Maths investigation
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Part 1: Rows Pattern

From the first 5 rows, I observe a pattern which is repeated on the subsequent row. First, the number 1 stands at the pivot of the pyramid and begins and ends each row. Second, the numbers follow a sequence 1,2,3,….both diagonally and downwards, and are same per row on the right and left sides of the pyramid, and the first number coincides with the number of the row. A similar sequence follows next (1,3,6..) on both sides, and then (1,4,10…) etc. Third, every number on each row (excluding the 1's) is the sum of the two numbers sited immediately above it. Four, on every other row (i.e. the even number rows), there is a centrally-located number (2,6,20..), and this also has a sequence; it corresponds to the sum of the same number (1+1,3+3,10+10). The rows that fall in between consist of and even number of digits (i.e. no middle number here), but the two middle numbers are always the same (1,1; 3,3; 10,10;..). Therefore you have a small pyramid which is exactly symmetrically shaped in the middle such that a sequence shows every time a new row is formed. You can see the pattern 2,3,4. I can predict that the next one will be five and the one after six and so on. Each number in the triangle is the sum of the two directly above it. So the next row will be. 1, 5, 10, 10, 5, 1. this is because you put 1 first then add 1 + 4 which is 5, then you add 4 + 6 which is 10 and 4 + 6 again. then you 1+4 again and you add a 1 to the outside. The next row will be. 1, 6, 15, 2 0, 15, 6, 1. This is the seventh row. And the eight row will be 1, 7, 21, 35, 35, 21, 7, 1. You can clearly see a pattern occurring. You always add up the two numbers above together and this is how you will be able to find out what the next row is. See below:

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1

Part 2: Sum Pattern

1. The sum of the numbers in each row for the first six rows are as follows:

0^{th} row = 1

1^{st} row 1+1 = 2

2^{nd} row = 1+2+1 = 4

3^{rd} row = 1+3+3+1 = 8

4^{th} row = 1+4+6+4+1 = 16

5^{th} row = 1+5+10+10+5+1= 32

6^{th} row = 1+6+15+20+15+6+1= 64

2. I observed that the sum of every row is the double that of the previous number

3. A general formula is that the Sum of the n^{th} row = 2^n

Maths formula per row would therefore be:

Row 0:(x+1)^0 = 1
Row 1:(x+1)^1 = 1 + x
Row 2:(x+1)^2 = 1 + 2x + x^2
Row 3:(x+1)^3 = 1 + 3x + 3x^2 + x^3
Row 4:(x+1)^4 = 1 + 4x + 6x^2 + 4x^3 + x^4
Row 5:(x+1)^5 = 1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5 ...
Row n: (x+1)^n = 1 + nx + 2nx^2 + 2nx^3 + nx^4 + x^n ...

Part 2a: Alternating Sum Patterns

1.) Below are the alternating sums for each of the first six rows:

0^{th} row 1

1^{st} row 1 1 = 0

2^{nd} row 1 2+1 = 0

3^{rd} row 1 3+3 1 = 0

4^{th} row 1 4+6 4+1 = 0

5^{th} row 1 5+10 10+5 1 = 0

6^{th} row 1 6+15 20+15 6+1 = 0

2.) I notice that they all end in 0.

3.) Alternating sum in row n = 0.

Part 3: Multiple Pattern

1. If you add 2 numbers that are multiples of n, then also the sum will be a multiple, and an interesting pattern appears when certain multiples are highlighted. For example, multiples of 2 and 3 (see diagrams below).

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

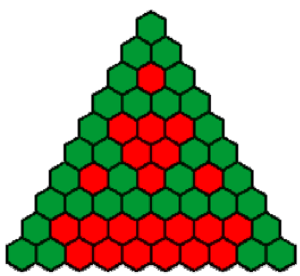
1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

1 8 28 56 70 56 28 8 1

1 9 36 84 126 126 84 36 9 1

The sum of two odd numbers is an even number, i.e. the number occurring below; and the sum of two even numbers is an even number, seen below; but the sum of an odd number and an even number is an odd number which changes the sequence in this case. During my research, I included two patterns for Pascal's multiples. The red coloration in the first picture shows a symmetrical pattern that is formed when all the even numbers are coloured differently.



The pattern for multiples of 3's and 5's were as follows:

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

1 8 28 56 70 56 28 8 1

1 9 36 84 126 126 84 36 9 1

1 10 45 120 210 252 210 120 45 10 1

1 11 55 165 330 462 462 330 165 55 11 1

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1

On shading multiples of 3 (pink), there were inverted triangles that formed a pattern around the middle of the pyramid. The smaller triangles stood on a base of 2 numbers, and the base of two triangles was separated by one number (20). The height of the triangle is also two rows.

On shading multiples of 5, we have an inverted triangle which again is symmetrical around the middle on a base of four numbers. This pattern is then repeated but this time on two sides of the pyramid, and the base is again separated by one number (252). In this case, the height of the triangle is four rows.

Explanation: Perhaps the triangular patterns are always formed on a base and height of (n-1) in a symmetrical pattern, like two sides of an equilateral triangle.

Part 4: Pascal Petals

1.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1

Cell 21 (grey petal):

6x28x35 = 5880 = (2x3)x(2x2x7)x(5x7)

15x7x56 = 5880 = (3x5)x(1x7)x(2x2x2x7)

i.e. perhaps there are 21 ways of writing this sets of factors

Cell 6 (turquoise petal):

3x4x10 = 120 = (1x3)x(2x2)x(2x5)

4x10x3 = 120 = (2x2)x(2x5)x(1x3)

i.e. perhaps there are 6 ways of arranging this sets of factors

A		B		C		D
	A+B		B+C		C+D	
		A+2B+C		B+2C+D		

Every time you follow the flower-like order, you get the same answers. But I noticed that you have to follow this particular order other wise it you will end up with another (or a wrong) answer.
My explanation: I think the explanation here is that factors when multiplied always result in the same numbers unlike when added or squared up.

Part 5: Sideways Pattern

1st triangular number: The first triangular number will be 1. And if you add 2 you will get the 2nd triangular number.

2nd triangular number: Here as I told you had to add 2 more to one that will bring you to 3. And now you have to add three. You see the pattern every time you have to plus the number with one more.

3rd triangular number: is 6 and then you add 4.

4th triangular number: it will be 10 and you add 5 this time.

5th triangular number: will be 15 and after this you have to add 6.

And so on so on. This pattern will stay for the rest of Pascal's triangle.

	Row 4	Row 3	Row 2	Row1
N1	1			
		+3		
N2	4		+3	
		+6		+1
N3	10		+4	
		+10		+1
N4	20		+5	
		+15		+1
N5	35		+6	
		+21		
N6	56			

Extra:

1. $(a+b)^6$

Since $(a+b)^3 = a^3+3a^2b+3ab^2+b^3$ i.e. row 3 pattern of …… 1 3 3 1

And row 6 pattern is….1 6 15 20 15 6 1

Therefore, $(a+b)^6 = a^6+6a^5b+15a^4b^2+20a^3b^3+15a^2b^4+6ab^5+b^6$

2. $(a-b)^7$

Since row 7 pattern is….1 7 21 35 35 21 7 1

Therefore, $(a-b)^7 = a^7-7a^6b+21a^5b^2-35a^4b^3+35a^3b^4-21a^2b^5+7ab^6-b^7$

3. $(2x-1)^3$

And $(a+b)^3 = a^3+3a^2b+3ab^2+b^3$; where a=2x and b=-1

Answer: $(2x)^3 + 3.4x^2.-1 + 3.2x.1 + (-1)^3 = 8x^3- 12\ x^2+ 6x.1\ -1.$

4. $(x+1/x)^4$

Since row 7 pattern is….1 7 21 35 35 21 7 1

And $(a+b)^4 = a^4+4a^3b+6a^2b^2+4ab^3+b^4$; where a=x and b=1/x

Answer: $x^4+ 4x^3/x + 6x^2/x^2+ 4x/x^3+1/x^4 = x^4+ 4x^2 + 6 + 4/x^2+1/x^4$

$= x^4+ 4x^2 + 6 + 4/x^2+1/x^4$

Bibliography

1. http://mathforum.org/workshops/usi/pascal/elem.color_pascal.html