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Mathematics HL - Portfolio
IB – Year 12

Introduction

Different values measured can be represented on a scatter graph. To form a relation between the points that describes the value of each and every point may be a difficult task. In order to do this we take the Line or Curve Of Best Fit, that be st describes the points. However, this line or curve of best fit may be a Linear Function, Polynomial Function To The nth Degree or an Exponential Function. In order to see which best describes the plotted points, we must see the r^2 value of the lines or curves. The closer the r2 value is to the digit "1", the more accurate the line or curve of best fit is.

In this portfolio, we are going to investigate the most accurate line or curve of best fit, that best describes the plotted points, which is the Flow Ra te against Time of the Nolichucky River in Tennessee, between the 27th of October 2007, and 2nd November 2007.

Rivers carry a large Volume of water, at a particular Flow Rate at different moments of Time every day. All these details can be measured at a We ather Station. The Volume of the water can be measured in Cubic Feet, and the Time in Hours. This gives us the Flow Rate in Cubic Feet Per Hour, however, a more standard unit has been derived which calculates the time in seconds, giving us the Flow Rate in Cubic Feet Per Second (cfs).
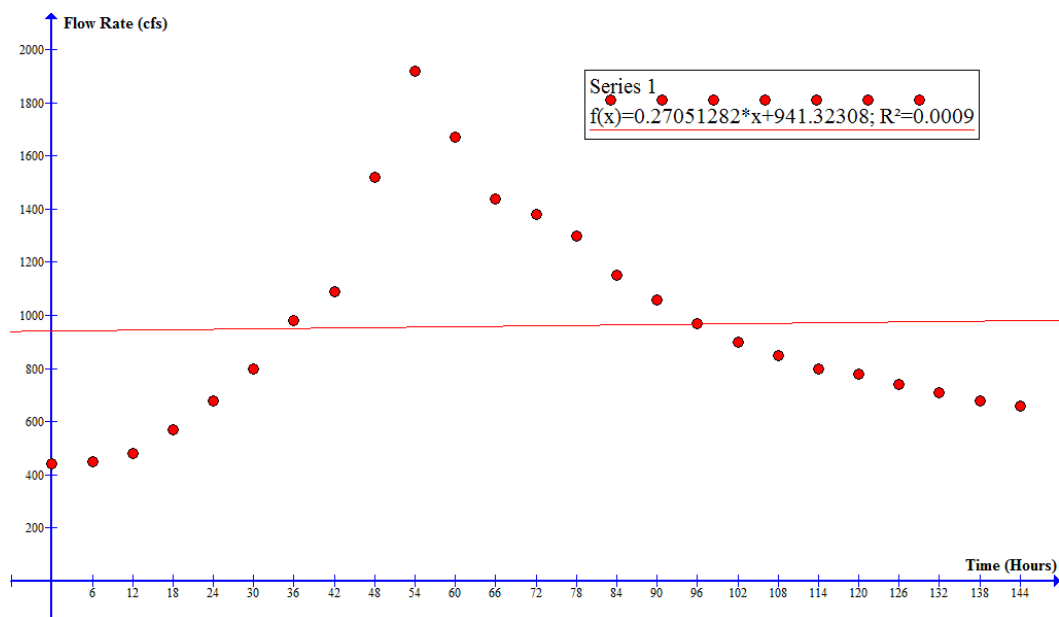
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Flow Rate of Nolichucky River between 27th October 2002 and 2nd November 2002.

Time	0	6	12	18	24	30	36	42	48	54	60	66	72
Flow Rate (cfs)	440	450	480	570	680	800	980	1090	1520	1920	1670	1440	1380

| Time	78	84	90	96	102	108	114	120	126	132	138	144
Flow Rate (cfs)	1300	1150	1060	970	900	850	800	780	740	710	680	660

Graph 1

This chart indicates the flow rate of the river against the time.

The software also gives the r^2 value of the graph that indicates the accura cy of the line best fit, in relation to the points that have been plotted. The closer the value to the digit "1", the more accurate it is. Here we get the r^2 value as 0.0009. This means that the line of best fit for this graph is very inaccurate.

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In order to get the most accurate line of best fit, we must try different types of line of best fit, such as Linear, Polynomial to the nth degree, and exponential.

Graph 2(a)
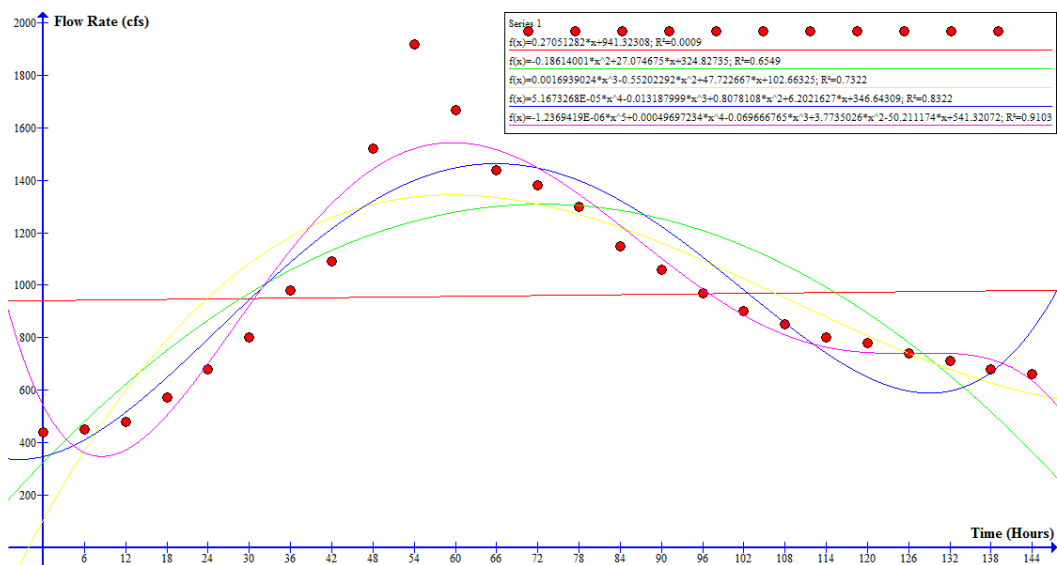
Linear Function → r2 = 0.0009

Polynomial Function To The nth Degree

$n = 2 \rightarrow r^2 = 0.6549$
$n = 3 \rightarrow r^2 = 0.7322$
$n = 4 \rightarrow r^2 = 0.8322$
$n = 5 \rightarrow r^2 = 0.9103$

We can see that as the value of "n" increases, the r^2 value approaches the digit "1". We can then hypothesize that as we increase our nth degree, we will get a more accurate line or curve of best fit. In order to prove this, we consider more examples of increasing "n" values in a Polynomial Function to the nth degree for the line or curve of best fit.
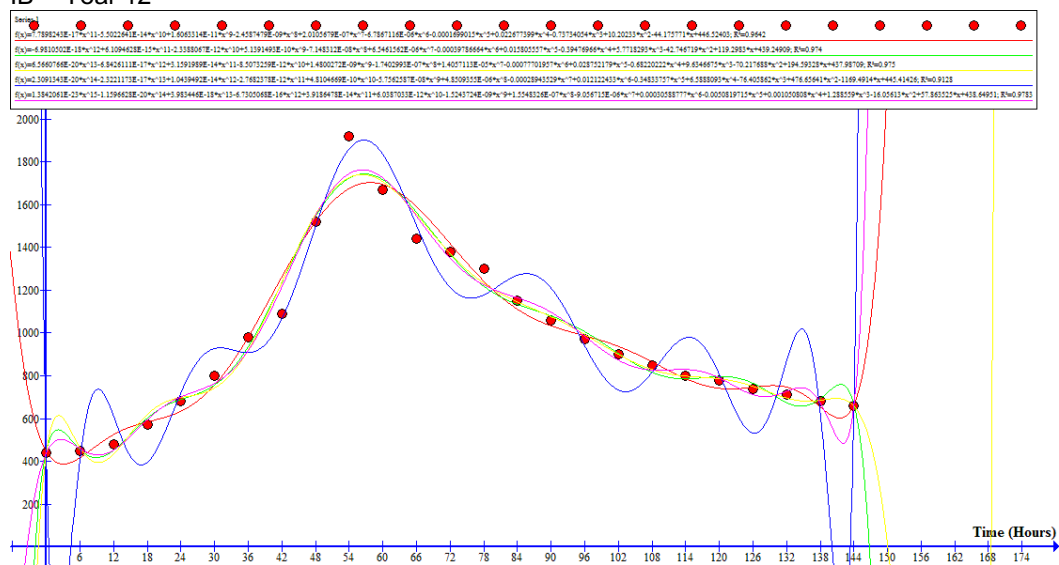
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Graph 2(b)
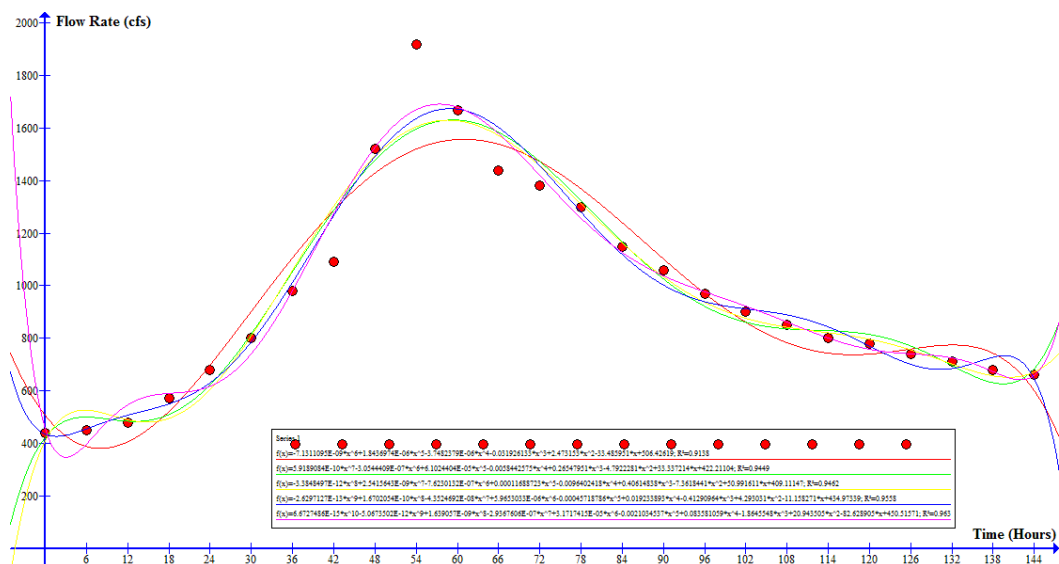
$n = 6 \rightarrow r^2 = 0.9135$
$n = 7 \rightarrow r^2 = 0.9449$
$n = 8 \rightarrow r^2 = 0.9452$
$n = 9 \rightarrow r^2 = 0.9558$
$n = 10 \rightarrow r^2 = 0.963$

Graph 2(c)

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$n = 11 \rightarrow r^2 = 0.9642$
$n = 12 \rightarrow r^2 = 0.9740$
$n = 13 \rightarrow r^2 = 0.9750$
$n = 14 \rightarrow r^2 = 0.9128$
$n = 15 \rightarrow r^2 = 0.9783$

Graph 2(d)

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Series 1

f(x)=-1.5474205E-26*x^16+2.7351001E-23*x^15-1.6409549E-20*x^14+4.795645E-18*x^13-6.9995489E-16*x^12+2.2565418E-14*x^11+9.9390967E

f(x)=-2.2482048E-27*x^17+1.9297068E-24*x^16-6.5513362E-22*x^15+9.6952713E-20*x^14+1.5653331E-19*x^13-2.2762059E-15*x^12+3.2253376

f(x)=-7.0943411E-29*x^18+5.4066831E-26*x^17-1.5018656E-23*x^16+1.2230365E-21*x^15+2.5336989E-19*x^14-6.4204401E-17*x^13+2.5139858

f(x)=-6.8771576E-32*x^19+4.0072318E-29*x^18-6.5223528E-27*x^17-2.832909E-25*x^16+8.7027584E-23*x^15+3.1322451E-20*x^14-7.3607672E

f(x)=5.2852835E-34*x^20-4.2326531E-31*x^19+1.1836463E-28*x^18-9.058851E-27*x^17-1.3180199E-24*x^16+3.9329675E-24*x^15+9.3693589E-

$n = 16 \rightarrow r^2 = 0.9785$
$n = 17 \rightarrow r^2 = 0.9825$
$n = 18 \rightarrow r^2 = 0.973$
$n = 19 \rightarrow r^2 = 0.9835$
$n = 20 \rightarrow r^2 = 0.9846$

Graph 2(e)

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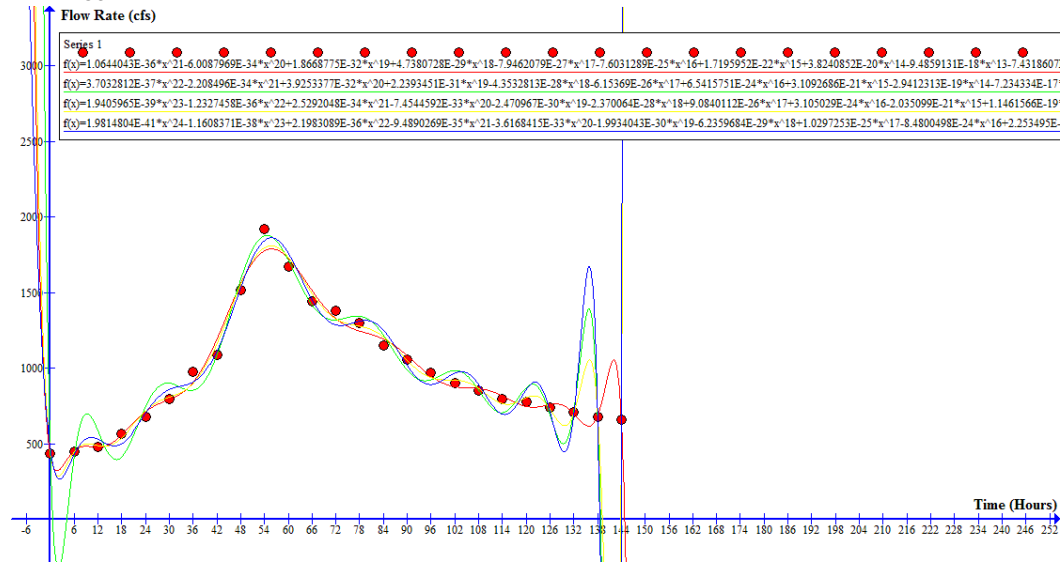
n = 21 → r^2 = 0.9841
n = 22 → r^2 = 0.9671
n = 23 → r^2 = 0.9871
n = 24 → r^2 = 0.9775
n = 25 → r^2 = No Result due to Software Limitation – Not Enough Points For Calculation

Graph 2(f)

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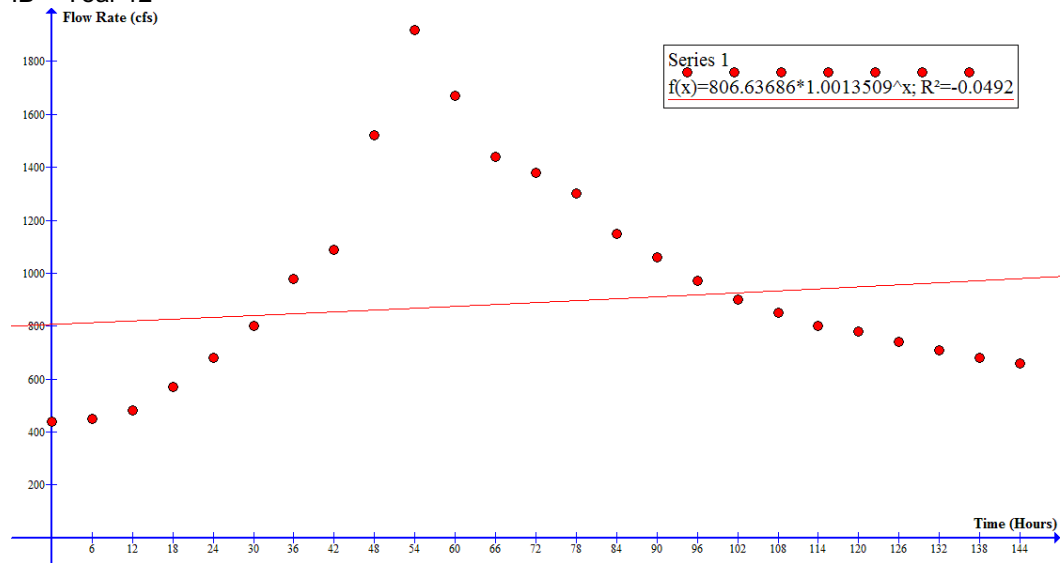
Exponential → r2 = 0.0492

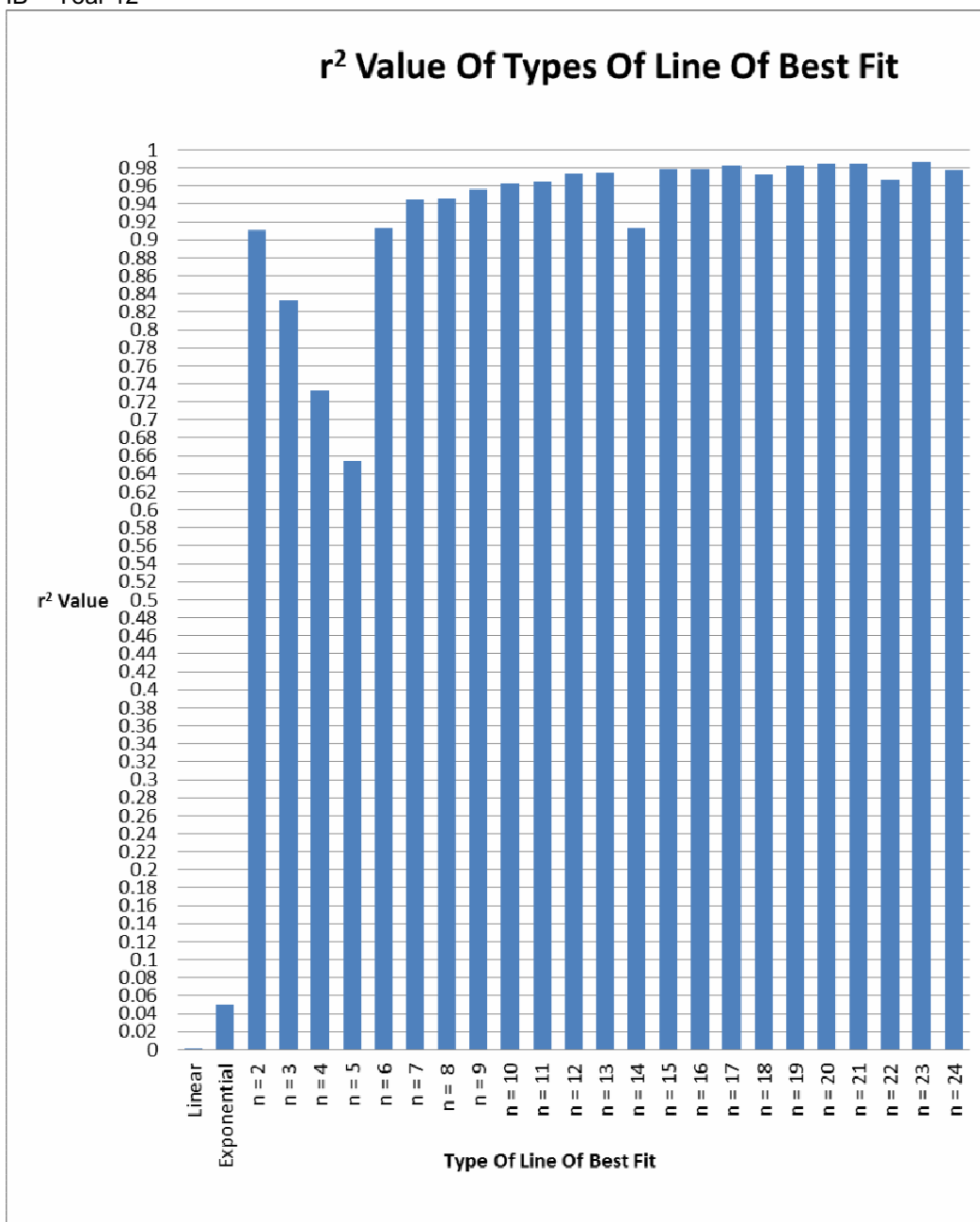| Type Of Line Of Best Fit	r^2 Value
Linear	0.0009

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Exponential	0.0492
Polynomial To The nth Degree:	
n = 2	0.9103
n = 3	0.8322
n = 4	0.7322
n = 5	0.6549
n = 6	0.9135
n = 7	0.9449
n = 8	0.9452
n = 9	0.9558
n = 10	0.9630
n = 11	0.9642
n = 12	0.9740
n = 13	0.9750
n = 14	0.9128
n = 15	0.9783
n = 16	0.9785
n = 17	0.9825
n = 18	0.9730
n = 19	0.9835
n = 20	0.9846
n = 21	0.9841
n = 22	0.9671
n = 23	0.9871
n = 24	0.9775

Graph 3

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From Graph 3 we can see that we get the most accurate line of best fit in a Polynomial Function to the 23rd degree, as our r^2 value is the highest. This means that the function of the plotted points is best described as:

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$f(x) = (1.9405965E-39)x^{23} - (1.2327458E-36)x^{22} + (2.5292048E-34)x^{21} - (7.4544592E-33)x^{20} - (2.470967E-30)x^{19} - (2.370064E-28)x^{18} + (9.0840112E-26)x^{17} + (3.105029E-24)x^{16} - (2.035099E-21)x^{15} + (1.1461566E-19)x^{14} - (1.7255951E-18)x^{13} + (3.2613424E-15)x^{12} - (7.9428277E-13)x^{11} + (9.7793277E-11)x^{10} - (8.7976202E-09)x^9 + (6.6934434E-07)x^8 - (4.1506499E-05)x^7 + (0.001891044)x^6 - (0.058446511)x^5 + (1.1527414)x^4 - (13.484413)x^3 + (82.955637)x^2 - (196.95227)x + 439.95795$

Therefore, we can derive the rate of change of this data, by taking the derivative of the above function.

$$\frac{d\, f(x)}{d(x)} = f'(x)$$

$f'(x) = (4.463372E-380)x^{22} - (2.7120407E-35)x^{21} + (5.3113301E-33)x^{20} - (1.4908918E-31)x^{19} - (4.6948373E-29)x^{18} - (4.2661153E-27)x^{17} + (1.5442819E-24)x^{16} + (4.9680464E-23)x^{15} - (3.0526485E-20)x^{14} + (1.6046193E-18)x^{13} - (2.2432736E-17)x^{12} + (3.9136109E-14)x^{11} - (8.7371105E-12)x^{10} + (9.7793277E-10)x^9 - (7.9178582E-08)x^8 + (5.3547547E-06)x^7 - (0.00029054549)x^6 + (0.011346264)x^5 - (0.29223255)x^4 + (4.6109657)x^3 - (40.453239)x^2 + (165.91127)x - 196.95227$

x	f(x)	f'(x)
0	439.9579	-196.9523
6	449.1375	35.4822
12	488.2414	-5.1602
18	543.3245	24.8299
24	714.6382	24.9330
30	809.0785	9.2756
36	897.8598	26.7526
42	1166.7390	61.1598
48	1558.5261	60.6303
54	1798.3029	14.3179
60	1728.8272	-32.8806
66	1490.6678	-38.7835
72	1324.8572	-15.2345
78	1277.4382	-5.5528
84	1205.0280	-20.5891
90	1049.4515	-26.8026
96	936.8251	-8.6036

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102	922.5893	-1.4237
108	865.0882	-18.2611
114	765.8757	-6.7330
120	806.9793	11.0478
126	727.5779	-39.6695
132	713.5226	80.3178
138	679.4246	-405.2977
144	660.0418	4613.0808
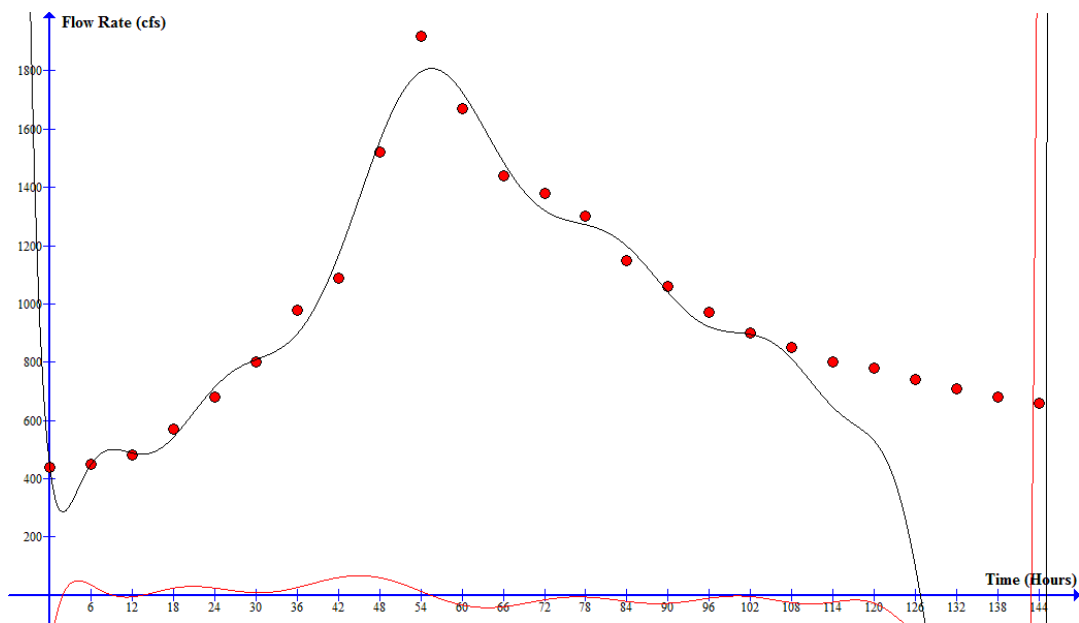
We can also find the rate of change between 2 points by using the formula $\frac{y_2 - y_1}{x_2 - x_1}$

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Range Of "x" Values	Rate Of Change
0 ≤ x ≥ 6	1.67
6 ≤ x ≥ 12	5.00
12 ≤ x ≥ 18	15.00
18 ≤ x ≥ 24	18.33
24 ≤ x ≥ 30	20.00
30 ≤ x ≥ 36	30.00
36 ≤ x ≥ 42	18.33
42 ≤ x ≥ 48	71.67
48 ≤ x ≥ 54	66.67
54 ≤ x ≥ 60	-41.67
60 ≤ x ≥ 66	-38.33
66 ≤ x ≥ 72	-10.00
72 ≤ x ≥ 78	-13.33
78 ≤ x ≥ 84	-25.00
84 ≤ x ≥ 90	-15.00
90 ≤ x ≥ 96	-15.00
96 ≤ x ≥ 102	-11.67
102 ≤ x ≥ 108	-8.33
108 ≤ x ≥ 114	-8.33
114 ≤ x ≥ 120	-3.33
120 ≤ x ≥ 126	-6.67
126 ≤ x ≥ 132	-5.00
132 ≤ x ≥ 138	-5.00
138 ≤ x ≥ 6	-3.33

Graph 4

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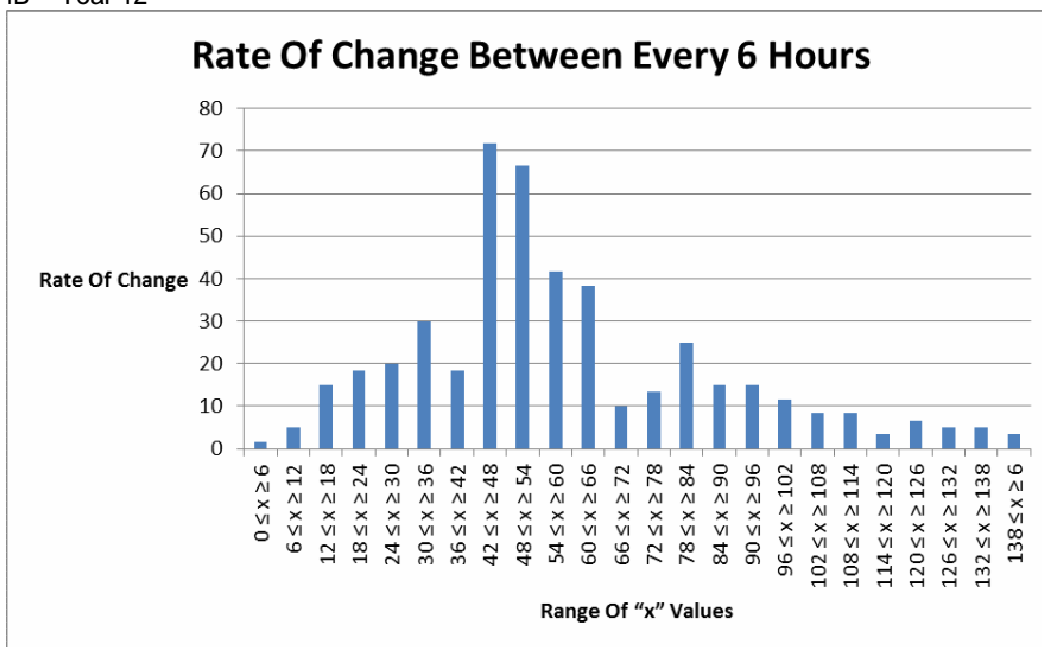
From Graph 4, we can see that between the time when x = 42 hours and x = 48 hours, the rate of change of the rate of flow of the river is the greatest.

The rate of flow when 42 ≤ x ≥ 48 =

y2 – y1
 x2 – x1

= 1520 – 1090
    48 – 42

= 71.67

From the data in Graph 1, we see that the rate of flow of the river increases exponentially, and then decreases exponentially. A weather pattern that could represent this kind of data would be of a Scanty Rainy Season, where there is an input of water in the form of rain to the river, increasing the rate of flow for a particular time period.

Between 28th October 2007 and 29th October 2007, a total volume of water flowed past the measuring station. In order to find this total volume, we can find the area under the graph, between the time 00:00 on the 28th and 00:00 on the 29th.

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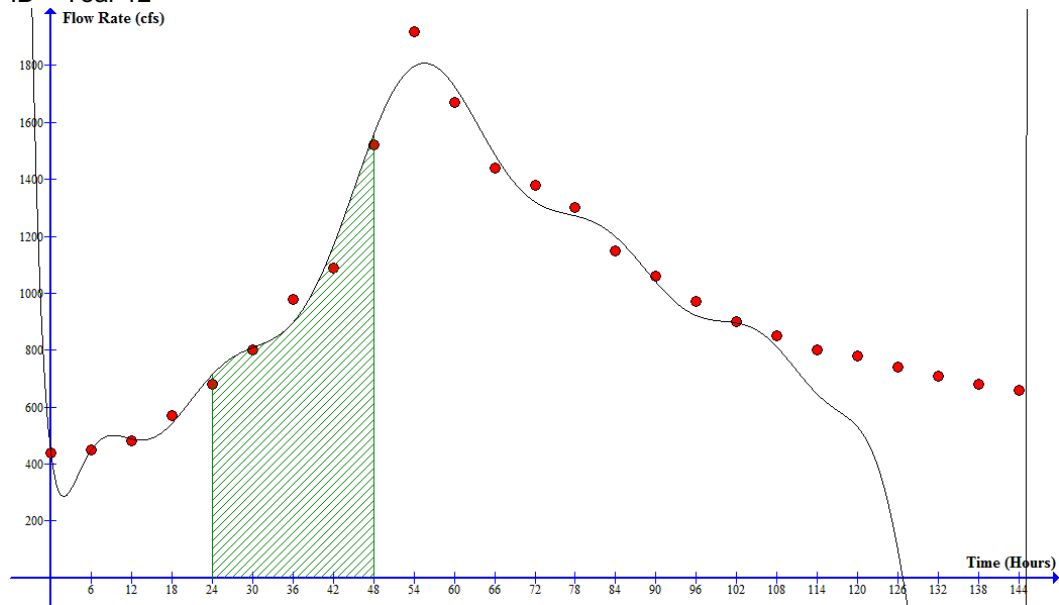
Area Under The Graph Between The Time 00:00 on the 28th (x = 24 hours), and 00:00 on the 29th (x = 48 hours) = 2.3947E+4 cubic feet (Calculated using Area Under Graph option on program "Graph" which uses the Simpson's Formula, i.e:

$$\int_a^b f(x)\, dx \approx \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$

The time when the flow rate is 1000cfs =

T = 38.8823 hours, T = 91.4791 hours

(Correct to 4 d.p.)

The amount of flowing water began to increase on the 27th of October, 2007, and kept increasing on each day until 6 am on the 29th of October, 2007.

The average Flow Rate of the River can be calculated by taking $\frac{\sum y}{n}$

Where y is the values of the rate of flow every 6 hours, and "n" is the number of terms being added.

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$\sum y =$
440+450+480+570+680+800+980+1090+1520+1920+1670+1440+1380+1300+1150+1060+970+900+850+800+780+740+710+680+660

= 24020

n = 25

Therefore Average Flow Rate = 960.8 cfs

Another way to find the Average Flow Rate is to take the Total Volume(Area Under The Graph) and divide it by the Total Time.

Total Volume = 7.6858E+4

Total Time = 144 Hours

Therefore Average Flow Rate = $\dfrac{7.6858E+4}{144}$

= 533.7361 cfs

Difference between the two Average Flow Rates = 427.0639 cfs

Let The average Flow Rate = 960.8

Therefore, T when Flow Rate = 960.8 =

T = 37.9385 Hours, T = 93.2561 Hours

Conclusion

The difference between the Average Flow Rate derived by the formula $\dfrac{\sum y}{n}$

and by dividing the area under the graph by the total time = 427.0639 cfs.

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This shows us the inaccuracy of the functio n of the line of best fit, which best describes the points plotted, even though the r^2 value is = 0.9871, which was the most close to the digit "1".

From this I can conclude, that in order to calculate the right values for the Average Flow Rate or the Total Volume, we must have the exact line of best fit, where the r^2 = 1. Even the smallest deviation, in this case the residual value that is $1 - r^2$ value = 1 – 0.9871 = 0.0129, should = 0 to give us the best results.