In this investigation I will be studying the case of an infection of particles. I will be looking into, and analyzing, how the particles work when they first enter the body, what effect the response of the immune system has, how medication is delivered and maintained, as well as death and recovery. Furthermore, I will be altering my investigation models to cater to a young child as opposed to an adult.

First I will look at the initial phase of the infection – the part where the particles enter the body and replicate yet none are expelled because the immune system hasn't responded. To determine how long it will take before the immune system responds I need to create a basic formula:

$a(r^n)$

In this formula, *a* represents the initial amount of particles and *r* represents the ratio at which they multiply every 4 hours – just like they are used in sequences. *n* represents how many times they multiply, which is once every 4 hours.

Considering the case of a young adult male, I presume that he is initially infected with 10,000 particles and that they double every hour. I also presume that the immune system responds when the particle count reaches 1,000,000. Therefore in order to find the time it takes for the immune system to respond I need to equate the formula to 1,000,000 and plug in the values I have presumed. After doing this I would be left with the following:

$10{,}000(2^n) = 1{,}000{,}000$

$2^n = 1{,}000{,}000 \div 10{,}000 = 100$

At this point I use log to determine the value of *n*:

$\text{Log}2^n = \text{Log}100$

$n\text{Log}2 = \text{Log } 100$

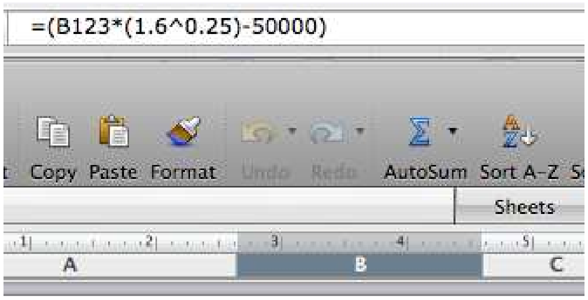
$n = \text{Log}100 \div \text{Log}2 = 6.64$

At this point I remember that *n* is the number of times the particle count doubles, and that it takes place once every 4 hours. Therefore I multiply my value of *n* with 4 to determine the time it takes for the immune system to respond, which is 26.58 hours.

At this point I decide that I want to find out how long the patient will live for if he goes untreated. I presume that once the immune system responds the particles will not double every four hours, but instead they will increase by 160%. I also presume that 50,000 particles will be eliminated from the body every hour because of the immune system. Furthermore, I assume that once the person reaches 10^{12} particles he will die. For this task I decide to create a basic formula that can be input into a spreadsheet:

$u_n = u_{n-1}(1.6^{0.25}) – 50{,}000$

I take u_n to be the amount of particles at a certain time, and u_{n-1} to be the amount of particles an hour before. I multiply u_{n-1} with $1.6^{0.25}$ to show the increase in particles (0.25 because this formula is for every hour, and the particles multiply by 1.6 every four hours, therefore 1 ÷ 4 = 0.25). Finally I subtract by 50,000 to illustrate the removal of particles by the immune system. This leaves me with the new number of particles at current time. I put this formula into Microsoft Excel to show me when the particle count reaches 10^{12}.



A	B	C
97	53373605215	
98	60028317774	
99	67512757532	
100	75930377075	
101	85397527734	
102	96045067827	
103	1.0802E+11	
104	1.21488E+11	
105	1.36636E+11	
106	1.53672E+11	
107	1.72832E+11	
108	1.94381E+11	
109	2.18617E+11	
110	2.45875E+11	
111	2.76531E+11	
112	3.1101E+11	
113	3.49787E+11	
114	3.93399E+11	
115	4.42449E+11	
116	4.97615E+11	
117	5.59659E+11	
118	6.29439E+11	
119	7.07919E+11	
120	7.96184E+11	
121	8.95454E+11	
122	1.0071E+12	ANSWER
123	1.13267E+12	
124	1.27389E+12	

The left hand column displays the number of hours since the immune system came into effect, and the right hand column displays the particle count. On top the formula input by me is displayed. 122 hours after the immune system responds the particle count is 1.0071×10^{12} which I feel is pretty close to 1×10^{12}, so I keep my answer as 122 hours. However, this is the amount of time it would take for the victim to die from the time the immune system responds. To determine the time of death from first infected I need to add 26.58 and 122, which gives us 148.6 (4 s.f.) hours.

At this point I decide to see the effect of an antiviral medication. I postulate that the medicine will not have an effect on the growth of the particles but it, along with the immune system, will eliminate 1,200,000 particles every hour. Despite the huge number of particles being expelled from the victim's system I believe that there is a point where the growth of the particles outnumber expulsion of particles leading to inevitable death. I assume the point when the number of particles are so large that the medicine is useless, is between 9 million and 10 million. Therefore I decide to plug both numbers into the following formula:

$u_n(1.6^{0.25}) - 1{,}200{,}000$

Plugging 9,000,000 I find that:

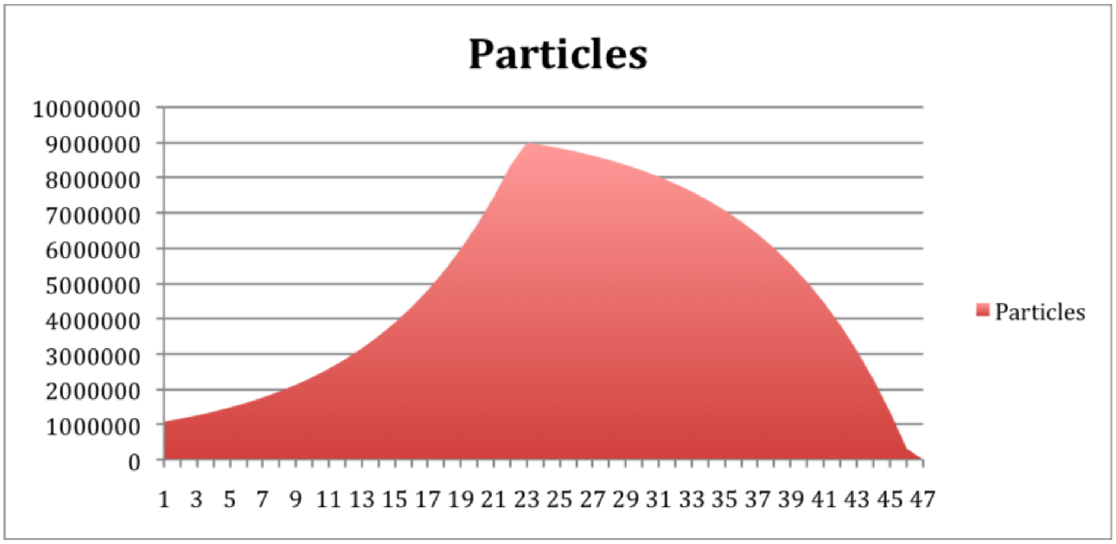
$9{,}000{,}000(1.6^{0.25}) - 1{,}200{,}000 = 8922000$ (4 s.f.)

This shows that the number of particles in the system have actually decreased implying that the victim is recovering.

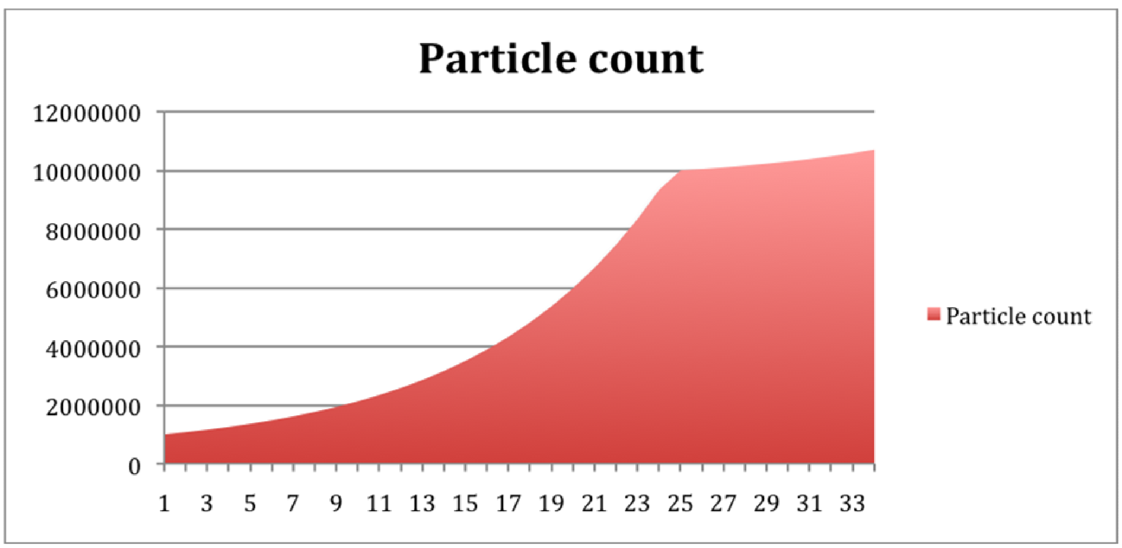
However when I plug 10,000,000 into the equation I find that:

$10{,}000{,}000(1.6^{0.25}) - 1{,}200{,}000 = 10{,}050{,}000$ (4 s.f.)

This, on the other hand, displays that despite the medicine being in the victim's system (seen by the 1,200,000 particle expulsion) the victim's particle count has still increased deeming the medicine useless. The following graphs can illustrate this further:



The graph above shows how if the medicine is administered at 9,000,000 particles, the victim begins to recover. However, the graph below shows how the victim's particle count still rises if the medicine is administered at 10,000,000 particles.



These two graphs display that the medicine MUST be administered before 9,000,000 particles are reached if the victim wants to make a full recovery.

Now I presume that realistically a person requires time to adapt to medicine so therefore I decided to give the victim a 4 hour time period to adapt to the medicine during which he is on continuous intravenous dosing, whereby he gets a drip of medicine every minute. However, I presume that during this time the kidney is removing 2.5% of the medicine per hour. I also assume that the victim requires at least 90 micrograms of medicine at the end of the 4 hours to begin and maintain the elimination of 1,200,000 particles. Using this information I decide to determine how much of the medicine the patient receives every minute and every hour to reach 90 micrograms by the end of 4 hours.

The victim loses 2.5% of the medicine every hour, but since takes place over the course of the hour I cannot simply subtract 2.5% from total amount of medicine at the end of every hour. However, finding the exact amount lost is very complicated so in order to find a decently accurate I presume that the decay is linear, therefore (2.5 ÷ 60 = 0.0416)% of the medicine is eliminated every minute. This enables to create the following equation:

$D(0.9996^{240}) + D(0.9996^{239}) + \dots + D(0.9996^{1}) + D$

This formula helps determine the dosage because *D* represents the dosage per minute, and multiplying with 0.999583 is the equivalent of removing 0.0416%. This number has a power of 240 because 0.0416% is removed 240 times by the end. For the next dose injected a minute later, this takes place 239 times because it is in the body for 1 minute less. Similarly, the dose injected 1 minute before the end of the 4 hours only has a power of 1 because 0.0416% is only removed one time before the end.

This equation can then be rearranged to put the smallest powers at the beginning and the largest powers at the end but either way I have a geometric sequence. I then use the formula to find the sum of a geometric sequence:

$a(r^n - 1) \div (r - 1)$ where *a* is the initial term (*D*, if the equation is rearranged) and *r* is the ratio of multiplication (0.9996, if the equation is rearranged) and *n* is the number of times the ratio is multiplied (240).

This equation must multiply out to 90 so that 90 micrograms are ensured in the system at the end of the four hours. Plugging the values in I get the following:

$D(0.9996^{240} - 1) \div (0.9996 - 1) = 90$

$D(-0.095) \div (-0.000416) = 90$

$D(-0.095) = 90 \times (-0.000416) = -0.038$

$D = (=0.038) \div (-0.095) = 0.39$ micrograms

I presume that once the level of medication has reached 90 micrograms, at the end of the 4 hours, the body will have adapted to the medicine therefore the victim will be taken off the intravenous phase and he will be given injections every 4 hours. Nevertheless, the kidneys will still be removing 2.5% of the medication every hour, therefore I need to calculate the extra dosage which must be given every 4 hours so that the amount of medicine in the victims bloodstream is never lower than 90 micrograms. The following equation can help me come up with my answer:

$(90 + D) \times 0.975^4 \geq 90$

$(90 + D) \times 0.904 \geq 90$

$90 + D \geq 90 \div 0.904 \geq 99.59$

$D \geq 99.59 - 90$

$D \geq 9.59$

I used this equation because I presumed that a dose, D, was added before the 4 hour period begun, then 2.5% was removed every hour, for 4 hours.

Now I decide to go into more detail regarding the last point when the victim can take medicine and survive. To do this first I have to find out the number of particles the person must have in his body at this point. To do this I looked at each aspect of the following formula:

$u_n(1.6^{0.25}) - 1{,}200{,}000$

This formula shows the number of particles in the next hour. If I have u_n as the starting amount of particles and the finishing amount of particles is 1 less than u_n, then u_n is the number of particles the person would have in his body at the last possible time to take medicine and recover. In other words, the gain in particles would be 1,199,999 and the loss would be 1,200,000 so 1 particle is lost starting the recovery process. Using this I can equate the formula above to:

$u_n(1.6^{0.25}) - 1{,}200{,}000 = u_n - 1$

$u_n(1.6^{0.25}) - 1{,}200{,}000 - u_n = -1$

$u_n(1.6^{0.25}) - u_n = 1{,}199{,}999$

$u_n(1.6^{0.25} - 1) = 1{,}199{,}999$

$u_n(0.1246) = 1{,}199{,}999$

$u_n = 9624426$

Using this knowledge I can utilize the general formula to calculate the last possible time from the onset of the infection to start the medication.

General formula: $a(r^{(n)}) - e[(r^{(n)} - 1) \div (r - 1)]$

In this formula, *a* is the initial term, *r* is the ratio of multiplication, *n* is the number of hours, and *e* is the amount being eliminated by the body every hour.

I insert values in this formula to determine the time it takes to reach 9624426 from the moment the immune system responds to the virus:

$a = 1,000,000$

$r = 1.6^{0.25}$

$e = 50,000$

$r^n = 1.6^{0.25n}$ because of indices rules

$1,000,000(1.6^{(0.25n)}) – 50,000[(1.6^{(0.25n)} – 1) \div (1.6^{0.25} – 1)] = 9624426$

Factorizing and clearing this up, I get:

$20(1.6^{(0.25n)}) – [(1.6^{(0.25n)} – 1) \div (0.1246)] = 192.5$ (4 s.f.)

$(1.6^{(0.25n)} – 1) \div (0.1246) = 20(1.6^{(0.25n)}) – 192.5$

$(1.6^{(0.25n)} – 1) = 2.49(1.6^{(0.25n)}) – 23.99$

$(1.6^{(0.25n)} – 1) - 2.49(1.6^{(0.25n)}) = - 23.99$

$1.6^{(0.25n)} - 2.49(1.6^{(0.25n)}) = -22.99$

$1.6^{(0.25n)}(1 – 2.49) = -22.99$

$1.6^{(0.25n)}(-1.49) = -22.99$

$1.6^{(0.25n)} = (-22.99) \div (-1.49) = 15.43$

Using log:

$(0.25n)\log1.6 = \log15.43$

$0.25n = \log15.43 \div \log1.6 = 5.82$

$n = 23.28$ hours (last possible time from response of immune system to start medication)

However, I need to find the last possible time from the onset of infection to start the medication therefore I need to add 26.58 to 23.28 which gives us 49.87 hours.

In order to complete the analysis of this victim I also need to calculate the time it will take him to recover from this point in time. In order to do this I will manipulate the general formula again and equate it to 0 (giving the time he has 0 particles left in his system):

$9624426(1.6^{0.25n}) – 1,200,000[(1.6^{0.25n} – 1) \div (1.6^{0.25} – 1)] = 0$

$1,200,000[8.0203(1.6^{0.25n}) – [(1.6^{0.25n} – 1) \div (1.6^{0.25} – 1)] = 0$

$8.02(1.6^{0.25n}) – [(1.6^{0.25n} – 1) \div (0.1246)] = 0$

$8.02(1.6^{0.25n}) = (1.6^{0.25n} – 1) \div (0.1246)$

$0.9999(1.6^{0.25n}) = (1.6^{0.25n}) – 1$

$0.9999(1.6^{0.25n}) - (1.6^{0.25n}) = – 1$

$(1.6^{0.25n})(0.9999 – 1) = – 1$

$(1.6^{0.25n})(-0.0000881) = - 1$

$(1.6^{0.25n}) = (- 1) \div (-0.00000881) = 1135073$

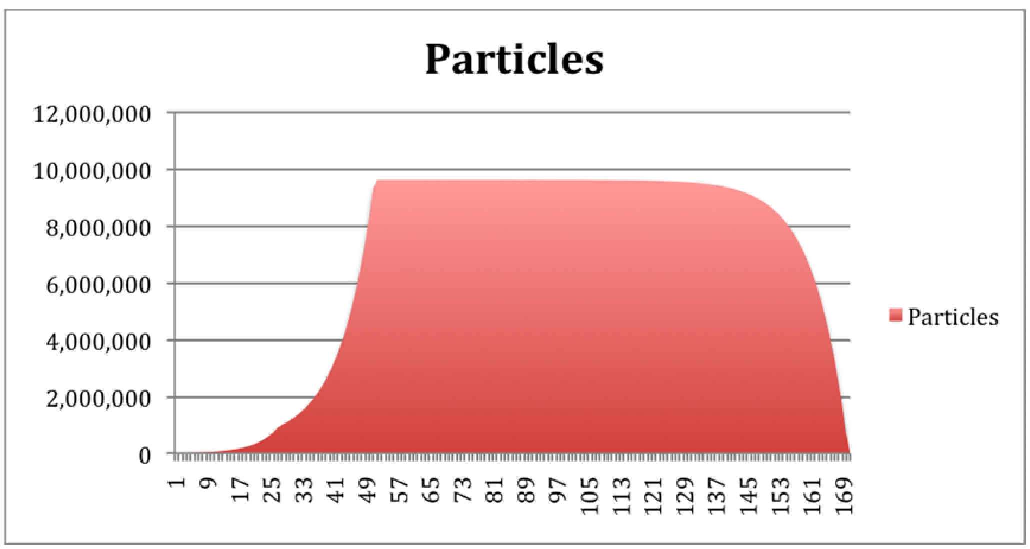
Using log:

$(0.25n)\log1.6 = \log1135073$

$0.25n = \log1135073 \div \log 1.6 = 29.66$ hours

$n = 118.65$ hours

This displays that it will take the victim 118.65 hours from the start of the regimen of medicine to recover completely. From this I can determine that the victim is infected for a total time of 118.65 + 23.28 + 26.58 = 168.51 hours.



In my investigation I have made many assumptions which I have listed before utilizing them, throughout the course of the investigation. I think the strength of the model is that it gives each answer very precisely as the general formula doesn't give an approximation, however this is also a weakness because it is highly unlikely that any syringe can drop 0.39 micrograms of medicine every minute. There are a lot of variables in this investigation that would make the answers very unrealistic – deeming the investigation very unreliable.

I have displayed how this affects a young adult, but this model can be modified if the victim was a child instead. This can be done by changing many of the variables – the initial amount could be 25000 instead of 10000 since a child has lower resistance, the rate of multiplication could be 3 instead of 2 at the beginning for the same reason. The immune system could kick in at 3,000,000 instead of 1,000,000 because it would take longer for the body to acknowledge the presence of the virus. The amount of particles expelled could be 20000 instead of 50000 as the child's immune system isn't as strong. Furthermore, the child – having a smaller body – would take it a lot less medicine dropping all those figures even further. Finally, the amount of particles needed for death would be a lot lower, 10^9 instead of 10^{12}.

I will now explain how I derived the general formula. First I presume that I are just finding a general formula for the time period after the immune system kicks in so I get the following:

$\{[1,000,000(1.6^{0.25}) – 50,000](1.6^{0.25}) – 50,000\}(1.6^{0.25}) – 50,000 \ldots$

Opening this up I get the following:

$1,000,000(1.6^{0.25})(1.6^{0.25})(1.6^{0.25})–50,000(1.6^{0.25})(1.6^{0.25})–50,000(1.6^{0.25})–50,000 \ldots$

For the first part I can immediately make out that the number of $(1.6^{0.25})$ attached to the 1,000,000 are equal to the number of hours passed making it, where *n* is the number of hours passed:

$1,000,000(1.6^{0.25n})$

For the second half we can factorize 50,000 out leaving us with:

$X – 50,000\{(1.6^{0.25})(1.6^{0.25}) + (1.6^{0.25}) + 1\}$

$X – 50,000\{(1.6^{0.25(2)}) + (1.6^{0.25(1)}) + (1.6^{0.25(0)})\}$

I then see a geometric equation forming with *a* being 1 because $(1.6^{0.25(0)}) = 1$ and *r* being $(1.6^{0.25})$. Using the sum of a geometric formula:

$a(r^n – 1) \div (r – 1)$

$1[(1.6^{0.25n}) – 1] \div [(1.6^{0.25}) – 1]$

From this we can derive the general formula after the immune system kicks in to be:

$1,000,000(1.6^{0.25n}) – 50,000[(1.6^{0.25n}) – 1] \div [(1.6^{0.25}) – 1])$

We can then use this format to derive a general formula that would work for any numbers:

$P(r^{0.25n}) – e[(r^{0.25n} – 1) \div (r – 1)]$

In this formula P is the starting number of particles (so 10,000 if we start from when he was first affected, or 1,000,000 if we start from when the immune system kicks in). r is the rate of multiplication every 4 hours, and e is the number of particles eliminated from the body every hour. From this formula we can plug in any values for any of the factors and determine the number of particles.