Math Portfolio Project

In this portfolio project, the task at hand is to investigate the relationship between G-force and time by developing models that represent the tolerance of human beings to G-forces over time. G-force is used to describe the acceleration that results when different forces are applied to an object. For example the g-force acting on a stationary object resting on the earth's surface is 1 g as it results from the Earth surface's resistance to gravity. Because there are no other forces acting on the object besides gravity, the g-force acting on a stationary object is equal and opposite to gravity.

However, when the object is under acceleration, the two different types of g-force are vertical g-force and horizontal g-force. Horizontal g-force is the measure of acceleration perpendicular to the spine which can either be forward or backward acceleration. In this portfolio I will be looking at the human tolerance for forward acceleration where the g-force pushes the body backwards. Experiments have shown that untrained humans can tolerate up to 17 g of forward acceleration for several minutes without losing consciousness. Upwards vertical g-force, which drives the blood downwards to the feet, away from the head is typically a lot less tolerable for humans because it poses problems for the brain and eyes. On average humans can handle approximately 5 g before losing consciousness.

With the data given, I will develop individual functions that model the relationship between the time tolerable for humans versus the various measurements of forward horizontal g-force on a subject, as well as the relationship between time and upward vertical g-force. I will compare hand generated and computer generated functions to see how well the models fit the data, and discuss any limitations to the models.

The models will be based on the data:

Time (minutes)	+Gx (grams)
0.01	35
0.03	28
0.1	20
0.3	15
1	11
3	9
10	6
30	4.5

TIme (minutes)	+Gz
0.01	18
0.03	14
0.1	11
0.3	14
1	7
3	6
10	4.5
30	3.5

In both these sets of data I will define the independent variable as the G-force represented along the y-axis, and the dependent variable as the time represented on the x-axis:

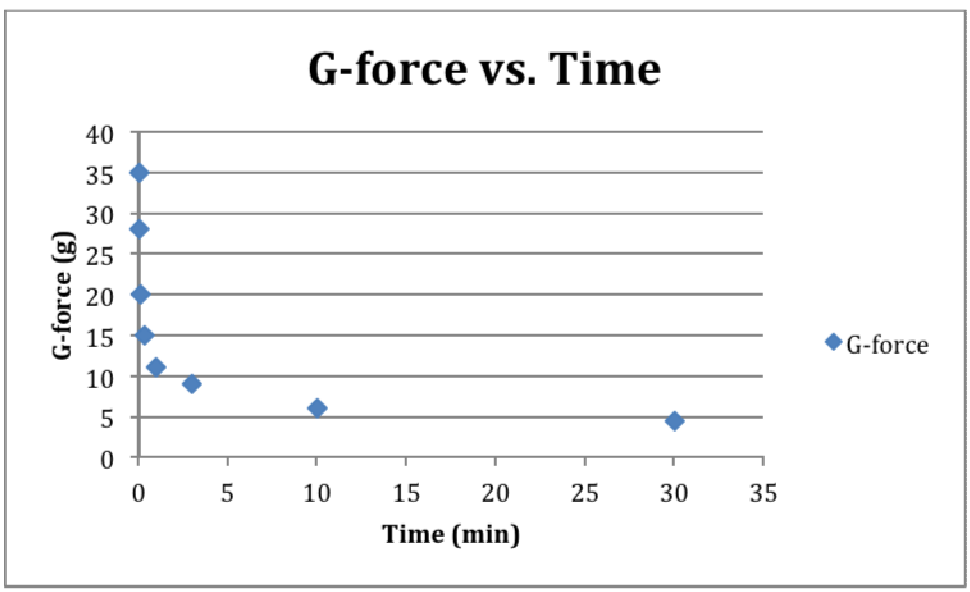
- where +Gx represents the positive acceleration in the horizontal direction in grams.
- where +Gz represents the positive acceleration in the vertical direction in grams.
- where t represents the time in minutes that the humans are exposed to the g-force given

There are two parameters for which the data holds true:

1. That there is a scaling factor that moves the value of time up as it increases. There is already a trend when looking at how t increases. Each consecutive value seems to rise by .02 and .07, for which the values then seem to move a decimal place to the right to .2 and .7, and then to 2 and 7, and lastly and increase of 20.
2. That there is an exponent or a power that must determine the functions rate of growth or decay. From the clear decrease in +Gx the exponent is likely to be negative, and whether it is an integer or a rational or irrational number, will also determine the function's overall shape and behavior.

The constraint to this data is that the value along the x-axis, time is always to be positive. Because time is not tangible but a representation of the duration of the day, it has come to the conclusion that time cannot be negative. Therefore whether or not this function holds true for a negative t value is irrelevant because of the real life situation at hand. Another constraint is that the data does not hold in account that there is a moment when g-force is zero in a weightless situation or a vacuum.

Initial Data Plot:



From the data points plotted on the graph one can see that there is a clear trend that as the amount of g-force decreases (+Gx), the time that humans can tolerate the given level of g-force (t) increases. Also it seems as though humans have a natural tolerance for g-force up until the g-force level reaches 9 g, where the time humans can tolerate the amount of g-force suddenly decreases rapidly.

Keeping the second parameter discussed earlier in mind, as one can see now, the function is clearly decreasing meaning that the power must indeed me negative and as the function also seems to be concave up, the exponent is also less than zero.

This graph seems to model a power function because of the parameters that the data holds true for. The function needs two constants, one to use as a scaling factor increasing time, and secondly a power, to decrease the function exponentially as +Gx decreases. Therefore a power function will fit the data the best: $+Gx = at^b$ where a is a positive constant and b is a negative exponent.

To find an equation that fits the data points, one must use substitution in order to be able to solve for one of the variables:

Here, the constant a was cancelled out through division, so logs can be used to continue to solve for b.

Potentially, a could be solved for with b, but the model would not be very accurate if only two sets of data points were held into account. Therefore for each consecutive data points, b must be solved for.

Data points:	b value
(0.01,35) (0.03,28)	-.2031120136
(0.03,28) (0.1,20)	-.2794683031
(0.1,20) (0.3,15)	-.261859507
(0.3,15) (1,11)	-.2576095799
(1,11) (3,9)	-.1826583386
(3,9) (10,6)	-.3367726469
(10,6) (30,4.5)	-.261859507

Average b: -.2547628423

Substitute this average in the equation $+Gx = at^b$ with values of +Gx and t for all 8 consecutive data points:

For the data points (0.01,35) and (0.03,28), solve for a with substitution:

$$\frac{35}{01^{-.25476}}$$

The value cannot be rounded just yet, because it will limit the accuracy significantly. Only after all the points are taken into account can 3 significant figures be taken.

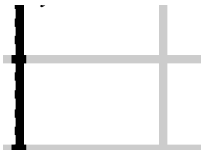
Data points:	a value
(0.01,35)	10.82785322
(0.03,28)	11.46001309
(0.1,20)	11.12415812
(0.3,15)	11.03776602
(1,11)	11
(3,9)	11.90680597
(10,6)	10.78733318
(30,4.5)	10.70355692

Average a: 11.10593582

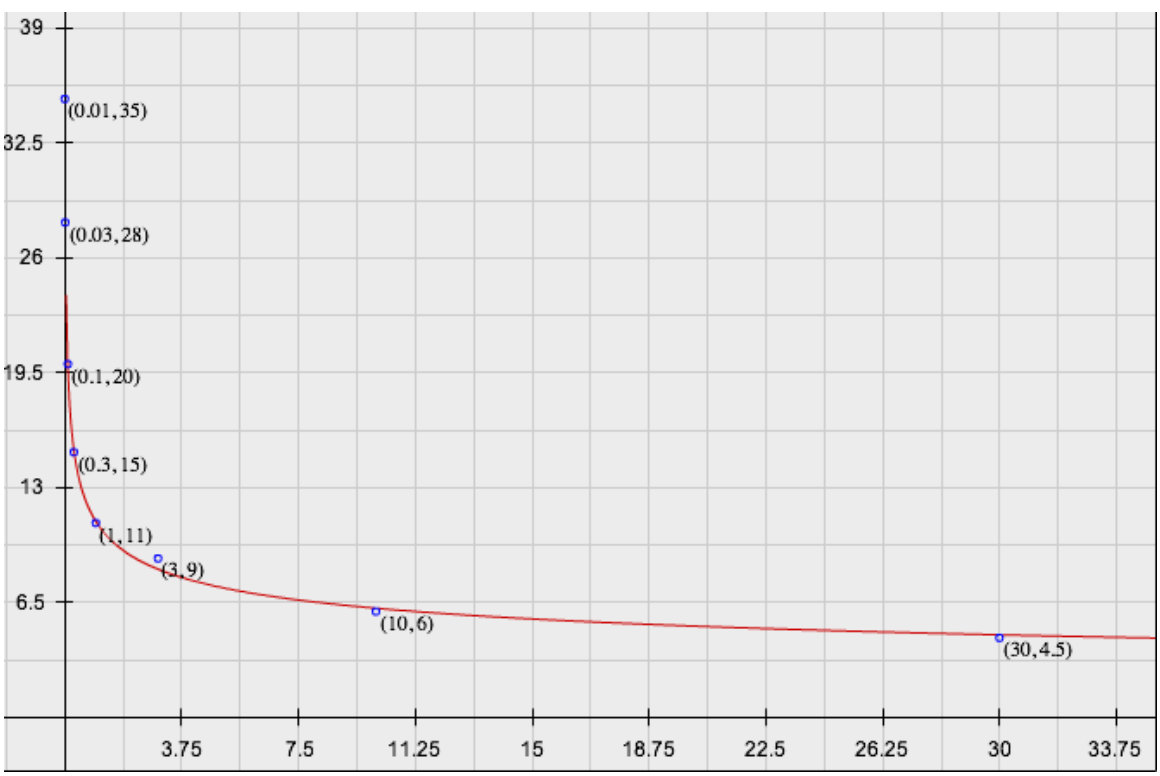
Function:

Before graphing the function, it should already be known that there are certain limitations due to the method of finding the function. Using the different data points to solve for the exponent (b) is a valid method, but not all the data points were taken into account. There are many combinations of data points that could have been taken and now only seven were taken. Because the value of the exponent is not 100 percent accurate, it also means that when the value is substituted into the equation when solving for a, there will be more inaccuracy.

Hand calculated model of :



Graph of Hand Calculated Model with Original Data Points:

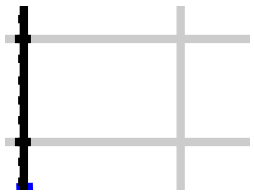


From the above models it becomes apparent that just like the data plots that as +Gx increases the amount of time humans can tolerate that force decreases. This is because g-force has serious physical effects on humans, which include loss of eyesight, lack of breath as well as unconsciousness.

It appears that the hand calculated graph is very accurate because almost all the original data points fit the function. The only point that doesn't fit the data is (3,9), which implies that the method of substitution to find a function is quite accurate.

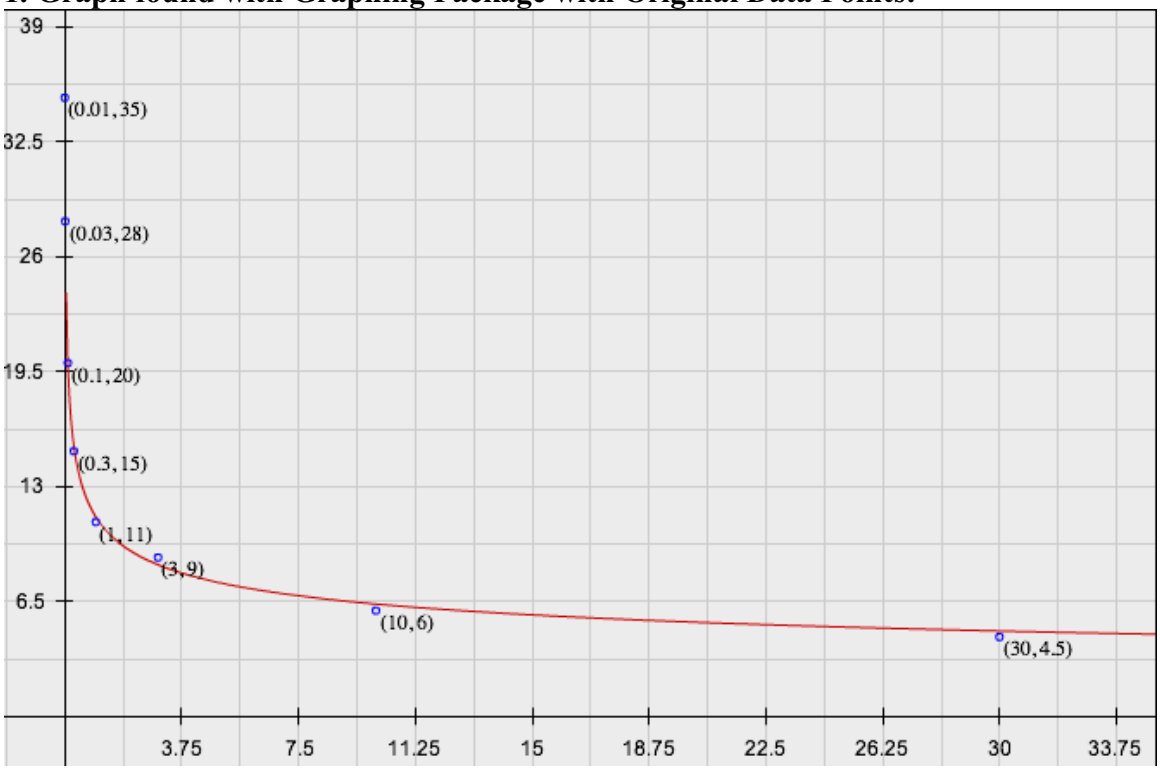
To find another model that fits the data, use a graphing package that takes all the data points in account and tries to find the best fit function. In this case it is a power function because most of the data points fit. The equation $\mathbf{y=11.255x^{-.24988}}$ is calculated when all the data is inserted into the graphing package. Three significant figures are taken of both constants to produce the equation: $\mathbf{+Gx=11.3t^{-.250}}$.

Regression model using graphing package of: $+Gx = 11.3t^{-.250}$
(scale: 35 min x 40 g)



Like the function drawn by hand, the function illustrates that as +Gx decreases, the time that humans can tolerate the force increases. This function also seems to show that once g-force is below 8 or 9 g, the time that humans can tolerate the force starts to increase less rapidly.

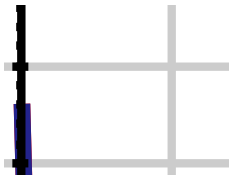
1. Graph found with Graphing Package with Original Data Points:



2. **Hand calculated graph & technology found graph** where the red indicates the technology found graph and the purple indicates the hand calculated graph.

→ **red: $+Gx = 11.3t^{-.250}$**

→**purple: $+Gx = 11.1t^{-.255}$**



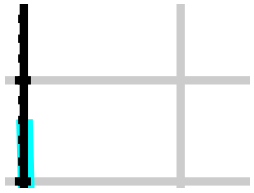
In the graph labeled 1, the function clearly shows more discrepancies between the original data plots and the function found on the graphing package. The point (3,9) is closer to the function than the hand calculated function because it illustrates the rapid decrease in g-force for a small change in time. However, the points (10,6) and (30,4.5) do not fit the function very well because they are just under the modeled function. This means that the g-force will actually decrease more than the function shows. However, because the point (3,9) is very clearly above the function, it implies that the g-force does not decrease as fast as the function shows. This could also be because this point is an outlier in the data and that it is due to an experimental error.

In the graph labeled 2, functions found by hand and technology are graphed on the same axes. The functions are very similar and both represent the data quite accurately. The biggest difference that can be seen from the graph is that in the hand calculated function the g-force decreases faster in respect to the time, as the line representing the function is displayed under the function found with technology. In the hand calculated function the g-force also decreases more after the same amount of time (35 minutes).

After solely looking at horizontal g-force, vertical g-force must also be considered. This graph displays the hand calculated function of the second set of data for vertical g-force (using the same substitution method): $\mathbf{+Gz = 7.02t^{-.239}}$ as well as the first hand calculated function of horizontal g-force: $\mathbf{+Gx = 11.1t^{-.255}}$.

→ **blue display +Gx**

→ **pink displays +Gz**



From the above graph it can be

seen immediately that as both positive horizontal and vertical g-force decreases, the time that a human can tolerate that force increases.
However, when comparing horizontal versus vertical g-force, it has to be noted that humans can tolerate a high amount of horizontal g-force longer than vertical g-force, which is seen from the bigger decrease in +Gz than +Gx. Although, both forces pose great danger to the body when exposed to high amounts, the effects they have on the human body are different.

With positive vertical g-force the tolerance is lower because the person exposed to it will have trouble breathing because the force will pull down on the rib cage, which pushes down on the lungs, emptying the air. More importantly, because of the force, the blood gets pulled away from the brain and moves towards the feet. Because of the lost blood, oxygen and other nutrients don't get supplied to the brain and the person exposed to the g-force will start to lose peripheral vision and be left with tunnel vision and experience black outs as well as unconsciousness. After longer exposure to g-force all vision can become impaired. Other symptoms include impairment of speech, limb muscle strength and the senses. Because of the force pulling down on the body, the organs also get pulled down towards the feet, which may lead to the malfunction of certain organs.

Humans have a higher tolerance towards horizontal g-force because instead of the blood getting rushed towards the feet, the blood gets rushed to the back of the head. This is why the term "eyeballs in" was created. As the blood flows towards the back of the brain, eyesight is also likely to become impaired. The lack of oxygen to certain parts of the brain will also eventually cause unconsciousness but not as quickly, as seen from the models.

Although coherent models have been produced from the data given, it still remains too insufficient to make full conclusions. This is because the effect of g-force on the human body changes with the person's posture as well as the room conditions they are in. There is a way of resisting the effects of g-force, which is called "strain". "Strain" involves that the person has to contract their muscles below the waist and relax the upper body in order to breathe properly. The person's shoulders have to be able to drop in order for the diaphragm to be relaxed. This can be helpful for situations on an airplane right before turns so that people can prepare for the g-force effects.

The room conditions in which the g-force is applied make a difference in the effect of the g-force because if the g-force acting on the subject is in a weightless environment the g-force will be zero. This is a clear limitation in the models, because they do not illustrate a x-intercept whereas in a real life situation it is plausible that the g-force is zero if the object were free falling in a vacuum. Another constraint of this data is that it only is modeling positive g-force, while negative g-force is just as plausible.

Although there are limitations to the data as well as to the models, never the less the power function remains the best solution. The functions found by hand as well as by technology both are almost completely accurate because they intercept all data points but one: (3,9). All the functions show that as +Gx decreases, the time that humans can

tolerate that given g-force becomes longer.

From the models created from the data, it can be concluded that humans should not be exposed to high amounts of g-force or g-force for long periods of time because there are a lot of health risks. At low levels of g-force, humans are able to tolerate it for longer amounts of time but will still suffer from unconsciousness and other side effects. This means that when engineers are designing rockets, formula one cars, roller coasters and other objects that have very high rates of acceleration, they must think about the effects of the g-force on humans and how that changes with the time of exposure to the force.