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Q1) Find a formula for  $a_{n+1}$  in terms of  $a_n$ .

Answer)

The following expression is an example of an infinite surd:

This means that can be shown as=

$$a_1 =$$

$$a_2 =$$

$$a_3 =$$

A formula for  $a_{n+1}$  in terms of  $a_n$  would be:

$$a_2 =$$

$$a_3 =$$

Square on both sides

$$(a_3)^2 = ()^2$$

$$(a_3)^2 = 1 + a_2$$

$$a_2 = (a_3)^2 - 1$$

$$a_n = (a_{n+1})^2 - 1$$

$$(a_{n+1})^2 = a_n + 1$$

Therefore,

$$a_{n+1} =$$

Q2) Calculate the decimal values of the first ten terms of the sequence. Using technology, plot the relation between 'n' and 'a'. Describe what you notice. What does this suggest about the value of  $a_n - a_{n+1}$  as 'n' gets very large? Use your results to find the exact value for this infinite surd.

Ans)

➤ <u>n</u>	<u>Term (<math>a_n</math>)</u>	<u><math>a_n - a_{n+1}</math></u>
➤ 1	1.414213562	-

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➤ 2	1.553773974	0.139560412
➤ 3	1.598053182	0.044279208
➤ 4	1.611847754	0.013794572
➤ 5	1.616121207	0.004273452
➤ 6	1.617442799	0.001321592
➤ 7	1.617851291	0.000408492
➤ 8	1.617977531	0.000126241
➤ 9	1.618016542	$3.90113 \times 10^{-5}$
➤ 10	1.618028597	$1.20552 \times 10^{-5}$

The difference between each successive term is beginning to decrease and is almost at zero.

→ As 'n' keeps increasing, the value of  $a_n - a_{n+1}$  keeps decreasing, and gradually at that.

The exact value of this infinite surd is **1.62**.

Q3) Consider another infinite surd, where the first term is Repeat the entire process above and find the exact value for this surd.

Answer) Basically repeat the same process for the first infinite surd that had 1, instead of 2.

So, in this case we take the surd as a sequence of terms, where is;

$$a_1 =$$

so,

$$a_2 =$$

$$a_3 =$$

also,

$$a_2 =$$

$$a_3 =$$

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Squaring on both sides

$$(a_3)^2 = ()^2$$

$$(a_3)^2 = 2 + a_2$$

$$a_2 = (a_3)^2 - 2$$

$$a_n = (a_{n+1})^2 - 2$$

Therefore,

$$a_{n+1} =$$

Again, we have to find the terms' exact values.

So,

➤ <u>n</u>	<u>Term (<math>a_n</math>)</u>	<u><math>a_n - a_{n-1}</math></u>
➤ 1	1.847759	1.847759
➤ 2	1.961571	0.113811
➤ 3	1.990369	0.028799
➤ 4	1.997591	0.007221
➤ 5	1.999398	0.001807
➤ 6	1.999849	0.000452
➤ 7	1.999962	0.000113
➤ 8	1.999991	$2.82 \times 10^{-5}$
➤ 9	1.999998	$7.06 \times 10^{-6}$
➤ 10	1.999999	$1.76 \times 10^{-6}$

And now a graph of the relation between n and the nth term:

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The difference between each successive term is decreasing, and eventually it will reach zero.

Since 'n' keeps increasing, the value of - keeps decreasing, gradually.

The exact value of this surd is 2.

Q4) Now consider the general infinite surd where the first term is . Find an expression for the exact value of this general infinite surd in terms of 'k'.

Answer)

In the general infinite surd:

$$a_1 =$$

$$a_2 =$$

also,

$$a_2 =$$

$$a_3 =$$

Squaring both sides,

$$(a_3)^2 = ()^2$$

$$(a_3)^2 = a_2 + k$$

$$a_2 = (a_3)^2 - k$$

Therefore,

$$a_{n-1} = (a_n)^2 - k$$

$$\text{so, } k = (a_n)^2 - a_{n-1}$$

Q5) Find some value of 'k' that makes the expression an integer. Find the general statement that represents all the values of 'k' for which the expression is an integer.

➔ I used Apple iWork'09 Numbers to find the values of k that are integers.

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k	a1	a2	a3	A4	A5	A6	A7	A8	A9	a10	a11
1	1.414214	1.553774	1.598053	1.611848	1.616121	1.617443	1.617851	1.617978	1.618017	1.618029	1.618032
2	1.847759	1.961571	1.990369	1.997591	1.999398	1.999849	1.999962	1.999991	1.999998	1.999999	2
3	2.175328	2.274935	2.296723	2.301461	2.30249	2.302714	2.302762	2.302773	2.302775	2.302775	2.302776
4	2.44949	2.539585	2.557261	2.560715	2.561389	2.561521	2.561547	2.561552	2.561553	2.561553	2.561553
5	2.689994	2.773084	2.788025	2.790703	2.791183	2.791269	2.791284	2.791287	2.791288	2.791288	2.791288
6	2.906801	2.984426	2.997403	2.999567	2.999928	2.999988	2.999998	3	3	3	3
7	3.105761	3.178956	3.190448	3.192248	3.19253	3.192574	3.192581	3.192582	3.192582	3.192582	3.192582
8	3.290658	3.360157	3.370483	3.372015	3.372242	3.372275	3.37228	3.372281	3.372281	3.372281	3.372281
9	3.464102	3.530453	3.539838	3.541163	3.541351	3.541377	3.541381	3.541381	3.541381	3.541381	3.541381
10	3.627985	3.69161	3.700218	3.70138	3.701538	3.701559	3.701562	3.701562	3.701562	3.701562	3.701562
11	3.783732	3.844962	3.852916	3.853948	3.854082	3.854099	3.854102	3.854102	3.854102	3.854102	3.854102
12	3.932442	3.991546	3.998943	3.999868	3.999983	3.999998	4	4	4	4	4
13	4.074991	4.13219	4.139105	4.13994	4.140041	4.140053	4.140055	4.140055	4.140055	4.140055	4.140055
14	4.212085	4.267562	4.274057	4.274817	4.274905	4.274916	4.274917	4.274917	4.274917	4.274917	4.274917
15	4.344305	4.398216	4.404341	4.405036	4.405115	4.405124	4.405125	4.405125	4.405125	4.405125	4.405125
16	4.472136	4.524614	4.53041	4.53105	4.53112	4.531128	4.531129	4.531129	4.531129	4.531129	4.531129
17	4.595988	4.647148	4.65265	4.653241	4.653304	4.653311	4.653312	4.653312	4.653312	4.653312	4.653312
18	4.71621	4.766153	4.771389	4.771938	4.771995	4.772001	4.772002	4.772002	4.772002	4.772002	4.772002
19	4.833104	4.881916	4.886913	4.887424	4.887476	4.887482	4.887482	4.887482	4.887482	4.887482	4.887482
20	4.946932	4.99469	4.999469	4.999947	4.999995	4.999999	5	5	5	5	5
21	5.057922	5.104696	5.109275	5.109724	5.109767	5.109772	5.109772	5.109772	5.109772	5.109772	5.109772
22	5.166277	5.212128	5.216524	5.216946	5.216986	5.21699	5.216991	5.216991	5.216991	5.216991	5.216991
23	5.272175	5.317159	5.321387	5.321784	5.321822	5.321825	5.321825	5.321825	5.321825	5.321825	5.321825
24	5.375777	5.419943	5.424015	5.424391	5.424425	5.424429	5.424429	5.424429	5.424429	5.424429	5.424429
25	5.477226	5.520618	5.524547	5.524902	5.524935	5.524938	5.524938	5.524938	5.524938	5.524938	5.524938
26	5.576649	5.61931	5.623105	5.623442	5.623472	5.623475	5.623475	5.623475	5.623475	5.623475	5.623475
27	5.674165	5.716132	5.719802	5.720123	5.720151	5.720153	5.720153	5.720153	5.720153	5.720153	5.720153
28	5.769879	5.811186	5.814739	5.815044	5.81507	5.815073	5.815073	5.815073	5.815073	5.815073	5.815073
29	5.863886	5.904565	5.908009	5.9083	5.908325	5.908327	5.908327	5.908327	5.908327	5.908327	5.908327
30	5.956276	5.996355	5.999696	5.999975	5.999998	6	6	6	6	6	6

I used the formula;

$$a_1 =$$

$$a_2 =$$

$$a_3 =$$

and so on.

Therefore,

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When  $k = 2$ , the equation gets a value as an integer at  $a_{12}$  When  $k = 6$ , the equation gets a value as an integer at  $a_9$

So,

When  $k = 12$ , the equation gets a value as an integer at  $a_8$

When  $k = 20$ , the equation gets a value as an integer at  $a_8$

When  $k = 30$ , the equation gets a value as an integer at  $a_7$

The values of 'k' that make the expression an integer are:

2, 6, 12, 20, 30...

Based on all this, I got to my general statement, and it is;

$$a_n = 2(a_{(n-1)} + 1) - a_{n-2}$$

Q6) Test the validity of your statement by using different values of 'k'.

Answer)

$$a_5 = 2(a_{(5-1)} + 1) - a_{(5-2)}$$

$$= 2(a_4 + 1) - a_3$$

$$= 2(20 + 1) - 12$$

$$= 42 - 12$$

$$= 30$$

$$a_4 = 2(a_{(4-1)} + 1) - a_{(4-2)}$$

$$= 2(a_3 + 1) - a_2$$

$$= 2(12 + 1) - 6$$

$$= 26 - 6$$

$$= 20$$

So, the general statement that we concluded is completely valid.

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Q7) Discuss the scope and/or limitation of your general statement.

Answer) To know any of these terms, we need to know about its 2 preceding terms, and because of this, we may not be able to find the first 2 terms. They have to be given, as there are no terms that precede the first 2 terms. I only found values up to the 5<sup>th</sup> term; therefore I drew this conclusion from my analysis of the behaviour of those terms.

Q8) Explain how you arrived at your general statement.

Answer)

As is said above, the successive term can be obtained by the relation;

$$a_n = 2(a_{n-1} + 1) - a_{n-2}$$

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