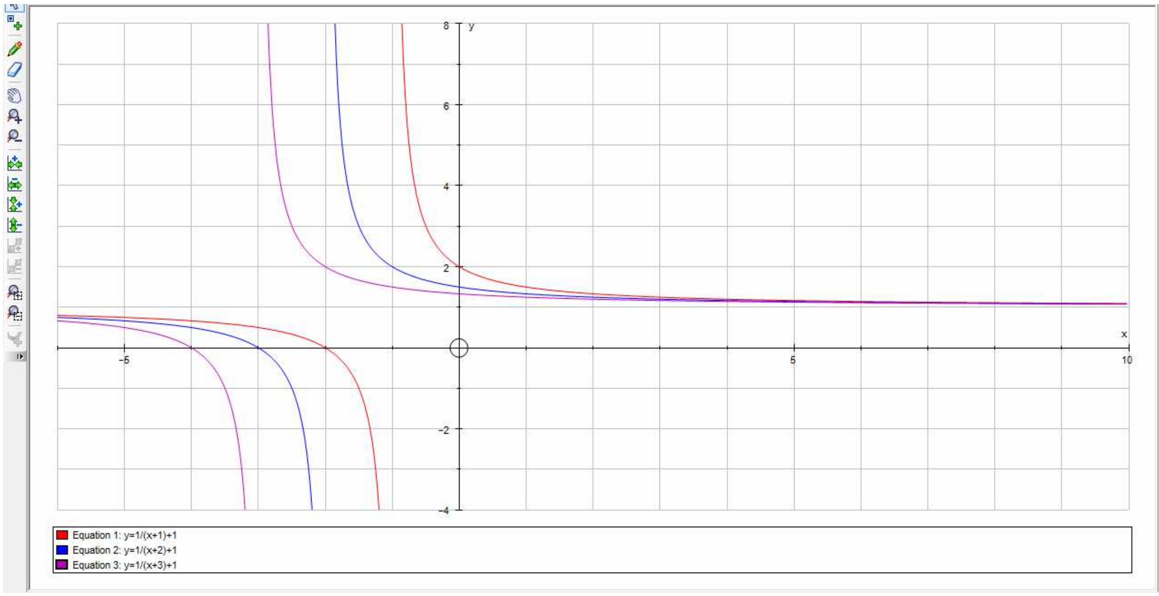


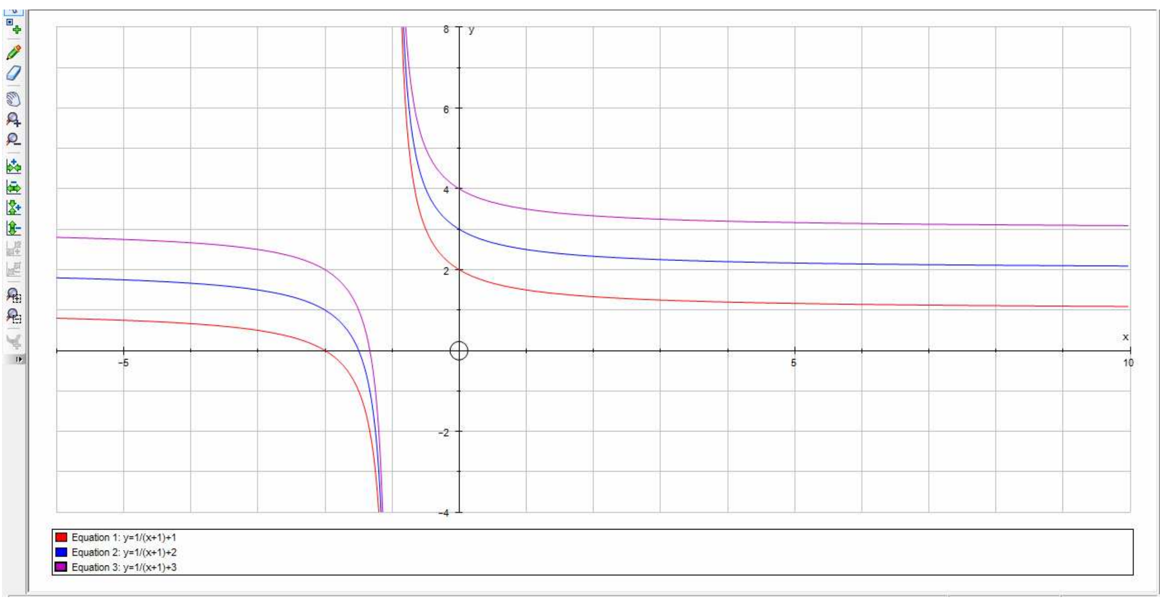

| **Height of drop** | 1.7 | 2 | 2.9 | 4.1 | 5.6 | 6.3 | 7 | 8 | 10 | 13.9 |
|---|---|---|---|---|---|---|---|---|---|---|
| **Number of drops** | 42 | 32 | 14.9 | 6.7 | 4 | 3.6 | 3.4 | 3.3 | 3.2 | 3.2 |

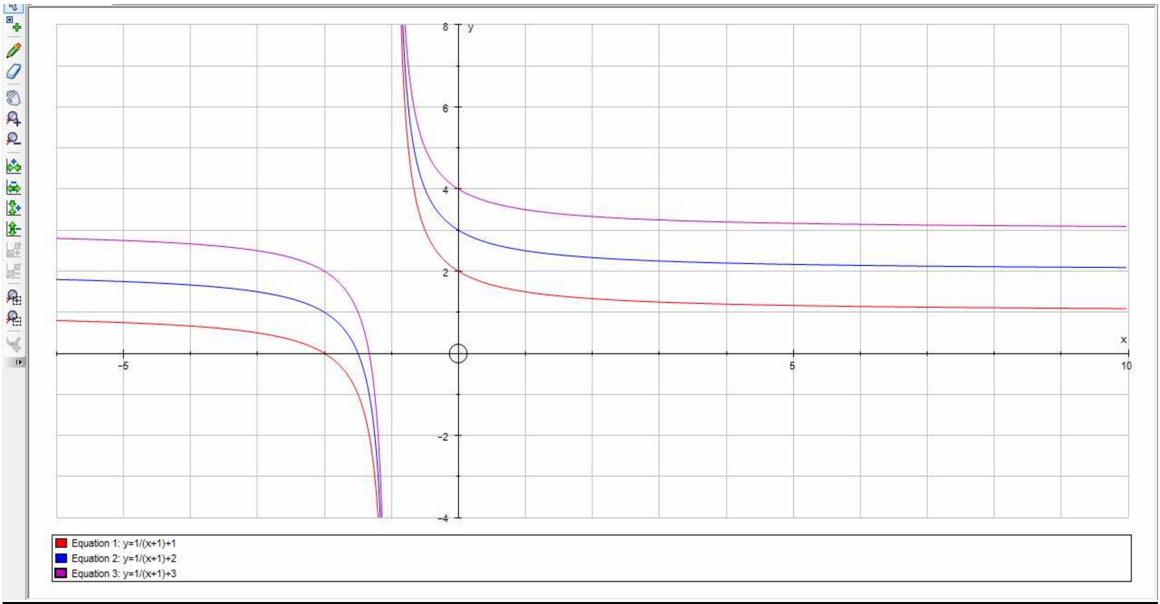
The curve of my graph is off-set to the bottom right therefore just coming of the line, also my model is not steep enough compared with the original. My model does cross through 3 points with the original graph showing how close it is. My model also shows that it is on the correct axis as the x axis asymptote, though not the y-axis asymptote. The accuracy of my graph is not too satisfying therefore using trial and error for $y = \frac{1}{x}$ will give me a better answer, because through using more parameters I will be able to mold the equation perfectly with the original.

**Trial and error**

The equation I will be using will be $y = \frac{a}{x+b} + c$ where $a, b, c$ are the parameters and I will be altering to find the equation needed.


Equation 1: y=1/(x+1)+1
Equation 2: y=1/(x+2)+1
Equation 3: y=1/(x+3)+1


Equation 1: y=1/(x+1)+1
Equation 2: y=1/(x+1)+2
Equation 3: y=1/(x+1)+3


Equation 1: y=1/(x+1)+1
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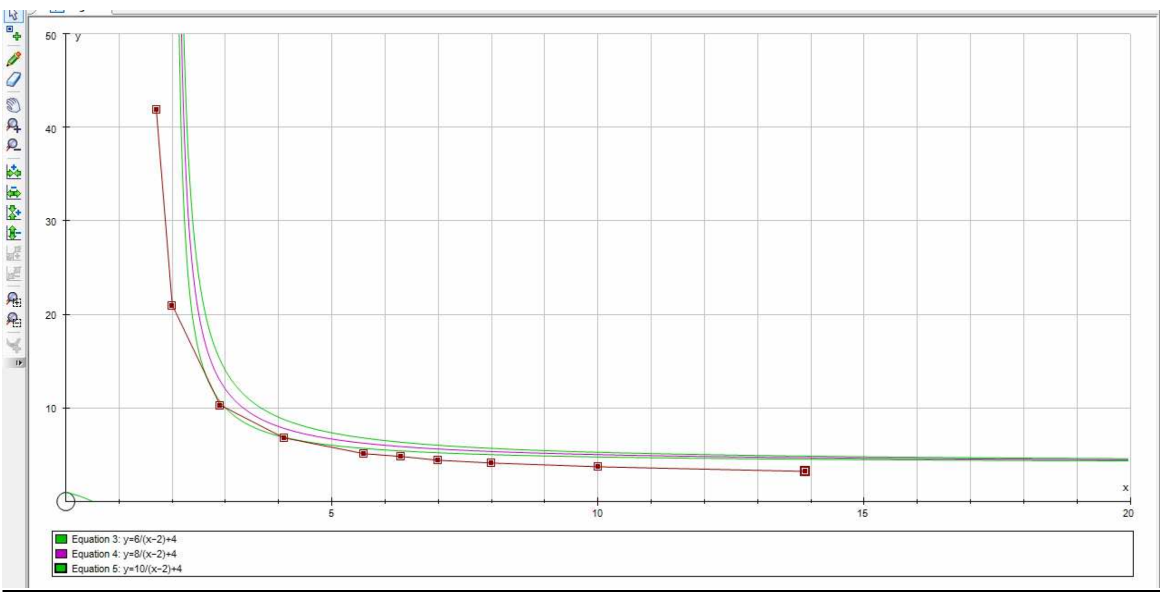
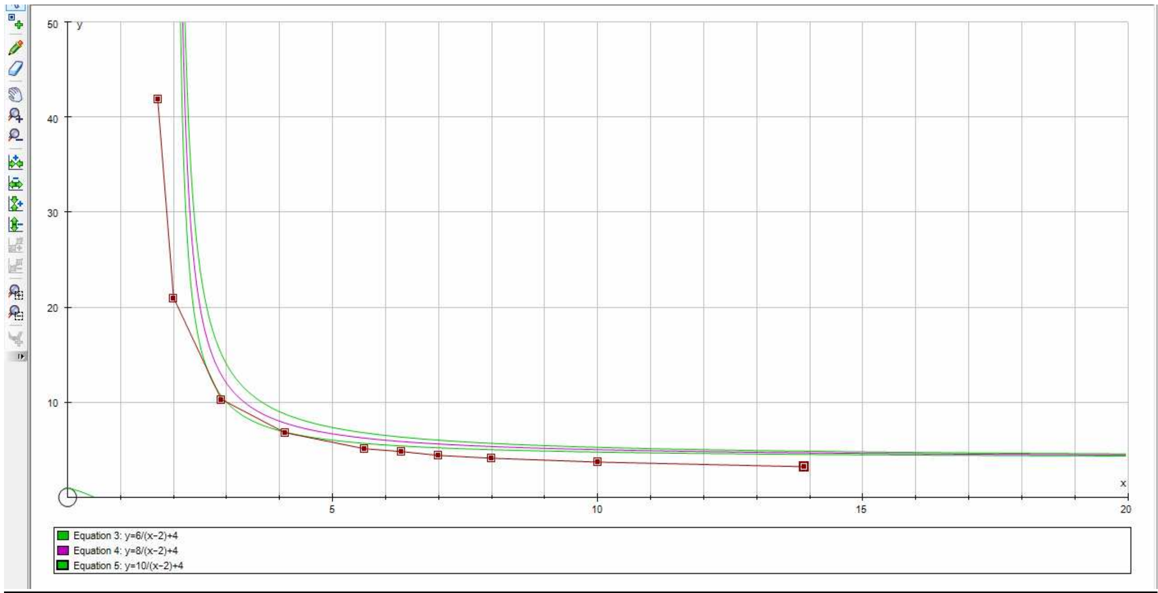

Equation 3: y=6/(x-2)+4
Equation 4: y=8/(x-2)+4
Equation 5: y=10/(x-2)+4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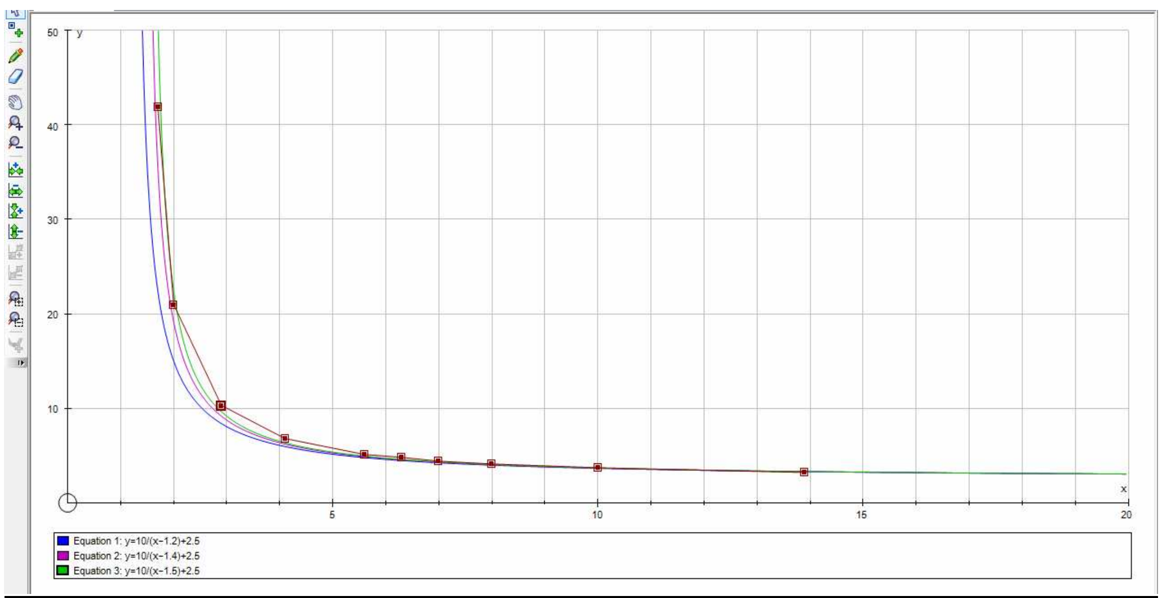
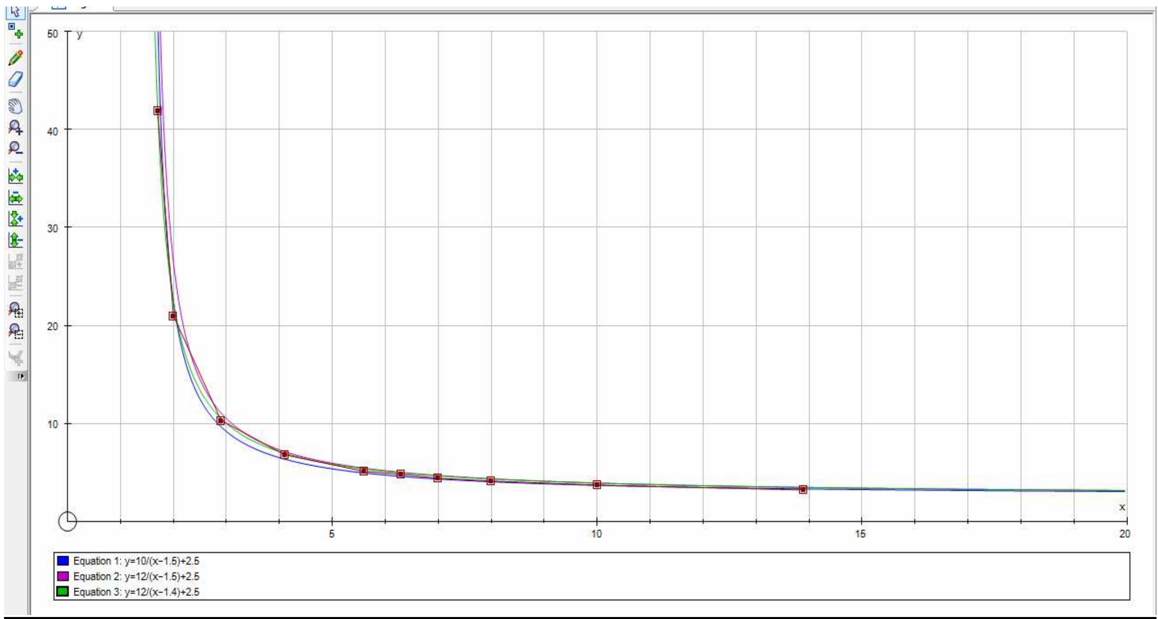
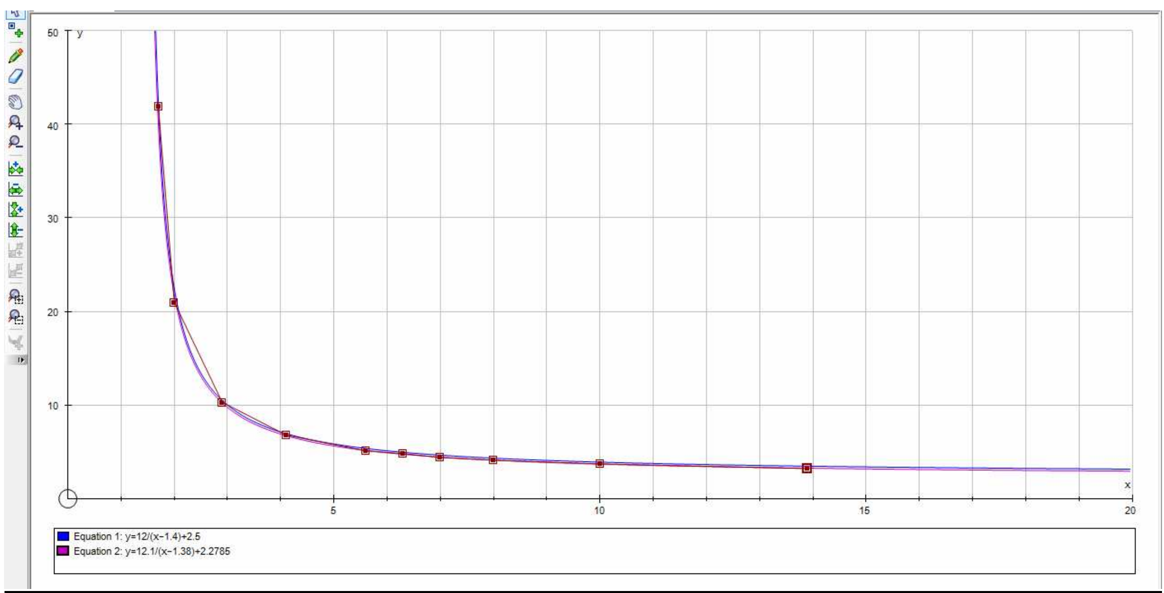
Equation 5: y=10/(x−2)+4


Equation 1: y=10/(x−1.2)+2.5
Equation 2: y=10/(x−1.4)+2.5
Equation 3: y=10/(x−1.5)+2.5

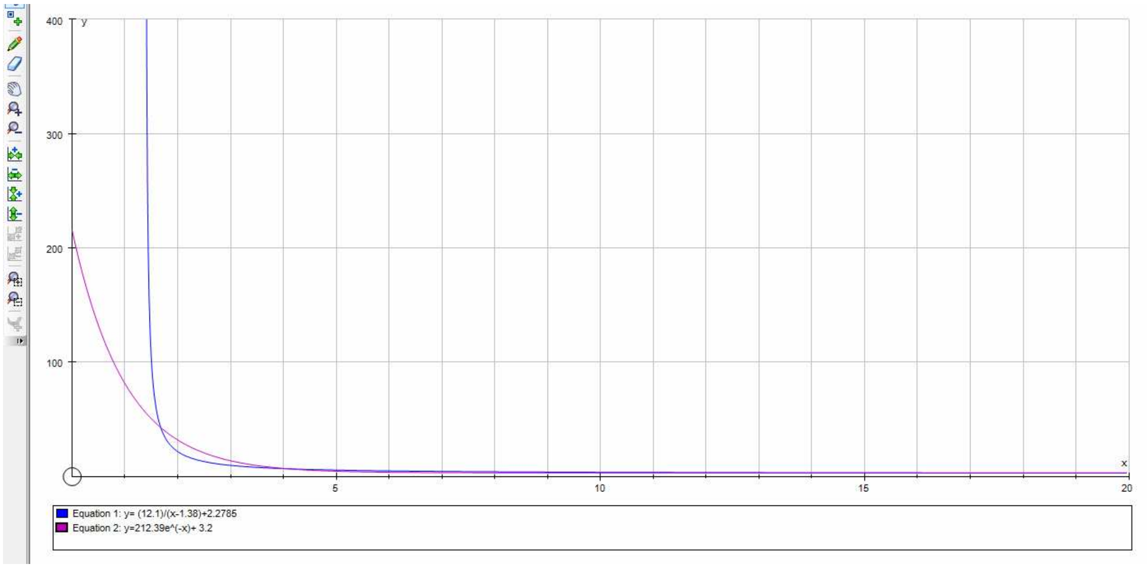


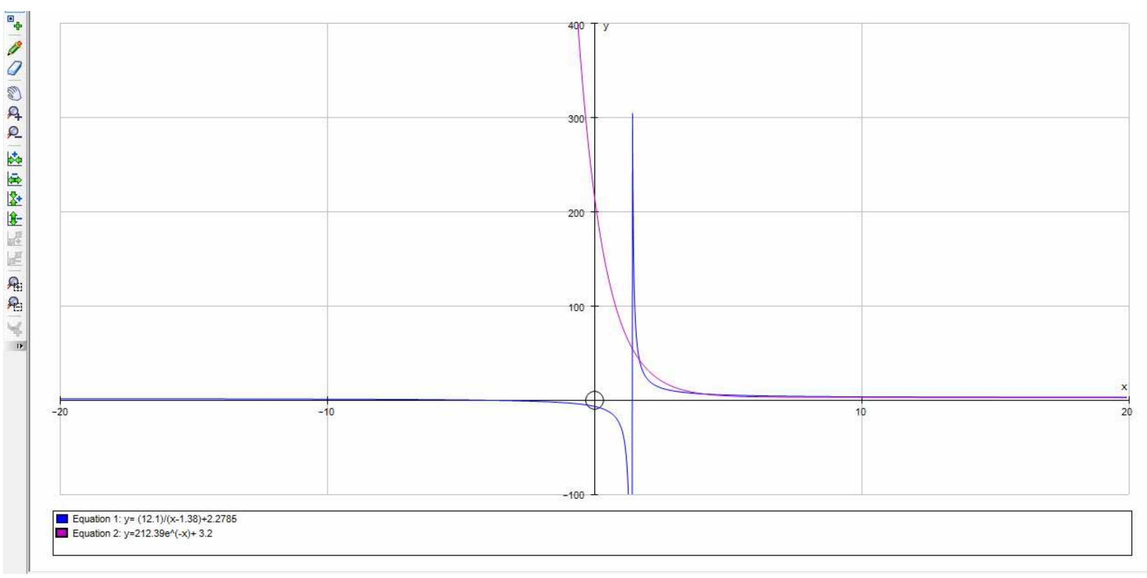

**<u>End Result:</u>**

$$y = \frac{12.1}{x-1.38} + 2.2785 \qquad 1.7 \geq x \leq 13.9$$

**Comparing the two functions:**




The first clear difference between the functions is that one crosses the y-axis, the inverse exponential graph crosses the y-axis on (0,212) therefore showing a clear limitation to the graph. Whilst the $y = \frac{1}{x}$ shows that it has two asymptotes just as the original graph does, therefore portrays it more accurately. Another difference between the graphs is their steepness, where the inverse exponential is less steep then the $y = \frac{1}{x}$ therefore it does not cover the top half of points on the original graph. Thought the horizontal asymptote is similar, therefore showing their relevance towards the original graph.

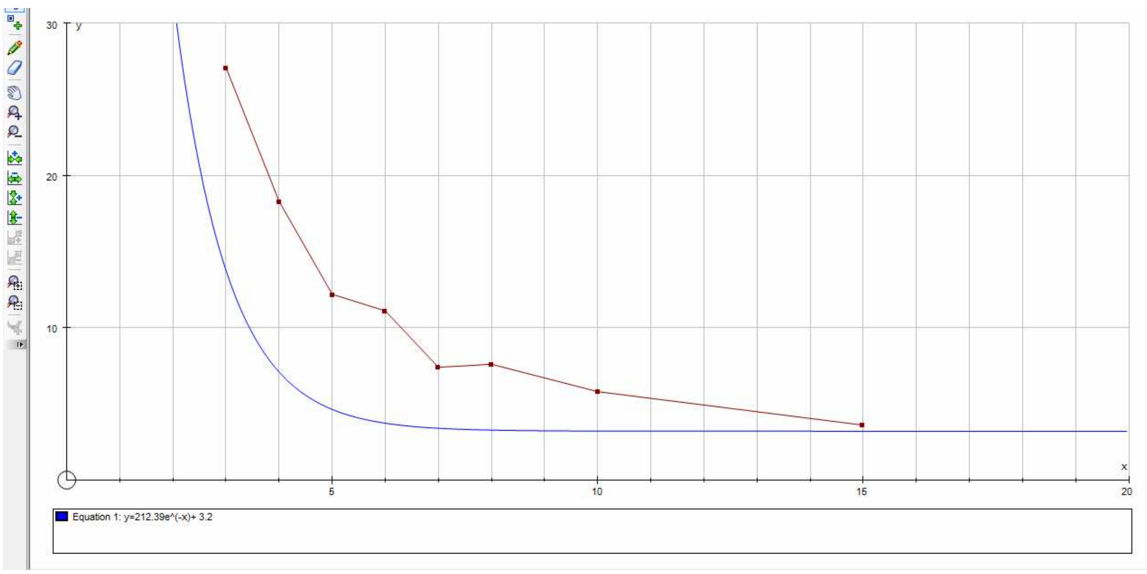


Furthermore the $y = \frac{1}{x}$ has a reflected graph into the negative axis, and this cannot apply to the original graph, as it is impossible to have a negative height, unless it's underground, nor negative frequency, hence setting limitations for the graph then this will be fixed. The area under $y = \frac{1}{x}$ is closer matched to the original compared with the inverse exponential graph, where the:

- Original has 74.91
- $y = \frac{1}{x}$ has 72.17
- Inverse exponential has 77.84

**<u>Different sized nuts</u>**

**Medium nuts:**




My first model is not applicable with the medium sized nuts, and this is because of the changes in the variables as the larger nuts where heavier and stronger, hence harder to break and had to be dropped more frequently from longer distances. The changes that will have to be made to my model to fit this, as the parameters a and b.

$$y = ae^{-x} + b$$

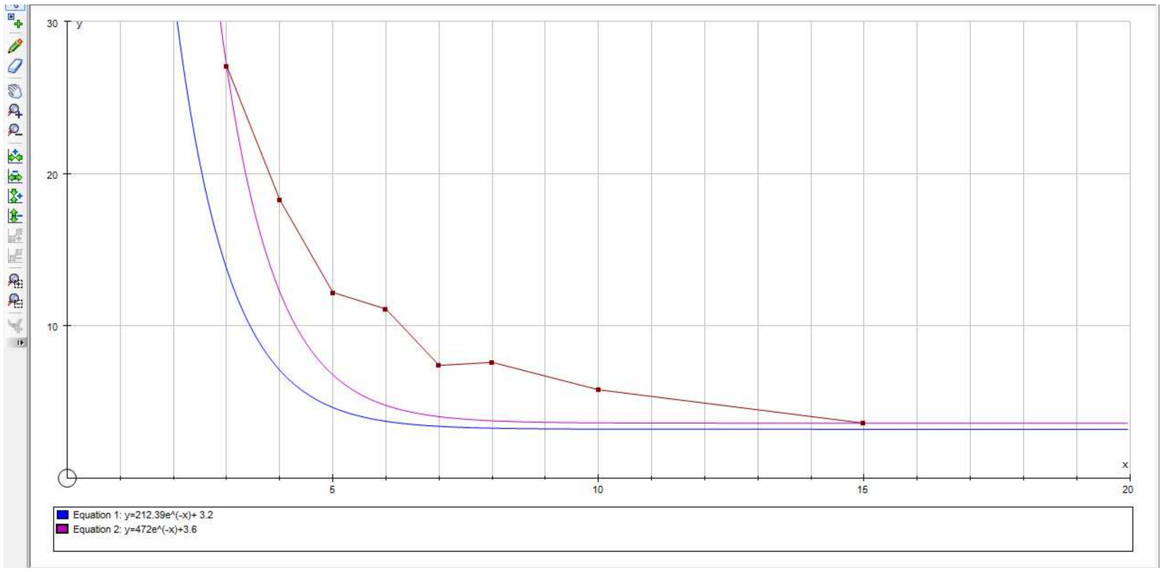
$$27.1 = ae^{-3} + b$$

$$3.6 = ae^{-15} + b$$

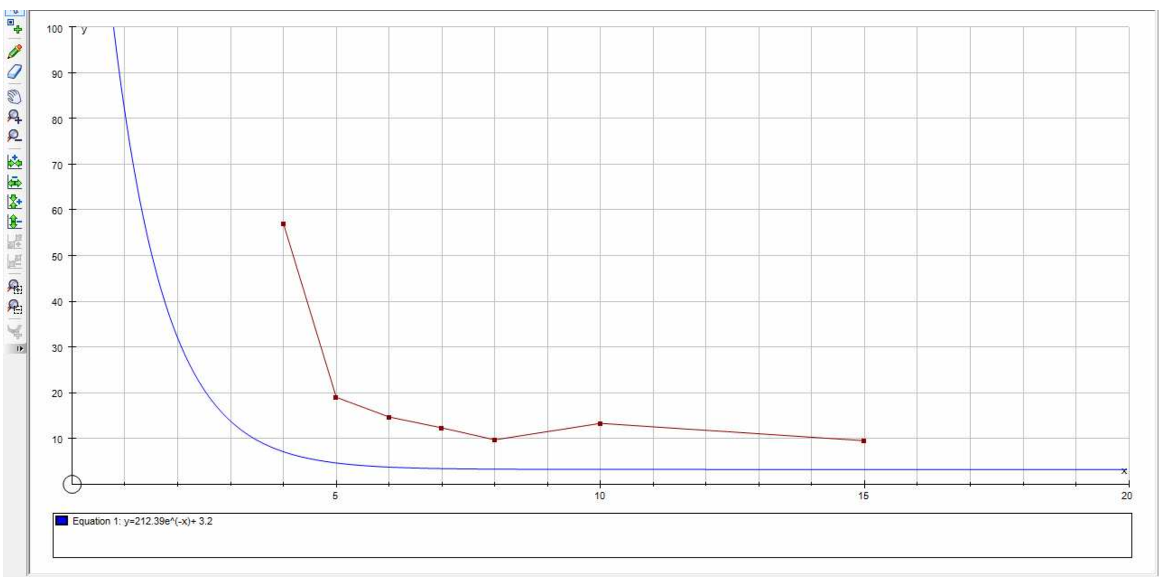
Through simultaneous equation we will be able to find a and b for the equation which is:

- A = 472
- B = 3.6

$$y = 472e^{-x} + 3.6$$




**Small nuts:**




$$y = ae^{-x} + b$$

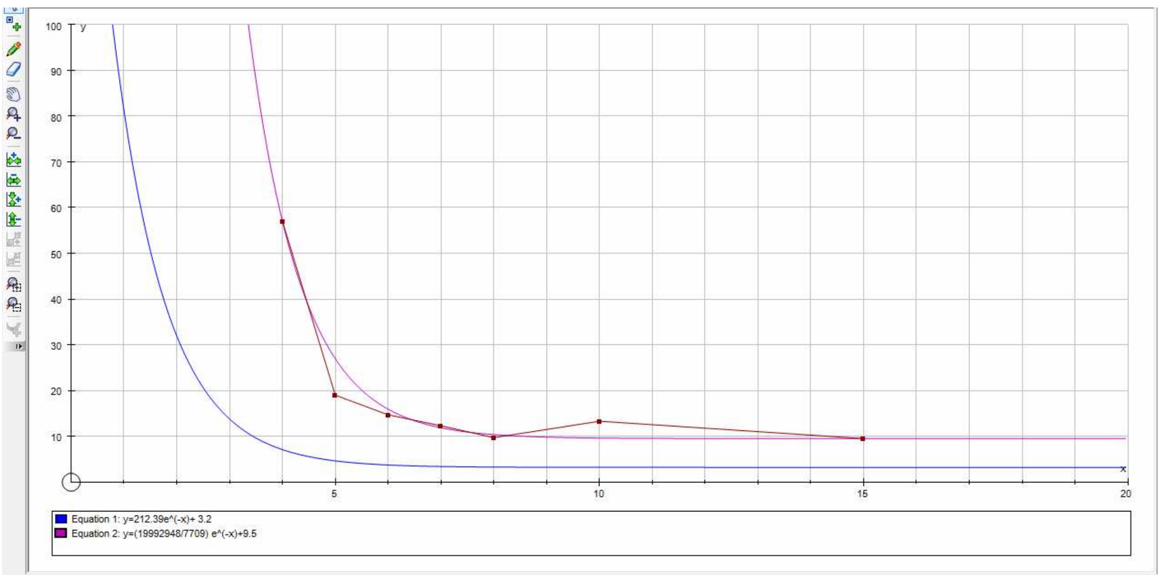
$$57 = ae^{-4} + b$$

$$9.5 = ae^{-15} + b$$

Through simultaneous equation we will be able to find a and b for the equation which is:

- A = 19992948/7709
- B = 9.5

$$y = \left(\frac{19992948}{7709}\right)e^{-x} + 9.5$$




I was only able to get the equation to go through the two end points of the equation, which means that the match is not very accurate and does not portray the medium nuts. The limitation of my model is the middle section, where the curve of my model does not meet with the medium nuts, therefore creating a lot of uncertainty. The medium nut graph does not have a real curve, where its more of a line graph, therefore creating a perfect model for this would be difficult and the closest I can is the inverse exponential graph: $\boldsymbol{y = 472e^{-x} + 3.6.}$

My first model still does not come close to depicting the original graph as the difference in variables, which leads to differences in the asymptotes and the curvature of the graph. Though through using simultaneous equations the model $\boldsymbol{y = \left(\frac{19992948}{7709}\right)e^{-x} + 9.5}$ comes very close to depicting the original, with four points matching and 2 not shows the accuracy of the graph. Through the limitation of the model is that its not crossing the rest of the points, where the curvature does not allow for the graph to include the two other points. This model will also cross the y-axis which will mean that the frequency will go in negative, hence in real life this model would not have worked.