Mathematics SL Portfolio Assignment 1

Title: Matrix Powers

Type 1

1. Consider the matrix **M** = $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

Calculate $\boldsymbol{M}^n$ for n = 2, 3, 4, 5, 10, 20, 50.

$$M^2=\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} * \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 2*2+0*0 & 2*0+0*2 \\ 0*2+2*0 & 0*0+2*2 \end{pmatrix} = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$$

$$M^3=\begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} * \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 4*2+0*0 & 4*0+0*2 \\ 0*2+4*0 & 0*0+2*2 \end{pmatrix} = \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix}$$

$$M^4=\begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix} * \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 8*2+0*0 & 8*0+0*2 \\ 0*2+8*0 & 0*0+8*2 \end{pmatrix} = \begin{pmatrix} 16 & 0 \\ 0 & 16 \end{pmatrix}$$

$$M^5=\begin{pmatrix} 16 & 0 \\ 0 & 16 \end{pmatrix} * \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 16*2+0*0 & 16*0+0*2 \\ 0*2+16*0 & 0*0+16*2 \end{pmatrix} = \begin{pmatrix} 32 & 0 \\ 0 & 32 \end{pmatrix}$$

$$M^{10}=\begin{pmatrix} 32 & 0 \\ 0 & 32 \end{pmatrix} * \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} 32*4+0*0 & 32*0+0*4 \\ 0*4+32*0 & 0*0+32*4 \end{pmatrix} = \begin{pmatrix} 1024 & 0 \\ 0 & 1024 \end{pmatrix}$$

$$M^{20}=\begin{pmatrix} 1024 & 0 \\ 0 & 1024 \end{pmatrix} * \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} 1024*4+0*0 & 1024*0+0*4 \\ 0*4+1024*0 & 0*0+1024*4 \end{pmatrix} = \begin{pmatrix} 1048576 & 0 \\ 0 & 1048576 \end{pmatrix}$$

$$M^{50}=\begin{pmatrix} 1024 & 0 \\ 0 & 1024 \end{pmatrix} * \begin{pmatrix} 32 & 0 \\ 0 & 32 \end{pmatrix} = \begin{pmatrix} 1024*32+0*0 & 1024*0+0*32 \\ 0*32+1024*0 & 0*0+1024*32 \end{pmatrix} =$$

$$\begin{pmatrix} 1.125899907*10\text{^}15 & 0 \\ 0 & 1.125899907*10\text{^}15 \end{pmatrix}$$

- *To determine the matrices above I have used (a*e+b*g), (a*f+b*h), (c*e+d*g), (c*f+d*h) as shown in the general formula seen below*

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} (ae+bg) & (af+bh) \\ (ce+dg) & (cf+dh) \end{pmatrix}$$

Describe in words any pattern you observe.

In the above matrices the pattern I observed is shown in relationship to the exponents and the numbers within the matrix. As the exponent increases consecutively the matrix is in turn multiplied by two.

Use this pattern to find a general expression for the matrix *M^n* in terms of *n*.

As a result of the pattern expressed, the General Formula for these matrices is **$M^n=M*2^{n-1}$**

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2. Consider the matrices $P=\begin{pmatrix}3 & 1\\ 1 & 3\end{pmatrix}$ and $S=\begin{pmatrix}4 & 2\\ 2 & 4\end{pmatrix}$

Calculate P^n and S^n for other values of n and describe any pattern(s) you observe.

$P^2=\begin{pmatrix}3 & 1\\ 1 & 3\end{pmatrix}*\begin{pmatrix}3 & 1\\ 1 & 3\end{pmatrix}=\begin{pmatrix}10 & 6\\ 6 & 10\end{pmatrix}=\boxed{2\begin{pmatrix}5 & 3\\ 3 & 5\end{pmatrix}}$ **Determinant: 100-36=64**

$P^3=\begin{pmatrix}10 & 6\\ 6 & 10\end{pmatrix}*\begin{pmatrix}3 & 1\\ 1 & 3\end{pmatrix}=\begin{pmatrix}36 & 28\\ 28 & 36\end{pmatrix}=\boxed{4\begin{pmatrix}9 & 7\\ 7 & 9\end{pmatrix}}$ **Determinant: 1296-784=512**

$P^4=\begin{pmatrix}36 & 28\\ 28 & 36\end{pmatrix}*\begin{pmatrix}3 & 1\\ 1 & 3\end{pmatrix}=\begin{pmatrix}136 & 120\\ 120 & 136\end{pmatrix}=\boxed{8\begin{pmatrix}17 & 15\\ 15 & 17\end{pmatrix}}$ **Determinant: 18496-14400=4096**

$P^5=\begin{pmatrix}136 & 120\\ 120 & 136\end{pmatrix}*\begin{pmatrix}3 & 1\\ 1 & 3\end{pmatrix}=\begin{pmatrix}528 & 496\\ 496 & 528\end{pmatrix}\boxed{16\begin{pmatrix}33 & 31\\ 31 & 33\end{pmatrix}}$ **Determinant: 278784-246016=32768**

$S^2=\begin{pmatrix}4 & 2\\ 2 & 4\end{pmatrix}*\begin{pmatrix}4 & 2\\ 2 & 4\end{pmatrix}=\begin{pmatrix}20 & 16\\ 16 & 20\end{pmatrix}=\boxed{2\begin{pmatrix}10 & 8\\ 8 & 10\end{pmatrix}}$ **Determinant: 400-256=144**

$S^3=\begin{pmatrix}20 & 16\\ 16 & 20\end{pmatrix}*\begin{pmatrix}4 & 2\\ 2 & 4\end{pmatrix}=\begin{pmatrix}112 & 104\\ 104 & 102\end{pmatrix}=\boxed{8\begin{pmatrix}19 & 13\\ 13 & 19\end{pmatrix}}$ **Determinant: 12544-10816=1728**

S□ $= \begin{pmatrix} 112 & 104 \\ 104 & 102 \end{pmatrix} * \begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} 656 & 640 \\ 640 & 656 \end{pmatrix} = \boxed{16 \begin{pmatrix} 41 & 40 \\ 40 & 41 \end{pmatrix}}$ **Determinant: 430336-409600=<u>20736</u>**

S□ $= \begin{pmatrix} 656 & 640 \\ 640 & 656 \end{pmatrix} * \begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} 3904 & 3872 \\ 3872 & 3904 \end{pmatrix} = \boxed{32 \begin{pmatrix} 122 & 121 \\ 2 & 4 \end{pmatrix}}$

Determinant: 15241216-14992384=<u>248832</u>

- [illegible] $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

Patterns Found:

The patterns portrayed in matrices **Pⁿ** and **Sⁿ** mainly correspond to the coefficient shown in the box and the determinant in each individual series. In the series of **Pⁿ**, as each exponent progress consecutively the coefficients associated with that particular matrix (in factored form) are multiplied by two **(2, 4, 8, and 16)**. Also, each determinant found in the **Pⁿ** series is multiplied by 8 as each exponent progress consecutively (64, 512, 4096, 32768). In the **Sⁿ** each determinant found yielded is multiplied by 12 as each exponent progress consecutively (512, 1728, 20736, 248832.) However, unlike the pattern found in the **Pⁿ** series, there are no consistencies in regards to coefficients of the matrixes factored form.

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3. Now consider matrices of the form $\begin{pmatrix} k+1 & k-1 \\ k-1 & k+1 \end{pmatrix}$

Steps 1 and 2 contain examples of these matrices for k = 1, 2 and 3

K=1: $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ **Determinant: 4-0=<u>4</u>** **K=2:** $\begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$ **Determinant: 9-1=<u>8</u>**

K=3: $\begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix}$ **Determinant: 16-4=<u>12</u>**

Consider other values of k, and describe any pattern(s) you observe.

K=4: $\begin{pmatrix} 4+1 & 4-1 \\ 4-1 & 4+1 \end{pmatrix}$**=**$\begin{pmatrix} 5 & 3 \\ 3 & 5 \end{pmatrix}$ **Determinant: 25-9=<u>16</u>**

K=5: $\begin{pmatrix} 5+1 & 5-1 \\ 5-1 & 5+1 \end{pmatrix}$**=**$\begin{pmatrix} 6 & 4 \\ 4 & 6 \end{pmatrix}$ **Determinant= 36-16=<u>20</u>**

K=6: $\begin{pmatrix} 6+1 & 6-1 \\ 6-1 & 6+1 \end{pmatrix}$**=**$\begin{pmatrix} 7 & 5 \\ 5 & 7 \end{pmatrix}$ **Determinant= 49-25=<u>24</u>**

K=7: $\begin{pmatrix} 7+1 & 7-1 \\ 7-1 & 7+1 \end{pmatrix}$**=**$\begin{pmatrix} 8 & 6 \\ 6 & 8 \end{pmatrix}$ **Determinant= 64-32=<u>28</u>**

Patterns found:

The pattern that was evident in the above matrices corresponds to the determinant value as well as the value of the consecutive matrices. As each value for K is raised consecutively the numbers in the associated matrix is then raised by one. The determinant also followed a pattern by raising the K value consecutively the determinant also rose by four.

Generalize these results in terms of k and n.

n=exponent value

Results for k, n=2:

$$\begin{pmatrix} k+1 & k-1 \\ k-1 & k+1 \end{pmatrix}^2 = \begin{pmatrix} k+1 & k-1 \\ k-1 & k+1 \end{pmatrix} * \begin{pmatrix} k+1 & k-1 \\ k-1 & k+1 \end{pmatrix} =$$

$$\begin{pmatrix} (k+1)(k+1)+(k-1)(k-1) & (k+1)(k-1)+(k-1)(k+1) \\ (k-1)(k+1)+(k+1)(k-1) & (k-1)(k-1)+(k+1)(k+1) \end{pmatrix} = \boxed{\begin{pmatrix} 2k^2+2 & 2k^2-2 \\ 2k^2-2 & 2k^2+2 \end{pmatrix}}$$

Results for k, n=3: $\begin{pmatrix} k+1 & k-1 \\ k-1 & k+1 \end{pmatrix}^3 = \begin{pmatrix} 2k^2+2 & 2k^2-2 \\ 2k^2-2 & 2k^2+2 \end{pmatrix} * \begin{pmatrix} k+1 & k-1 \\ k-1 & k+1 \end{pmatrix} = \boxed{\begin{pmatrix} 4k^3+4 & 4k^3-4 \\ 4k^3-4 & 4k^3+4 \end{pmatrix}}$

Results for k, n=4 $\begin{pmatrix} k+1 & k-1 \\ k-1 & k+1 \end{pmatrix}^4 = \begin{pmatrix} 4k^3+4 & 4k^3-4 \\ 4k^3-4 & 4k^3+4 \end{pmatrix} * \begin{pmatrix} k+1 & k-1 \\ k-1 & k+1 \end{pmatrix} = \boxed{\begin{pmatrix} 8k^4+8 & 8k^4-8 \\ 8k^4-8 & 8k^4+8 \end{pmatrix}}$

Results for k, n=5 $\begin{pmatrix} k+1 & k-1 \\ k-1 & k+1 \end{pmatrix}^5 = \begin{pmatrix} 8k^4+8 & 8k^4-8 \\ 8k^4-8 & 8k^4+8 \end{pmatrix} * \begin{pmatrix} k+1 & k-1 \\ k-1 & k+1 \end{pmatrix} =$

$$\boxed{\begin{pmatrix} 16k^5+16 & 16k^5-16 \\ 16k^5-16 & 16k^5+16 \end{pmatrix}}$$

- Therefore, based on the matrices shown above, the generalized formula of this pattern is:

$$\begin{pmatrix} 2^{n-1}k^n + 2^{n-1} & 2^{n-1}k^n - 2^{n-1} \\ 2^{n-1}k^n - 2^{n-1} & 2^{n-1}k^n + 2^{n-1} \end{pmatrix}$$

4. **Use technology to investigate what happens with further values of k and n. State the scope or limitations of k and n.**

Results for k=2, n=-(-2) $\begin{pmatrix} 2^{-2-1}2^{-1}+2^{-2-1} & 2^{-2-1}2^{-1}-2^{-2-1} \\ 2^{-2-1}2^{-1}-2^{-2-1} & 2^{-2-1}2^{-1}+2^{-2-1} \end{pmatrix} = \begin{pmatrix} .1875 & -.0625 \\ -.0625 & .1875 \end{pmatrix}$

Results for k=2, n=-(-3) $\begin{pmatrix} 2^{-3-1}2^{-1}+2^{-3-1} & 2^{-3-1}2^{-1}-2^{-3-1} \\ 2^{-3-1}2^{-1}-2^{-3-1} & 2^{-3-1}2^{-1}+2^{-3-1} \end{pmatrix} = \begin{pmatrix} .09375 & -.03125 \\ -.03125 & .09375 \end{pmatrix}$

Results for k=(-2), n=-(2) $\begin{pmatrix} 2^{2-1}-2^2+2^{2-1} & 2^{2-1}-2^2-2^{2-1} \\ 2^{2-1}-2^2-2^{2-1} & 2^{2-1}-2^2+2^{2-1} \end{pmatrix} = \begin{pmatrix} 10 & 6 \\ 6 & 10 \end{pmatrix}$

Results for k=(-3), n=-(2) $\begin{pmatrix} 2^{2-1}-3^{2}+2^{2-1} & 2^{2-1}-3^{2}-2^{2-1} \\ 2^{2-1}-3^{2}-2^{2-1} & 2^{2-1}-3^{2}+2^{2-1} \end{pmatrix} = \begin{pmatrix} 20 & 16 \\ 16 & 20 \end{pmatrix}$

Results for k=(.5), n=-(2) $\begin{pmatrix} 2^{2-1}-3^{2}+2^{2-1} & 2^{2-1}-3^{2}-2^{2-1} \\ 2^{2-1}-3^{2}-2^{2-1} & 2^{2-1}-3^{2}+2^{2-1} \end{pmatrix} = \begin{pmatrix} 2.75 & -1.25 \\ -1.25 & 2.75 \end{pmatrix}$

Results for k=(2), n=-(.5)

$$\begin{pmatrix} 2^{.5-1}2^{.5}+2^{.5-1} & 2^{.5-1}2^{.5}-2^{.5-1} \\ 2^{.5-1}2^{.5}-2^{.5-1} & 2^{.5-1}2^{.5}+2^{.5-1} \end{pmatrix} = \begin{pmatrix} 1.707106781\ \ 2 & .2928932188 \\ .2928932188 & 1.707106781\ \ 2 \end{pmatrix}$$

- *According to the numbers [illegible] in the problem above, [illegible] the scope of the problem [illegible] fractions. [illegible]*

5. **Explain why your results hold true in general.**

Referring back to work shown in question three, my results for these statements hold true because of the amount of times the equations were carried out. Overall, the steadiness of the results for each of the problems ensures the validity of each statement. A wide array of aspects was used to prove these statements true ranging from negative integers to fractions. The patterns seen were also consistent with the results which reinforce the strength of my statements. Therefore, I conclude that my results hold true in general.