

## Mathematics Portfolio

### Type II

## **Creating a Logistic Model**

### **Description**

A geometric population model takes the form  $u_{n+1} = r \cdot u_n$  where  $r$  is the growth factor and  $u_n$  is the population at year  $n$ . For example, if the population were to increase annually by 20%, the growth factor is  $r = 1.2$ , and this would lead to an exponential growth. If  $r = 1$  the population is stable. A logistic model takes a similar form to the geometric, but the growth factor depends on the size of the population and is variable. The growth factor is often estimated as a linear function by taking estimates of the projected initial growth rate and the eventual population.

### **Part 1**

Information which has been given in *Part 1*: -

- a) 10,000 fishes are introduced into a lake
- b) The population increase if 10,000 fishes are introduced into the lake would be by 50% in the first year.
- c) The long term sustainable limit in this case would be 60,000

It has been given that the geometric population growth model takes the form  $u_{n+1} = r \cdot u_n$ . Now, if we have to find out the ordered pair  $(u_0, r_0)$ : -

- It is mentioned in the description that  $u_n$  is the population at year  $n$ . Therefore,  $u_0$  will be the population at year 0, or the initial population of the lake which is 10,000.

Therefore,  $u_0 = 10,000$  ----- (1)

- $r$  is the growth factor as mentioned in the description. As the population would increase by 50% in the first year, so  $r_0 = 1 + 1 \times \frac{50}{100}$   
 $= 1 + 0.5$   
 $= 1.5$  ----- (2)

So the first ordered pair  $(u_0, r_0)$  as shown in (1) and (2) would be  $(10000, 1.5)$ .

Now, if we have to find out the ordered pair  $(u_n, r_n)$ : -

- It is given in the question that  $u_n = 60,000$ .

Therefore,  $u_n = 60,000$  ----- (3)

- $r$  is the growth factor as mentioned in the description. As found out in (3) that  $u_n = 60,000$  which shows that the population has reached its long term sustainable limit where population is stable.

Therefore,  $r_n = 1$  ----- (4)

So the second ordered pair  $(u_n, r_n)$  as shown in (3) and (4) would be  $(60000, 1)$

A general linear equation has the form  $(y - y_1) = m(x - x_1)$

where  $y$  and  $x$  are variables,  $y_1$  and  $x_1$  are the coordinates of a point on the curve and  $m$  is the slope of the curve. To find this equation we shall first find the slope ' $m$ ': -

The slope of a curve is given by =  $\frac{\text{Rise}}{\text{Run}}$

That is, the slope =  $\frac{Y_2 - Y_1}{X_2 - X_1}$

As we have found two ordered pairs  $(u_0, r_0)$  and  $(u_n, r_n)$  where: -

$$u_0 = X_1 = 10,000$$

$$r_0 = Y_1 = 1.5$$

$$u_n = X_2 = 60,000$$

$$r_n = Y_2 = 1$$

$$\text{Substituting these values in the formula for slope we get } \frac{Y_2 - Y_1}{X_2 - X_1} = \frac{1 - 1.5}{60000 - 10000} = \frac{0.5}{50000} = -1 \times 10^{-5}$$

Also, we have one pair of coordinates, that is,  $(u_0, r_0) = (10000, 1.5)$  which is equal to  $(x_1, y_1)$ .

Substituting all these values in the general linear equation, we get: -

$$(y - 1.5) = -1 \times 10^{-5}(x - 10000)$$

$$y - 1.5 = -1 \times 10^{-5}x + 1 \times 10^{-1}$$

$$y = -1 \times 10^{-5}x + 1.6$$

Here, the variable  $y$  can be replaced by  $r_n$  which represents the growth factor at some year  $n$  and the variable  $x$  can be replaced by  $u_n$  which represents the population at some year  $n$ . Therefore, the equation for the linear growth factor in terms of  $u_n$  will be: -

$$r_n = (-1 \times 10^{-5})u_n + 1.6$$

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	List 1	List 2	List 3	List 4
1	10000	1.5		
2	60000	1		
3				
4				
5				

10000

GRAPH CALC TEST INTR DISTR

Year	Population (millions)
1990	10.5
2000	11.5
2010	12.5

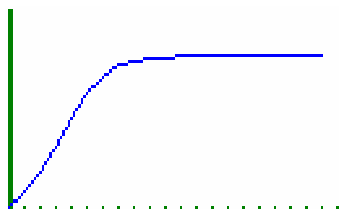
COPY DRAW

3

Thus, the logistic function can be modeled by the following recursive function: -

$$u_{n+1} = (-1 \times 10^{-5}) u_n^2 + 1.6u_n$$

The graph obtained is as follows:-



Window settings for the graph: -

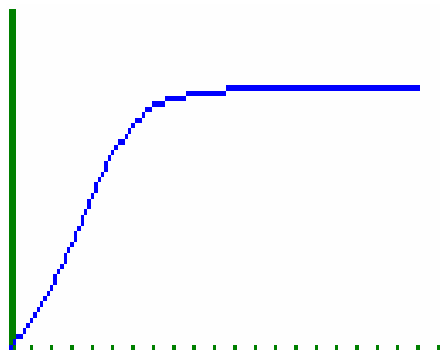
```
View Window
Xmin :0
max :22
scale:2
Ymin :0
max :63000
scale:10000
INIT TRIG STD STO RCL
```

### Part 3

Entering the recursive logistic function found in *Part 2* in the RECUR mode of the *GDC Casio CFX-9850GC PLUS*, we can find the values of the population over the next 20 years. This can be done by setting the range from 0 to 20. Thus, the following table is obtained: -

<i>n+1</i>	<i>u<sub>n+1</sub></i>	<i>n+1</i>	<i>u<sub>n+1</sub></i>
0	10000	10	59772
1	15000	11	59908
2	21750	12	59963
3	30069	13	59985
4	39069	14	59994
5	47246	15	59997
6	53272	16	59999
7	56856	17	59999
8	58643	18	59999
9	59439	19	59999
		20	59999

The following line graph is obtained using the *GDC Casio CFX-9850GC Plus*, which is as follows: -



```
View Window
Xmin :0
max :22
scale:2
Ymin :0
max :63000
scale:10000
INIT TRIG STD STO RCL
```

Window settings for the graph

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## Part 4

As speculated by the biologist, if the initial growth rates vary considerably, then the ordered pair  $(u_0, r_0)$  will change accordingly.

- when initial growth rate  $r_0 = 2$

The ordered pair  $(u_0, r_0)$  for the first pair when the predicted growth rate ' $r$ ' for the year is 2 will be  $(10000, 2)$ .

$(u_n, r_n) = (60000, 1)$  according to the description.

We can get the equation of the linear growth factor by entering these sets of ordered pairs in the STAT mode of the *GDC Casio CFX-9850GC PLUS*. The STAT mode looks as follows: -

List 1	List 2	List 3	List 4
1	10000	2	
2	60000	1	
3			
4			
5			

1

GRAPH CALC TEST INTR DIST

Thus, we obtain a linear graph which can be modeled by the following function: -



Therefore,

$$y = (-2 \times 10^{-5})x + 2.2$$

Here, the variable  $y$  can be replaced by  $r_n$  which represents the growth factor at some year  $n$  and the variable  $x$  can be replaced by  $u_n$  which represents the population at some year  $n$ . Therefore, the equation for the linear growth factor in terms of  $u_n$  will be: -

$$r_n = (-2 \times 10^{-5})u_n + 2.2$$

Now, to find the recursive logistic function, we shall substitute the value of  $r_n$  calculated above in the following form  $u_{n+1} = r_n \cdot u_n$

$$\begin{aligned} u_{n+1} &= [(-2 \times 10^{-5})u_n + 2.2]u_n \\ &= (-2 \times 10^{-5})u_n^2 + 2.2u_n \end{aligned}$$

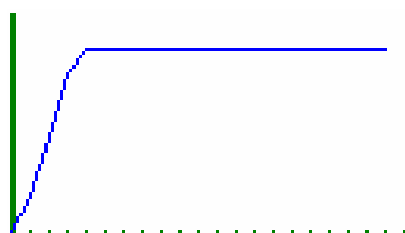
To find out the changes in the pattern, let us see the following table which shows the first 20 values of the population obtained according to the logistic function  $u_{n+1} = (-2 \times 10^{-5})u_n^2 + 2.2u_n$  just calculated: -

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$n+1$	$u_{n+1}$	$n+1$	$u_{n+1}$
0	10000	10	60000
1	20000	11	59999
2	36000	12	60000
3	53280	13	59999
4	60440	14	60000
5	59907	15	59999
6	60018	16	60000
7	59996	17	59999
8	60000	18	60000
9	59999	19	60000
		20	60000

The above tabulated data is represented by the following logistic function: -



```

View Window
Xmin : 0
max : 22
scale : 2
Ymin : 0
max : 62000
scale : 10000
INIT TRIG STD STO RCL

```

Window settings for the graph

Here, from the table and graph we can observe how the population rapidly grows beyond the sustainable limit (which is 60,000) to 60440 within 4 years itself but then comes down below 60000 to 59907 in the successive year. Such an increasing and decreasing trend in the population is observed in the following years although the population seems to come closer to the sustainable limit, i.e 60000 all the time. In the 8<sup>th</sup> year, the population is exactly at its sustainable limit and henceforth, the population stabilizes by only varying slightly until the 18<sup>th</sup> year.

- when the initial growth rate  $r_0 = 2.3$

The ordered pair  $(u_0, r_0)$  for the first pair when the predicted growth rate ' $r$ ' for the year is 2.3 will be  $(10000, 2.3)$ .

$(u_n, r_n) = (60000, 1)$  according to the description.

We can get the equation of the linear growth factor by entering these sets of ordered pairs in the STAT mode of the *GDC Casio CFX-9850GC PLUS*.

The STAT mode looks as follows: -

	List 1	List 2	List 3	List 4
1	10000	2.3		
2	60000	1		
3				
4				
5				

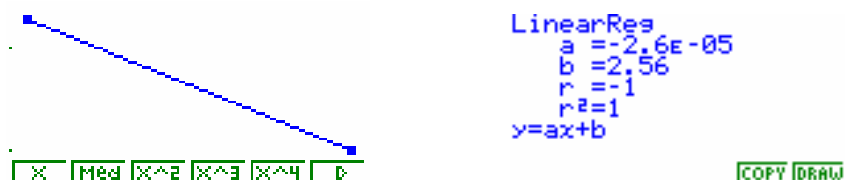
1

GRAPH CALC TEST INTR DIST D

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Thus, we obtain a linear graph which can be modeled by the following function: -



$$y = (-2.6 \times 10^{-5})x + 2.56$$

Here, the variable  $y$  can be replaced by  $r_n$  which represents the growth factor at some year  $n$  and the variable  $x$  can be replaced by  $u_n$  which represents the population at some year  $n$ . Therefore, the equation for the linear growth factor in terms of  $u_n$  will be: -

$$r_n = (-2.6 \times 10^{-5})u_n + 2.56$$

Now, to find the recursive logistic function, we shall substitute the value of  $r_n$  calculated above in the following form  $u_{n+1} = r_n \cdot u_n$

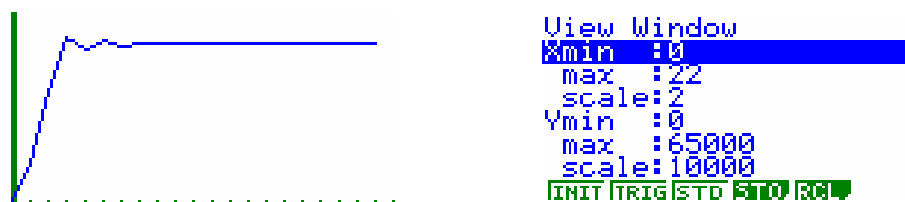
$$u_{n+1} = [(-2.6 \times 10^{-5})u_n + 2.56]u_n$$

$$= (-2.6 \times 10^{-5})u_n^2 + 2.56u_n$$

To find out the changes in the pattern, let us see the following table which shows the first 20 values of the population obtained according to the logistic function  $u_{n+1} = (-2.6 \times 10^{-5})u_n^2 + 2.56u_n$  just calculated: -

$n+1$	$u_{n+1}$	$n+1$	$u_{n+1}$
0	10000	10	59952
1	23000	11	60026
2	45126	12	59985
3	62577	13	60008
4	58384	14	59995
5	60837	15	60002
6	59513	16	59998
7	60266	17	60000
8	59848	18	59999
9	60084	19	60000
		20	59999

The above tabulated data can be represented by the following logistic function graph: -



Window settings for the graph

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- when the initial growth rate,  $r_0 = 2.5$

$(u_n, r_n) = (60000, 1)$  according to the description.

	List 1	List 2	List 3	List 4
1	10000	2.5		
2	60000			
3				
4				
5				

GFH1 GFH2 GFH3 SEL SET

LinearReg  
a = -3.1E-05  
b = 2.8  
r = -1  
r^2 = 1  
y = ax + b

Here, the variable  $y$  can be replaced by  $r_n$  which represents the growth factor at some year  $n$  and the variable  $x$  can be replaced by  $u_n$  which represents the population at some year  $n$ . Therefore, the equation for the linear growth factor in terms of  $u_n$  will be: -

$$r_n = (-3 \times 10^{-5})u_n + 2.8$$

$$\begin{aligned} u_{n+1} &= [(-3 \times 10^{-5})u_n + 2.8]u_n \\ &= (-3 \times 10^{-5})u_n^2 + 2.8u_n \end{aligned}$$



To find out the changes in the pattern, let us see the following table which shows the first 20 values of the population obtained according to the logistic function  $u_{n+1} = (-3 \times 10^{-5})u_n^2 + 2.8u_n$  just calculated: -

$n+1$	$u_{n+1}$	$n+1$	$u_{n+1}$
0	10000	10	58980
1	25000	11	60784
2	51250	12	59354
3	64703	13	60504
4	55573	14	59589
5	62953	15	60323
6	57375	16	59737
7	61892	17	60207
8	58378	18	59382
9	61218	19	60133
		20	59893

The above tabulated data can be represented by the following logistic function graph: -



The table and graph confirm how higher growth rates above 2 lead to less stability in population. Here, the variance from the long term sustainable limit (which is 60000) is much more (64703 in the 3<sup>rd</sup> year) compared to when the initial growth rate was 2.3 or 2. The successive year sees a similar larger fall to 55573 in population compared to the other two cases. Consequently, the table confirms how a high initial growth rate leads to longer time for the population to stabilize at the long term sustainable limit. In this case, it is not attained in the first 20 years. Rather, upon further calculations we can find out that the population becomes stable at the long term sustainable limit in the 41<sup>st</sup> year compared to the 17<sup>th</sup> year when the initial growth rate is 2.3 .

## Part 5

Let us see what happens when the initial growth rate is 2.9

The ordered pair  $(u_0, r_0)$  for the first pair when the predicted growth rate 'r' for the year is 2 will be  $(10000, 2.9)$ .

$(u_n, r_n) = (60000, 1)$  according to the description.

We can get the equation of the linear growth factor by entering these sets of ordered pairs in the STAT mode of the *GDC Casio CFX-9850GC PLUS*.

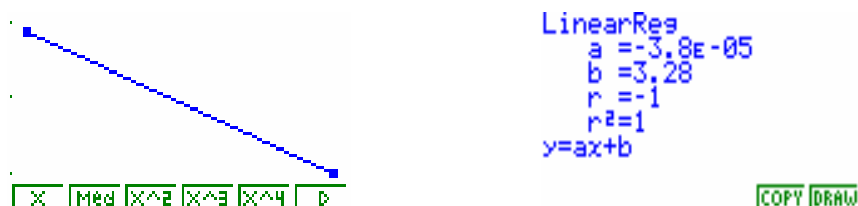
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	List 1	List 2	List 3	List 4
1	10000	2.9		
2	60000			
3				
4				
5				

1

GRAPH CALC TEST INTP DIST D

Thus, we obtain a linear graph which can be modeled by the following function: -



$$y = (-3.8 \times 10^{-5})x + 3.28$$

Here, the variable  $y$  can be replaced by  $r_n$  which represents the growth factor at some year  $n$  and the variable  $x$  can be replaced by  $u_n$  which represents the population at some year  $n$ . Therefore, the equation for the linear growth factor in terms of  $u_n$  will be: -

$$r_n = (-3.8 \times 10^{-5})u_n + 3.28$$

Now, to find the recursive logistic function, we shall substitute the value of  $r_n$  calculated above in the following form  $u_{n+1} = r_n \cdot u_n$

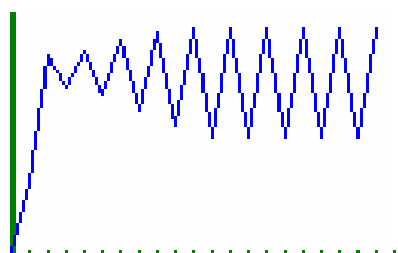
$$u_{n+1} = [(-3.8 \times 10^{-5})u_n + 3.28]u_n$$

$$= (-3.8 \times 10^{-5})u_n^2 + 3.28u_n$$

To find out the changes in the pattern, let us see the following table which shows the first 20 values of the population obtained according to the logistic function  $u_{n+1} = (-3 \times 10^{-5})u_n^2 + 2.8u_n$  just calculated: -

$n+1$	$u_{n+1}$	$n+1$	$u_{n+1}$
0	10000	10	70738
1	29000	11	41872
2	63162	12	70716
3	55572	13	41919
4	64922	14	70720
5	52779	15	41910
6	67261	16	70719
7	48701	17	41911
8	69611	18	70719
9	44187	19	41911
		20	70719

The above tabulated data can be represented by the following logistic function graph: -



```
View Window
Xmin :0
max :22
scale:2
Ymin :0
max :75000
scale:10000
INIT TRIG STD STO RCL
```

Window settings for the graph

We can infer from the data in the table and the graph how a high initial growth rate of 2.9 can lead to fluctuating populations every year. The long term sustainable limit of 60,000 is crossed at the end of the 2<sup>nd</sup> year itself, becoming 63162. But, in such a case, the environment (*in this case the lake*) cannot withhold such a population and thus there are a lot of fishes either dying out or migrating in the subsequent year which reduces the population to 55572. Yet, again in the next year, due to high growth rate, the population reaches well above even 63162 to 64922. Such a fluctuating trend is observed in the succeeding years until the end of the 16<sup>th</sup> and 17<sup>th</sup> year during and after which the population fluctuates between 70719 and 41911.

## Part 6

To initiate an annual harvest of 5000 fishes, let us first find out at what point the fish population stabilizes when the initial growth rate,  $r = 1.5$ .

The following table shows the growth in the population over the first 20 years taking  $r = 1.5$  and thus making use of the following recursive relation as found out in *Part 2*:-

$$u_{n+1} = (-1 \times 10^{-5}) u_n^2 + 1.6u_n$$

In this relation, the starting population is the initial population  $u_0 = 10000$ .

$n+1$	$u_{n+1}$	$n+1$	$u_{n+1}$
0	10000	10	59772
1	15000	11	59908
2	21750	12	59963
3	30069	13	59985
4	39069	14	59994
5	47246	15	59997
6	53272	16	59999
7	56856	17	59999
8	58643	18	59999
9	59439	19	59999
		20	59999

Here, we see that the population reaches a stable point, when  $r = 1$  at the end of the 16<sup>th</sup> year. According to the question, we shall initiate the harvest of 5000 fishes every year from this point onwards. The following changes are made in the original recursive relation:-

1. The starting population is taken to be 59999
2. In the relation  $u_{n+1} = (-1 \times 10^{-5}) u_n^2 + 1.6u_n$ , we subtract 5000 as this is the number which is removed every year. Thus, the new relation becomes  

$$u_{n+1} = (-1 \times 10^{-5}) u_n^2 + 1.6u_n - 5000$$

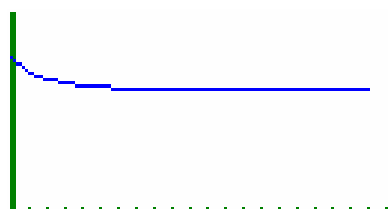
It should be noted that although the start population or  $u_0$  is taken to be 59999, the starting year is the 16<sup>th</sup> year.

Entering these values in the RECUR mode of the GDC Casio *CFX 9850GC Plus*, we get the following table:-

$n+1$	$u_{n+1}$	$n+1$	$u_{n+1}$
16	59999	26	50041
17	54999	27	50024
18	52749	28	50014
19	51574	29	50008
20	50919	30	50005
21	50543	31	50003
22	50323	32	50001
23	50192	33	50001
24	50115	34	50000
25	50069	35	50000

We see that there is a declining trend in the fish population with an annual harvest of 5000 fishes when the growth rate is 1.5. However, it should be noted that with this annual harvest, the population finally stabilizes in the 34<sup>th</sup> year when it becomes 50000.

The graph is as follows: -



```
View Window
Xmin : 0
max : 22
scale : 2
Ymin : 0
max : 70000
scale : 10000
INIT TRIG STD STO RCL
```

Window settings for the graph

Thus, we can conclude that it is feasible to initiate an annual harvest of 5000 fishes once the stable population of 59999 is reached in the 16<sup>th</sup> year with the growth rate being 1.5. Initially, a declining trend in the fish population is observed as 5000 fishes are harvested every year. Yet, from the 34<sup>th</sup> year onwards, we observe that the new stable fish population becomes 50,000 with an annual feasible harvest of 5000 fishes.

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## Part 7

Taking the same model in which the growth rate  $r = 1.5$ , we shall investigate other harvest sizes.

$n+1$	$u_{n+1}$	$n+1$	$u_{n+1}$
0	10000	10	59772
1	15000	11	59908
2	21750	12	59963
3	30069	13	59985
4	39069	14	59994
5	47246	15	59997
6	53272	16	59999
7	56856	17	59999
8	58643	18	59999
9	59439	19	59999
		20	59999

- When the harvest size = 2500 fishes

We shall initiate the harvest of 2500 fishes after the 16<sup>th</sup> year when the population stabilizes at 59999. The following changes are made in the original recursive relation:-

1. The starting population is taken to be 59999
2. In the relation  $u_{n+1} = (-1 \times 10^{-5}) u_n^2 + 1.6u_n$ , we subtract 2500 as this is the number which is removed every year. Thus, the new relation becomes  

$$u_{n+1} = (-1 \times 10^{-5}) u_n^2 + 1.6u_n - 2500$$

Entering these values in the RECUR mode of the GDC Casio *CFX 9850GC Plus*, we get the following table:-

Table Range n+1  
 Start:0  
 End :20  
 a0 :59999  
 b0 :0  
 anStr:0  
 bnStr:0  
 |a0|a1

$n+1$	$u_{n+1}$	$n+1$	$u_{n+1}$
16	59999	26	55498
17	57499	27	55496
18	56437	28	55495
19	55948	29	55495
20	55715	30	55495
21	55602	31	55495
22	55547	32	55495
23	55520	33	55495
24	55507	34	55495
25	55501	35	55495

The graph is as follows: -



We see that when the annual harvest is 2500, there is a declining trend which stabilizes quite early, that is from the 28<sup>th</sup> year onwards compared to when the harvest is 5000 fishes. Also, the new stable population, 55495 is higher compared to when the harvest if 5000 fishes. Thus, we can conclude that it is also feasible to initiate an annual harvest of 2500 fishes and the new stable population would be achieved from the 28<sup>th</sup> year onwards as 55495.

- When the harvest size = 7500

We shall initiate the harvest of 7500 fishes after the 16<sup>th</sup> year when the population stabilizes at 59999. The following changes are made in the original recursive relation:-

1. The starting population is taken to be 59999
2. In the relation  $u_{n+1} = (-1 \times 10^{-5}) u_n^2 + 1.6u_n$ , we subtract 7500 as this is the number which is removed every year. Thus, the new relation becomes  $u_{n+1} = (-1 \times 10^{-5}) u_n^2 + 1.6u_n - 7500$

The table settings are as follows: -

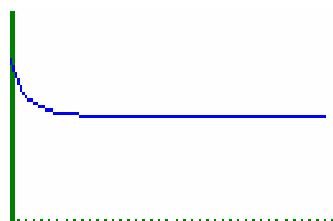
```
Table Range n+1
Start:0
End :40
a0 :59999
b0 :0
anStr:0
bnStr:0
ao | ai
```

Entering these values in the RECUR mode of the GDC Casio CFX 9850GC Plus, we get the following table:-

$n+1$	$u_{n+1}$	$n+1$	$u_{n+1}$	$n+1$	$u_{n+1}$	$n+1$	$u_{n+1}$
16	59999	26	42774	36	42278	46	42249
17	52499	27	42642	37	42270	47	42248
18	48937	28	42544	38	42265	48	42248
19	46851	29	42470	39	42260	49	42248
20	45511	30	42415	40	42257	50	42248
21	44605	31	42374	41	42255	51	42248
22	43972	32	42342	42	42253	52	42247
23	43520	33	42319	43	42251	53	42247
24	43192	34	42301	44	42250	54	42247
25	42951	35	42288	45	42249	55	42247

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The graph is as follows: -



```
View Window
Xmin : 0
max : 42
scale: 1
Ymin : 10000
max : 70000
scale: 1
INIT TRIG STD STO RCL
```

The above table and graph show that when the harvest is as high as 7500, the population has an increasing decreasing trend compared to when the harvest size is 5000. Also, the population does not seem to stabilize by the 35<sup>th</sup> year. By further investigating, the stable population is found to be in the 55<sup>th</sup> year.

Thus we can conclude that it is also feasible to initiate an annual harvest of 7500 fishes even though the stable population is lower than when the harvest size is 5000.

- When the harvest size = 4000 fishes

We shall initiate the harvest of 4000 fishes after the 16<sup>th</sup> year when the population stabilizes at 59999. The following changes are made in the original recursive relation:-

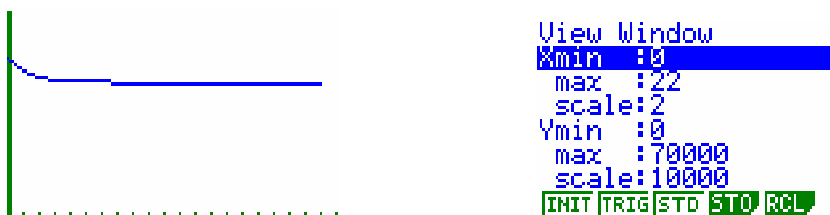
1. The starting population is taken to be 59999
2. In the relation  $u_{n+1} = (-1 \times 10^{-5}) u_n^2 + 1.6u_n$ , we subtract 4000 as this is the number which is removed every year. Thus, the new relation becomes  $u_{n+1} = (-1 \times 10^{-5}) u_n^2 + 1.6u_n - 4000$

Entering these values in the RECUR mode of the GDC Casio *CFX 9850GC Plus*, we get the following table:-

```
Table Range n+1
Start: 0
End : 20
a0 : 59999
b0 : 0
anStr: 0
bnStr: 0
[a0] [a1]
```

$n+1$	$u_{n+1}$	$n+1$	$u_{n+1}$
16	59999	26	52375
17	55999	27	52369
18	54239	28	52365
19	53364	29	52363
20	52905	30	52362
21	52658	31	52361
22	52524	32	52361
23	52451	33	52360
24	52410	34	52360
25	52388	35	52360

The graph is as follows: -



The above table shows that when the harvest is 4000, the population has a less decreasing trend compared to when the harvest size is 5000. Also, the population stabilizes by the 33<sup>rd</sup> year, an year earlier than when the harvest size is 5000.

Thus we can conclude that it is also feasible to initiate an annual harvest of 4000 fishes and the stable population is higher than when the harvest size is 5000.

- When the harvest size = 10000 fishes

We shall initiate the harvest of 10000 fishes after the 16<sup>th</sup> year when the population stabilizes at 59999. The following changes are made in the original recursive relation:-

1. The starting population is taken to be 59999
2. In the relation  $u_{n+1} = (-1 \times 10^{-5}) u_n^2 + 1.6u_n$ , we subtract 10000 as this is the number which is removed every year. Thus, the new relation becomes  $u_{n+1} = (-1 \times 10^{-5}) u_n^2 + 1.6u_n - 10000$

The table settings are as follows: -

```
Table Range n+1
Start:0
End:25
a:59999
b:0
anStr:0
bnStr:0
[a] [a]
```

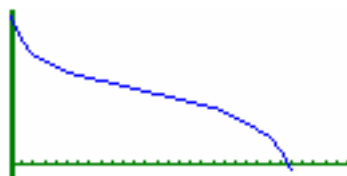
Entering these values in the RECUR mode of the GDC Casio *CFX 9850GC Plus*, we get the following table:-

$n+1$	$u_{n+1}$	$n+1$	$u_{n+1}$	$n+1$	$u_{n+1}$
16	59999	26	31204	37	18779
17	49999	27	30190	38	16520
18	44999	28	29189	39	13703
19	41749	29	28183	40	10047
20	39369	30	27150	41	5066
21	37491	31	26068	42	-2150
22	35930	32	24914		
23	34578	33	23655		
24	33368	34	22253		
25	32255	35	20653		

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The graph is as follows: -



```
View Window
Xmin : 0
max : 27
scale : 1
Ymin : 10000
max : 70000
scale : 1
INIT TRIG STD STO RCL
```

The above table and graph shows that when the harvest is as high as 10000, the population has an increasing decreasing trend compared to when the harvest size is 5000 or even 7500. The population does not stabilize at all, and in fact, dies out by the 42<sup>nd</sup> year when the value as shown by the calculator is -2150, and when the curve crosses or cuts the  $x$ -axis, indicating that the population is extinct or has died out.

Thus we can conclude that it is not feasible at all to initiate an annual harvest of 10000 fishes.

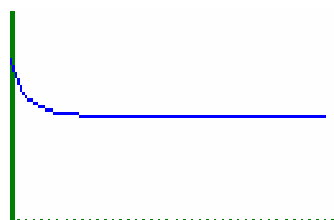
## Part 8

We shall use the 'Hit and Trial' method to find out the maximum annual sustainable harvest by examining different harvest sizes: -

The same methods shall be employed as in *Part 7* for different harvest sizes, that is: -

1. The starting population is taken to be 59999
  2. In the relation  $u_{n+1} = (-1 \times 10^{-5}) u_n^2 + 1.6u_n$ , we subtract  $x$  which is the harvest size taken in the particular case and is removed every year. Thus, the new relation becomes  $u_{n+1} = (-1 \times 10^{-5}) u_n^2 + 1.6u_n - x$
- We start by taking the harvest size,  $x = 7500$

The new recursive function becomes  $u_{n+1} = (-1 \times 10^{-5}) u_n^2 + 1.6u_n - 7500$  which we shall enter in the RECUR mode of the GDC *Casio CFX-9850GC Plus*. We have used the same table range as in *Part 7*.



```
View Window
Xmin : 0
max : 42
scale : 1
Ymin : 10000
max : 70000
scale : 1
INIT TRIG STD STO RCL
```

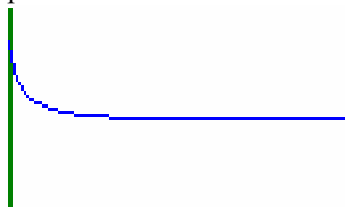
As found out already in *Part 7*, the population stabilizes in the 52<sup>nd</sup> year at 42247. Thus, there is scope for a higher harvest.

- Next, we take the harvest size,  $x = 8500$

The new recursive function becomes  $u_{n+1} = (-1 \times 10^{-5}) u_n^2 + 1.6u_n - 8500$  which we shall enter in the RECUR mode of the GDC Casio CFX-9850GC Plus.

```
Table Range n+1
Start:0
End :63
a0 :59999
b0 :0
anStr:0
bnStr:0
|a0|a1
```

The graph is as follows: -



```
View Window
Xmin :0
max :60
scale:1
Ymin :10000
max :70000
scale:1
|INIT|TRIG|STD|STD|RCL|
```

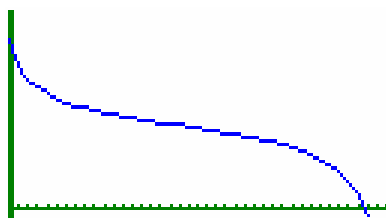
It is observed that the population stabilizes from the 75<sup>th</sup> year at 37071. We can try further higher values.

- When the harvest size,  $x = 9500$

The new recursive function becomes  $u_{n+1} = (-1 \times 10^{-5}) u_n^2 + 1.6u_n - 9500$  which we shall enter in the RECUR mode of the GDC Casio CFX-9850GC Plus. The table settings are: -

```
Table Range n+1
Start:0
End :38
a0 :59999
b0 :0
anStr:0
bnStr:0
|a0|a1
```

The graph is as follows: -



```
View Window
Xmin :0
max :40
scale:1
Ymin :-5000
max :70000
scale:1
|INIT|TRIG|STD|STD|RCL|
```

From the graph we see that the curve crosses or cuts the  $x$ -axis which indicated that the population dies out or becomes extinct during the 54<sup>th</sup> year. Therefore, we should try lower values.

- When the harvest size,  $x = 8900$

The new recursive function becomes  $u_{n+1} = (-1 \times 10^{-5}) u_n^2 + 1.6u_n - 8900$  which we shall enter in the RECUR mode of the GDC Casio CFX-9850GC Plus. The table settings are as follows: -

```
Table Range n+1
Start:0
End :150
a0 :59999
b0 :0
anStr:0
bnStr:0
|a0|a1
```

The graph is as follows: -



```
View Window
Xmin :0
max :150
scale:1
Ymin :10000
max :70000
scale:1
|INIT|TRIG|STD|STD|RCL|
```

It is observed that the population becomes stable only after the 150<sup>th</sup> year.  
We can try slightly higher values.

- When the harvest size,  $x = 9100$

The new recursive function becomes  $u_{n+1} = (-1 \times 10^{-5}) u_n^2 + 1.6u_n - 9100$  which we shall enter in the RECUR mode of the GDC Casio CFX-9850GC Plus. The table settings are as follows: -

```
Table Range n+1
Start:0
End :95
a0 :59999
b0 :0
anStr:0
bnStr:0
|a0|a1
```

The graph is as follows: -



```
View Window
Xmin :0
max :95
scale:1
Ymin :-5000
max :70000
scale:1
|INIT|TRIG|STD|STD|RCL|
```

It is observed that the population dies out and becomes extinct during the 109<sup>th</sup> year.  
Thus, the maximum sustainable harvest size must be somewhere between 8900 and 9100.

- Let us try with the harvest size,  $x = 9000$

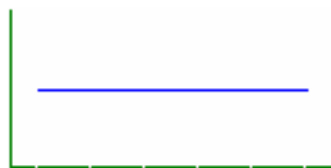
The new recursive function becomes  $u_{n+1} = (-1 \times 10^{-5}) u_n^2 + 1.6u_n - 9000$  which we shall enter in the RECUR mode of the GDC Casio CFX-9850GC Plus. The following are the table settings: -

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```
Table Range n+1
Start:0
End :1700
a0 :59999
b0 :0
anStr:0
bnStr:0
a0 | a1
```

The graph in the range 700<sup>th</sup> and 800<sup>th</sup> year is as follows: -

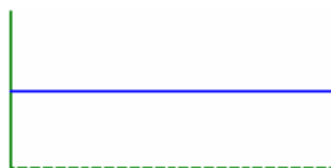


```
View Window
Xmin :690
max :810
scale:1
Ymin :0
max :60000
scale:1
INIT TRIG STD STO RCL
```

Let us also see some of the table values in this range: -

n+1	3n+1	n+1	3n+1	n+1	3n+1	n+1	3n+1
700	30141	740	30133	780	30126	800	30123
701	30140	741	30133	781	30126	797	30124
702	30140	742	30133	782	30126	798	30123
703	30140	743	30132	783	30126	799	30123

These values show that the population is decreasing in this range but at a very slow rate. Therefore, let us see whether the population becomes extinct during the 1700<sup>th</sup> and 1800<sup>th</sup> year:-



```
View Window
Xmin :1690
max :1700
scale:1
Ymin :0
max :60000
scale:1
INIT TRIG STD STO RCL
```

Also, some of the table values in this range are: -

n+1	3n+1	n+1	3n+1	n+1	3n+1	n+1	3n+1
1600	30062	1640	30060	1680	30059	1697	30058
1601	30062	1641	30060	1681	30059	1698	30058
1602	30062	1642	30060	1682	30059	1699	30058
1603	30062	1643	30060	1683	30059	1700	30058

The graph and these values above again show that the population is more or less stable between high years between the 1600<sup>th</sup> and 1700<sup>th</sup> year.

It is seen that the population does not die out even beyond the 1700<sup>th</sup> year or so. Moreover, the rate of decreasing of the population keeps on decreasing. Hence, we can assume that the population eventually becomes stable.

- To prove that 9000 is the maximum annual sustainable harvest, we shall check whether

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the population dies out or not when the harvest size,  $x = 9001$

The new recursive function becomes  $u_{n+1} = (-1 \times 10^{-5}) u_n^2 + 1.6u_n - 9001$  which we shall enter in the RECUR mode of the GDC Casio CFX-9850GC Plus. The table settings are as follows: -

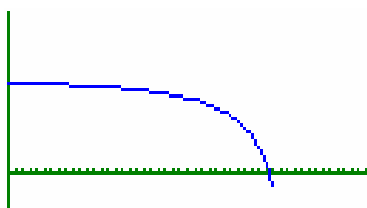
```
Table Range n+1
Start:0
End :990
a0 :59999
b0 :0
anStr:0
bnStr:0
|ao|ar
```

The graph between the 1<sup>st</sup> year and the 100<sup>th</sup> year is as follows: -



```
View Window
Xmin :0
max :100
scale:1
Ymin :0
max :60000
scale:1
|INIT|TRIG|STD|STO|RCL|
```

The following graph shows the values from the 950<sup>th</sup> to 990<sup>th</sup> year: -



```
View Window
Xmin :950
max :1000
scale:1
Ymin : -11612.903
max :48387.0967
scale:1
|INIT|TRIG|STD|STO|RCL|
```

The population is found to die out during the 1003<sup>th</sup> year which proves that 9000 is the maximum sustainable harvest which is possible.

## Part 9

Using the first model, that is, with the initial fish population of 10000, we shall determine whether it will be feasible to harvest immediately from the 1<sup>st</sup> year with different harvest sizes: -

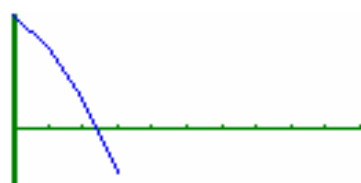
- Let us start by taking the maximum sustainable harvest of 8000 (as calculated in Part 8) and check whether it is feasible to start harvesting from the 1<sup>st</sup> year itself.

The new recursive relation will be  $u_{n+1} = (-1 \times 10^{-5}) u_n^2 + 1.6u_n - 8000$   
We set the following range for the table:

```
Table Range n+1
Start:0
End :3
a0 :10000
b0 :0
anStr:0
bnStr:0
[a0] [a1]
```

Entering the values in the RECUR mode of the GDC *Casio CFX-9850 GC Plus*, we get the following table: -

$n+1$	$u_{n+1}$
0	10000
1	7000
2	2710
3	0



```
View Window
Xmin :0
max :5
scale:1
Ymin :-5000
max :15000
scale:1
[INIT] [TRIG] [STD] [STO] [RCL]
```

The table and graph show how the population dies out or becomes extinct during the 3<sup>rd</sup> year and thus it would not be economically beneficial with a harvest size of 8000.

- Let us try with a lower value such as 6500

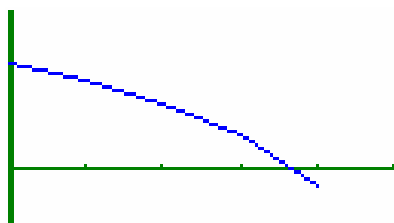
The new recursive relation will be  $u_{n+1} = (-1 \times 10^{-5}) u_n^2 + 1.6u_n - 6500$

We set the following range for the table:

```
Table Range n+1
Start:0
End :4
a0 :10000
b0 :0
anStr:0
bnStr:0
[a0] [a1]
```

Entering the values in the RECUR mode of the GDC *Casio CFX-9850 GC Plus*, we get the following table: -

$n+1$	$u_{n+1}$
0	10000
1	8500
2	6378
3	3297
4	0



```
View Window
Xmin : 0
max : 5
scale: 1
Ymin : -5000
max : 15000
scale: 1
[INIT] [TRIG] [STD] [STO] [RCL]
```

Again the table and the graph show how the population dies out during the 4<sup>th</sup> year proving that it will not be beneficial.

- Let us try with further lower value such as 5000.

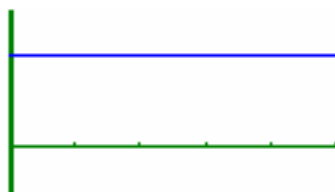
The new recursive relation will be  $u_{n+1} = (-1 \times 10^{-5}) u_n^2 + 1.6u_n - 5000$

We set the following range for the table:

```
Table Range n+1
Start: 0
End : 6
a0 : 10000
b0 : 0
anStr: 0
bnStr: 0
[a0] [a1]
```

Entering the values in the RECUR mode of the GDC *Casio CFX-9850 GC Plus*, we get the following table: -

$n+1$	$u_{n+1}$
0	10000
1	10000
2	10000
3	10000
4	10000
5	10000
6	10000



```
View Window
Xmin : 0
max : 5
scale: 1
Ymin : -5000
max : 15000
scale: 1
[INIT] [TRIG] [STD] [STO] [RCL]
```

Hence we see that the population sustains from the 1<sup>st</sup> year of harvest itself.

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To check whether 5000, is the maximum harvest that can be done from 10,000 so that the population sustains faster, we can take a slightly higher value and check whether that will be sustainable.

- Let us check with 5001

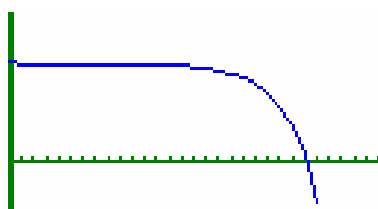
The new recursive relation will be  $u_{n+1} = (-1 \times 10^{-5}) u_n^2 + 1.6u_n - 5001$

We set the following range for the table: -

```
Table Range n+1
Start:0
End :25
a0 :10000
b0 :0
anStr:0
bnStr:0
[a0] [a1]
```

Entering the values in the RECUR mode of the GDC Casio CFX-9850 GC Plus, we get the following table: -

$n+1$	$u_{n+1}$	$n+1$	$u_{n+1}$
0	10000	13	9803
1	9999	14	9723
2	9998	15	9611
3	9996	16	9453
4	9983	17	9230
5	9976	18	8915
6	9984	19	8469
7	9976	20	7832
8	9966	21	6916
9	9951	22	5587
10	9930	23	3626
11	9901	24	669
12	9860	25	0



```
View Window
Xmin :0
max :30
scale:1
Ymin :-5000
max :15000
scale:1
[INIT] [TRIG] [STD] [STO] [RCL]
```

The graph and table shows how the population dies out during the 25<sup>th</sup> year itself. Thus, even 5001 is not sustainable.

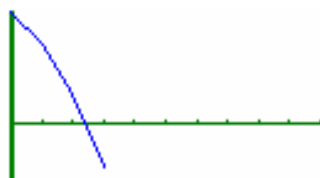
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- We start by taking the initial population size = 10000 fishes and initiating a harvest of 8000 fishes from the 1<sup>st</sup> year itself.

The new recursive function becomes  $u_{n+1} = (-1 \times 10^{-5}) u_n^2 + 1.6u_n - 8000$ . Also, the starting population,  $u_0 = 10000$ . This shall be entered in the GDC Casio CFX-9850GC Plus.

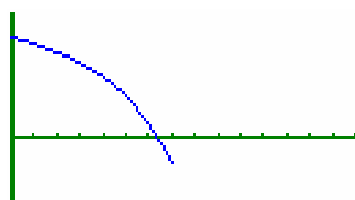


```
View Window
Xmin : 0
max : 10
scale: 1
Ymin : -5000
max : 10000
scale: 1
INIT TRIG STD STO RCL
```

In this case, we find out that the population dies out during the 3<sup>rd</sup> year which means that such a combination is not sustainable.

- Let us increase the initial population size = 16000 fishes and initiate a harvest of 8000 fishes from the 1<sup>st</sup> year itself.

The new recursive function becomes  $u_{n+1} = (-1 \times 10^{-5}) u_n^2 + 1.6u_n - 8000$ . Also, the starting population,  $u_0 = 16000$ . This shall be entered in the GDC Casio CFX-9850GC Plus.

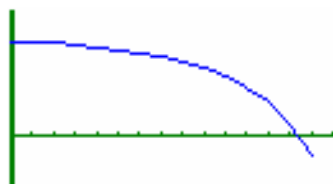


```
View Window
Xmin : 0
max : 15
scale: 1
Ymin : -10000
max : 20000
scale: 1
INIT TRIG STD STO RCL
```

In this case also, we find that the population dies put during the 7<sup>th</sup> year which means that such a combination also is not sustainable.

- Let us try with an initial population size of 19000 fishes and initiate a harvest of 8000 fishes from the 1<sup>st</sup> year itself.

The new recursive function becomes  $u_{n+1} = (-1 \times 10^{-5}) u_n^2 + 1.6u_n - 8000$ . Also, the starting population,  $u_0 = 19000$ . This shall be entered in the GDC Casio CFX-9850GC Plus.

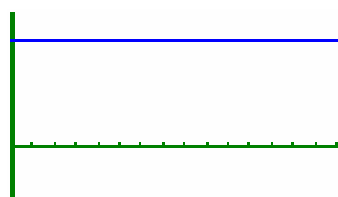


```
View Window
Xmin : 0
max : 15
scale: 1
Ymin : -10000
max : 25000
scale: 1
INIT TRIG STD STO RCL
```

The graph and GDC show that the population dies out during the 14<sup>th</sup> year which means that this combination is not sustainable.

- When the initial population size is 20000 fishes and a harvest of 8000 fishes is initiated from the 1<sup>st</sup> year itself.

The new recursive function becomes  $u_{n+1} = (-1 \times 10^{-5}) u_n^2 + 1.6u_n - 8000$ . Also, the starting population,  $u_0 = 20000$ . This shall be entered in the GDC Casio CFX-9850GC Plus.

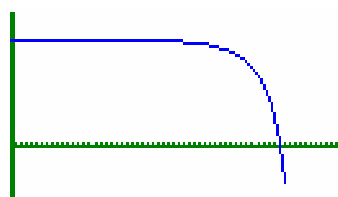


```
View Window
Xmin :0
max :15
scale:1
Ymin :-10000
max :25000
scale:1
INIT TRIG STD STO RCL
```

Using this combination, we have found out using the GDC that the population remains constant and stable starting from the 1<sup>st</sup> year itself. It neither depletes nor increases. Thus, this introductory fish population size of 20000 is the most sustainable for harvesting 8000 fishes.

- To prove that an initial population of 20000 fishes is the most suitable and sustainable for a harvest of 8000 fishes every year, we shall investigate using  $u_0 = 19999$ .

The new recursive function becomes  $u_{n+1} = (-1 \times 10^{-5}) u_n^2 + 1.6u_n - 8000$ . Also, the starting population,  $u_0 = 19999$ . This shall be entered in the GDC Casio CFX-9850GC Plus.



```
View Window
Xmin :0
max :62
scale:1
Ymin :-10000
max :25000
scale:1
INIT TRIG STD STO RCL
```

The GDC shows that in such a case, the population has a decreasing trend which slowly increases with increasing number of years and finally, dies out during the 52<sup>nd</sup> year. Hence, an initial population size of 19999 is not sustainable.

These results help us to infer that a minimum initial population size of 20000 fishes is required to sustain a harvest of 8000 fishes every year from the 1<sup>st</sup> year.