HI type 2

Math portfolio 2

Designing a freight elevator

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Math's Portfolio-Designing a Freight Elevator

In this portfolio I will attempt to design a freight Elevator model. Primarily I will study the given model to assess the usefulness and the strengths and weaknesses This will allow me to then form conditions for designing an effective and usefulelevator model.

Therefore I will now study the motion of the elevator by the given displacement equation : $y = 2.5t^3 - 15t^2$

Displacement

Checking to interpret straight line motion for time (t) =0,1,2,3,4,5,6

Substituting t=0

$y = (2.5)0^3 - (15)0^2$
$y = 0$

Substituting t=1

$y = (2.5)1^3 - (15)1^2$
$y = 2.5 - 15$
$y = -12.5$

Substituting t=2

$y = (2.5)2^3 - (15)2^2$
$y = 20 - 60$
$y = -40$

Substituting t=3

$y = (2.5)3^3 - (15)3^2$
$y = 67.5 - 135$
$y = -67.5$

Substituting t=4

$y = (2.5)4^3 - (15)4^2$
$y = 160 - 240$
$y = -80.0$

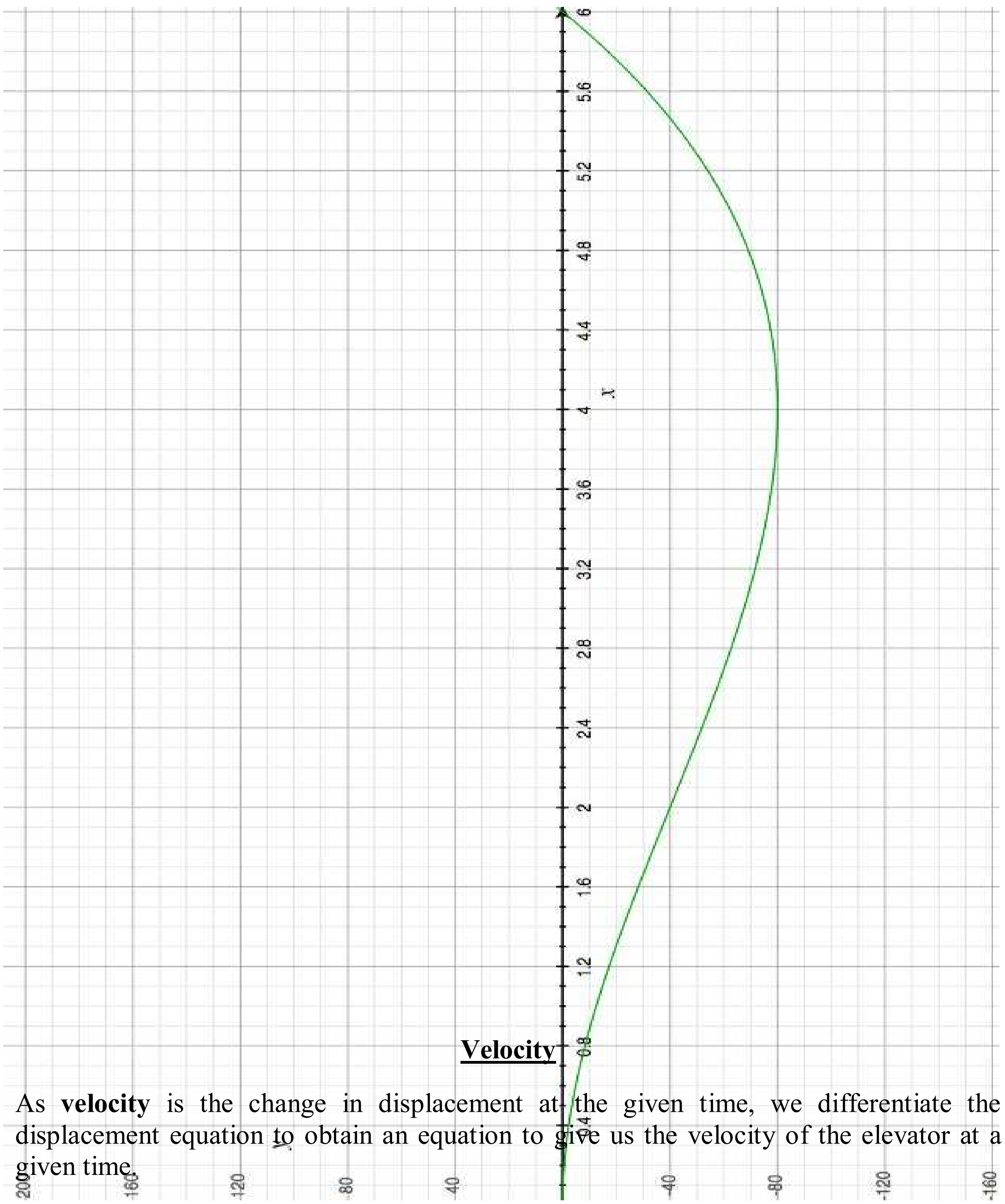
Substituting t=5

$y = (2.5)5^3 - (15)5^2$
$y = 312.5 - 375$
$y = -62.5$

Substituting t=6

$$y = (2.5)6^3 - (15)6^2$$
$$y = 540 - 540$$
$$y = 0.00$$

Displacement/Time Graph



Velocity

As **velocity** is the change in displacement at the given time, we differentiate the displacement equation to obtain an equation to give us the velocity of the elevator at a given time.

Therefore :

$$y = 2.5t^3 - 15t^2$$
$$v = \frac{dy}{dt} = 7.5t^2 - 30t$$

We now substitute r values of t = 0,1,2,3,4,5,6 to obtain the velocity at the given times

Substituting t=0

$$v = 7.5t^2 - 30t$$
$$v = 0 - 0$$
$$v = 0$$

Substituting t=1

$$v = 7.5t^2 - 30t$$
$$v = 7.5 - 30$$
$$v = -22.5$$

Substituting t=2

$$v = 7.5t^2 - 30t$$
$$v = 30 - 60$$
$$v = -30$$

Substituting t=3

$$v = 7.5t^2 - 30t$$
$$v = 67.5 - 90$$
$$v = -22.5$$

Substituting t=4

$$v = 7.5t^2 - 30t$$
$$v = 120 - 120$$
$$v = 0$$

Substituting t=5

$$v = 7.5t^2 - 30t$$
$$v = 187.5 - 150$$
$$v = 37.5$$

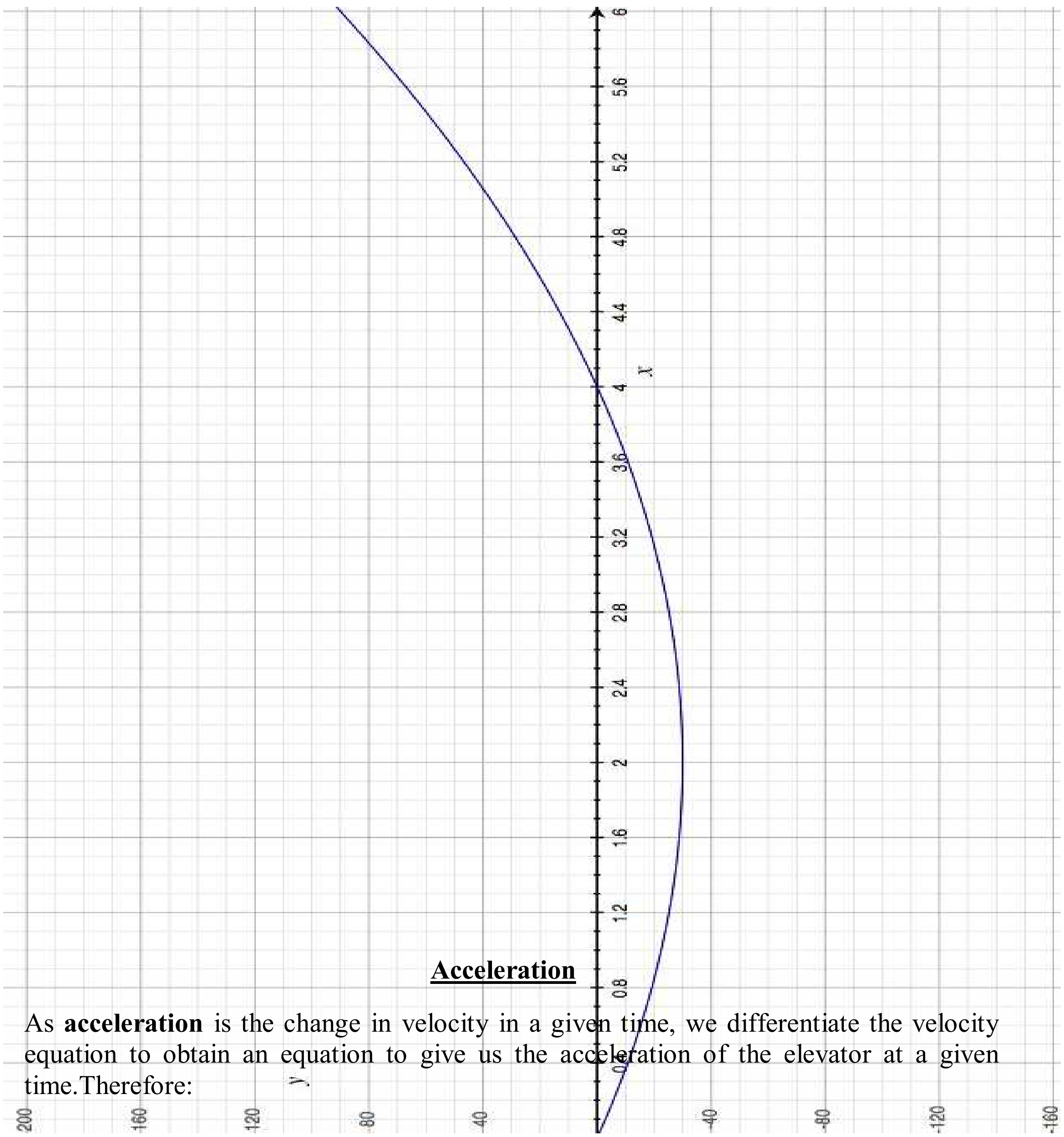
Substituting t=6

$v = 7.5t^2 - 30t$

$v = 270 - 180$

$v = 90$

Velocity/Time graph



Acceleration

As **acceleration** is the change in velocity in a given time, we differentiate the velocity equation to obtain an equation to give us the acceleration of the elevator at a given time.Therefore:

$v = 7.5t^2 - 30t$

$a = \dfrac{dv}{dt} = 15t - 30$

We now substitute r values of t = 0,1,2,3,4,5,6 to obtain the velocity at the given times

Substituting t=0

$a = 15t - 30$

$a = 0 - 30$

$a = -30$

Substituting t=1

$a = 15t - 30$

$a = 15 - 30$

$a = -15$

Substituting t=2

$a = 15t - 30$

$a = 30 - 30$

$a = 0$

Substituting t=3

$a = 15t - 30$

$a = 45 - 30$

$a = 15$

Substituting t=4

$a = 15t - 30$

$a = 60 - 30$

$a = 30$

Substituting t=5

$a = 15t - 30$

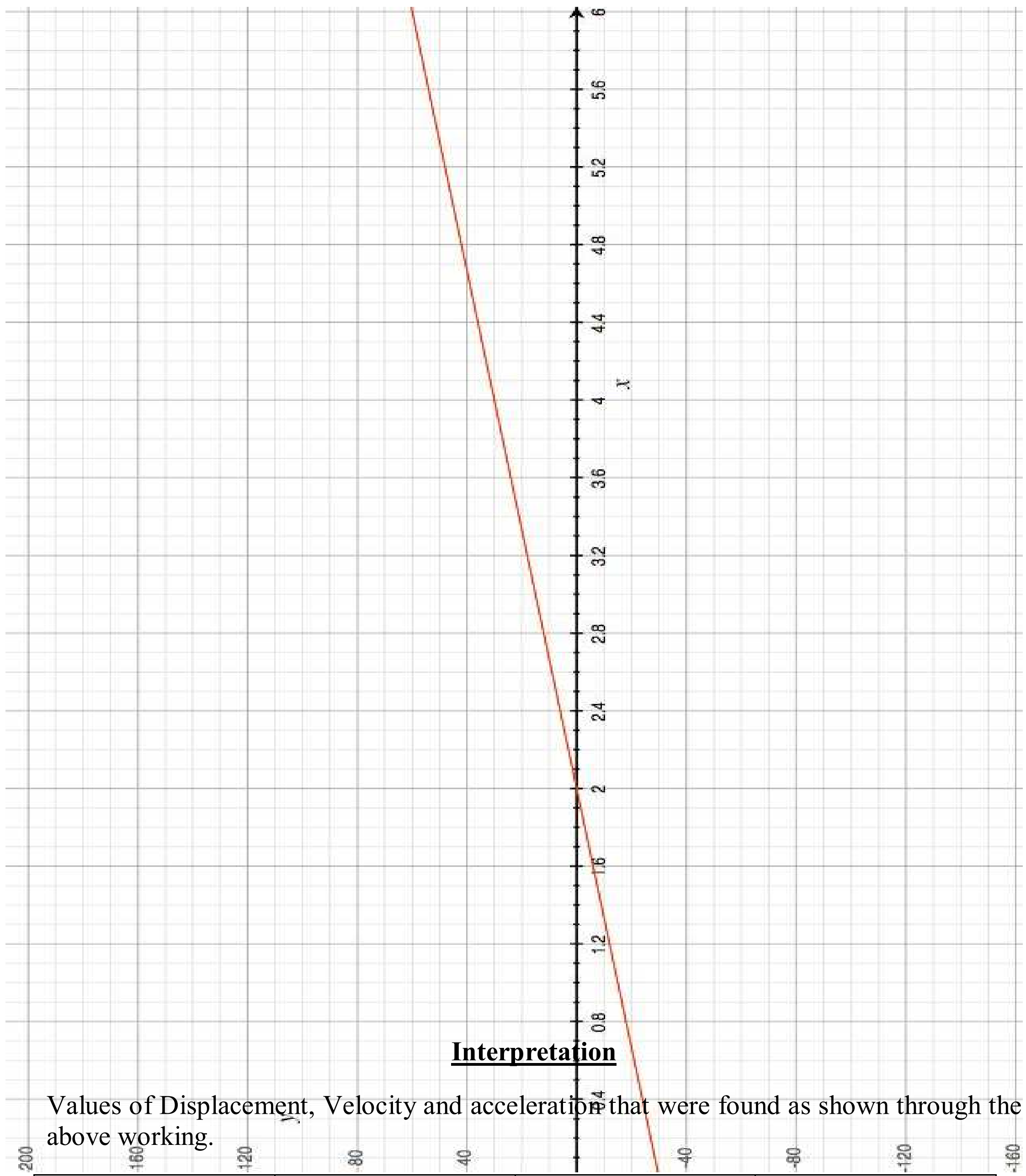
$a = 75 - 30$

$a = 45$

Substituting t=6

$$a = 15t - 30$$
$$a = 90 - 30$$
$$a = 60$$

<u>Accerleration/Time graph</u>

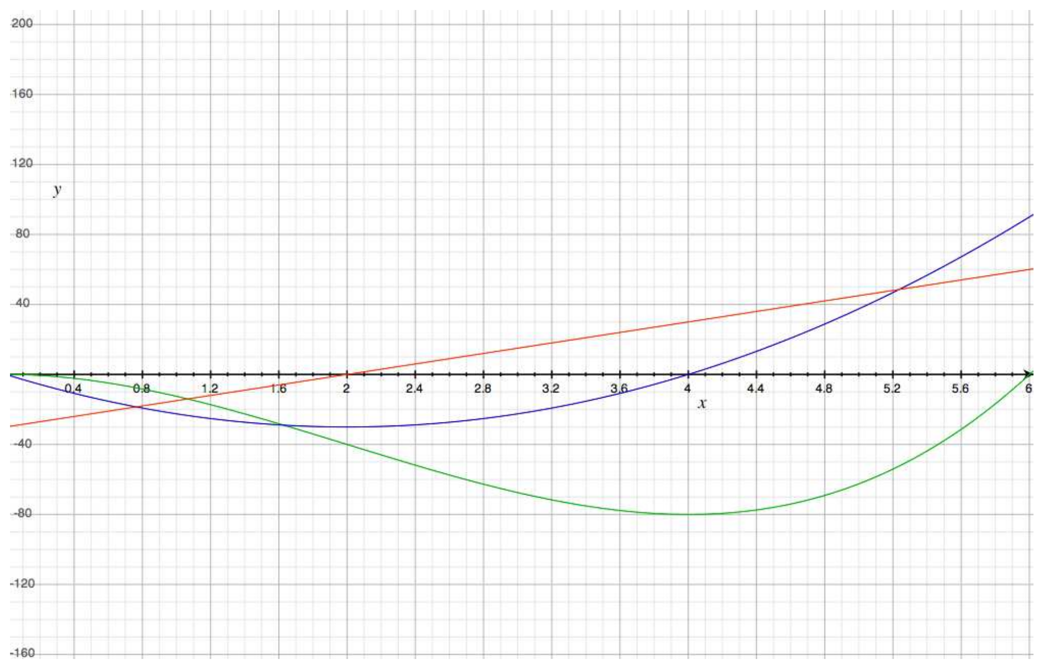


Interpretation

Values of Displacement, Velocity and acceleration that were found as shown through the above working.

Time	Displacement (s)	Velocity (v)	Acceleration (a)
0	0.00	0.00	-30.0
1	-12.5	-22.5	-15.0

2	-40.0	-30.0	0.00
3	-67.5	-22.5	15.0
4	-80.0	0.00	30.0
5	-62.5	37.5	45.0
6	0.00	90.0	60.0



Through the findings we can Interpret that the Elevator is going down the shaft due to the negative sign of the displacement values and it reaches a maximum displacement of 80 m. We see that the Elevator is not moving with a uniform velocity as we notice that the velocity values are different after each minute, also looking at the velocity graph we see that non symmetrical parabolic nature of the velocity equation reflects its non-uniform nature.

The motion of the elevator is such that at 0 minutes the elevator is not moving. From the 1^{st} to 3^{rd} minute we notice that the velocity values are negative. As velocity is a vector quantity i.e. that requires a magnitude and a direction to be fully described. We see that the negative sign shows the downward direction of the elevator. At the 4^{th} minute the elevator has a velocity of 0 which shows that the elevator has stopped moving, and the positive value of the 5^{th} and 6^{th} minute show that the elevator is moving upwards.

The acceleration as we see is increasing uniformly due to the linear nature of the curve. As the acceleration becomes positive the elevator starts to decelerate due to the opposite direction of acceleration and hence makes the velocity=0.Then the positive acceleration makes the lift then accelerate in the direction of the force .

We see that the problems of the model are:

1. The elevator does not go to the maximum height of 100m

2. The velocity of the elevator is 90m/s at the point where the elevator should stop.
3. The velocity is uneven through out the elevators movement as it takes 4 minutes to reach the bottom and 2 minutes to reach back up.
4. The acceleration of the elevator is 60 m/s at t = 6 , when the elevator is meant to stop.

Therefore, the elevator is not useful due to the reasons stated above.

Redesign of Elevator Model

Ideal Table

Time	Displacement (s)	Velocity (v)	Acceleration (a)
0	0.00	0.00	
3	-100.00	0.00	
6	0.00	0.00	0

These are the specifications of the Elevator Design that need to be followed to make the elevator useful. The reasons for this are:

1. At t=0 ,S=0 . This means the lift should be at the starting point.

2. At t=0 , v=0 as the lift is standing still at the starting position.
3. At t=3 the elevator should be at 0 m/s.
4. The maximum displacement should be of -100 and be at t = 3 to ensure uniform movement of elevator.
5. The velocity has to be 0 at t = 6 to ensure that the elevator will stop.
6. The acceleration has to be 0 at t = 6 minutes as to ensure that the elevator stops at the starting position.
7. At t=6 the displacement of the elevator should be 0, as it has reached back to the starting position

Conditions used to form Equation

The displacement equation will be determined and will be differentiated to find velocity and then further differentiated to find out the equation for acceleration.

Reason: To redesign this model I have 7 conditions to be satisfied. This means that I will have to phrase seven equations with 7 unknowns a,b,c,d,e, f and g respectively . Also as it is a vertical motion, it will be in terms of “y” as a function of time . As there is no horizontal motion, “x” will not be represented as a function of time.

Therefore the general equation for displacement (s) as a function of time is:

$$s(t) = at^6 + bt^5 + ct^4 + dt^3 + et^2 + ft + g$$

Differentiating to get general equation for velocity (v) as a function of time is:

$$s(t)' = v(t) = 6at^5 + 5bt^4 + 4ct^3 + 3dt^2 + 2et + f$$

Differentiating to get general equation for acceleration (a) as a function of time is:

$$s(t)'' = a(t) = 30at^4 + 20bt^3 + 12ct^2 + 6dt + 2e$$

I will now use these equations to form the desired movement of the elevator:

Substituting condition 1. t=0, s=0

$$s(0) = a0^6 + b0^5 + c0^4 + d0^3 + e0^2 + f0 + g$$

$$0 = g$$

Substituting condition 2. t=0,v=0

$$s(t)' = v(t) = 6a0^5 + 5b0^4 + 4c0^3 + 3d0^2 + 2e0 + f$$

$$0 = f$$

Substituting condition 3. t=3, s=-100

$$-100 = at3^6 + b3^5 + c3^4 + d3^3 + e3^2 + f3 + g$$

$$-100 = 729a + 243b + 81c + 27d + 9e + 3f + g$$

as f=0 and g=0

$$-100 = 729a + 243b + 81c + 27d + 9e$$

Substituting condition 4. t=3, v=0

$$0 = 6a3^5 + 5b3^4 + 4c3^3 + 3d3^2 + 2e3 + f$$

as f=0

$0 = 1458a + 405b + 108c + 27d + 6e$

Substituting condition 5. t=6, s=0

$0 = a6^6 + b6^5 + c6^4 + d6^3 + e6^2 + f6 + g$

$0 = 46656a + 7776b + 1296c + 216d + 36e + 6f$

As f=0

$0 = 46656a + 7776b + 1296c + 216d + 36e$

Substituting condition 6. t=6, v=0

$0 = 6a6^5 + 5b6^4 + 4c6^3 + 3d6^2 + 2e6 + f$

$0 = 46656a + 6480b + 864c + 108d + 12e + f$

As f=0

$0 = 46656a + 6480b + 864c + 108d + 12e$

Substituting condition 7. t=6, a=0

$0 = 30a6^4 + 20b6^3 + 12c6^2 + 6d6 + 2e$

$0 = 38880a + 4320b + 432c + 36d + 2e$

<u>Analysis</u>

As there are 5 variables: a, b, c, d, e and 5 equations with the variables. I have simultaneously equated them using technology.

Variables	Values
a	$\frac{100}{729}$
b	$-\frac{200}{81}$

c	$\frac{400}{27}$
d	$-\frac{800}{27}$
e, f, g	0

After substituting the values found above for a, b, c, d, e, f and g in the general equation. The model that I have final obtained is:

For Displacement (s) as a function of time

$$s(t)=\frac{100}{729}t^6+-\frac{200}{81}t^5+\frac{400}{27}t^4+-\frac{800}{27}t^3+0t^2+0t+0$$

For Velocity (v) as a function of time

$$v(t)=\frac{600}{729}t^5-\frac{1000}{81}t^4+\frac{1600}{27}t^3-\frac{800}{27}t^2+0t+0$$

For Acceleration (a) as a function of time

$$a(t)=\frac{3000}{729}t^4-\frac{4000}{81}t^3+\frac{4800}{27}t^2-\frac{4800}{27}t+0$$

I will now substitute the values of the variables in the equations below for t=0,1,2,3,4,5,6 to find the desired values for:

- Displacement
- Velocity
- Acceleration

Displacement (s)

When t=0

$$s(0)=\frac{100}{729}0^6+-\frac{200}{81}0^5+\frac{400}{27}0^4+-\frac{800}{27}0^3+0(0^2)+0(0)+0$$

$$s(0)=0$$

When t=1

$$s(1) = \frac{100}{729}1^6 + -\frac{200}{81}1^5 + \frac{400}{27}1^4 + -\frac{800}{27}1^3 + 0(1^2) + 0(1) + 0$$

$$s(1) = -17.15$$

When t=2

$$s(2) = \frac{100}{729}2^6 + -\frac{200}{81}2^5 + \frac{400}{27}2^4 + -\frac{800}{27}2^3 + 0(2^2) + 0(2) + 0$$

$$s(2) = -70.23$$

When t=3

$$s(3) = \frac{100}{729}3^6 + -\frac{200}{81}3^5 + \frac{400}{27}3^4 + -\frac{800}{27}3^3 + 0(3^2) + 0(3) + 0$$

$$s(3) = -100$$

When t=4

$$s(4) = \frac{100}{729}4^6 + -\frac{200}{81}4^5 + \frac{400}{27}4^4 + -\frac{800}{27}4^3 + 0(4^2) + 0(4) + 0$$

$$s(4) = -70.23$$

When t=5

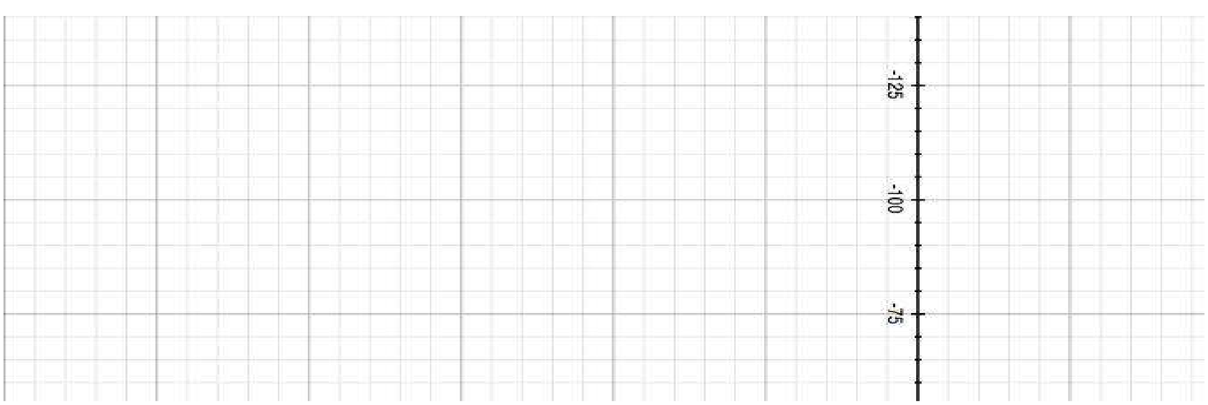
$$s(5) = \frac{100}{729}5^6 + -\frac{200}{81}5^5 + \frac{400}{27}5^4 + -\frac{800}{27}5^3 + 0(5^2) + 0(5) + 0$$

$$s(5) = -17.15$$

When t=6

$$s(6) = \frac{100}{729}6^6 + -\frac{200}{81}6^5 + \frac{400}{27}6^4 + -\frac{800}{27}6^3 + 0(6^2) + 0(6) + 0$$

$$s(6) = 0$$



Velocity (v)

When t=0

$$v(0) = 6.\frac{100}{729}0^5 + 5.-\frac{200}{81}.0^4 + 4.\frac{400}{27}0^3 + 3.-\frac{800}{27}0^2 + 2.0.0$$

$$v(0) = 0$$

When t=1

$$v(1) = 6.\frac{100}{729}1^5 + 5.-\frac{200}{81}.1^4 + 4.\frac{400}{27}1^3 + 3.-\frac{800}{27}1^2 + 2.0.1$$

$$v(1) = -41.15$$

When t=2

$$v(2) = 6.\frac{100}{729}2^5 + 5.-\frac{200}{81}.2^4 + 4.\frac{400}{27}2^3 + 3.-\frac{800}{27}2^2 + 2.0.2$$

$$v(2) = -52.67$$

When t=3

$$v(3) = 6.\frac{100}{729}3^5 + 5.-\frac{200}{81}.3^4 + 4.\frac{400}{27}3^3 + 3.-\frac{800}{27}3^2 + 2.0.3$$

$$v(3) = 0$$

When t=4

$$v(4) = 6.\frac{100}{729}4^5 + 5.-\frac{200}{81}.4^4 + 4.\frac{400}{27}4^3 + 3.-\frac{800}{27}4^2 + 2.0.4$$

$$v(4) = 52.67$$

When t=5

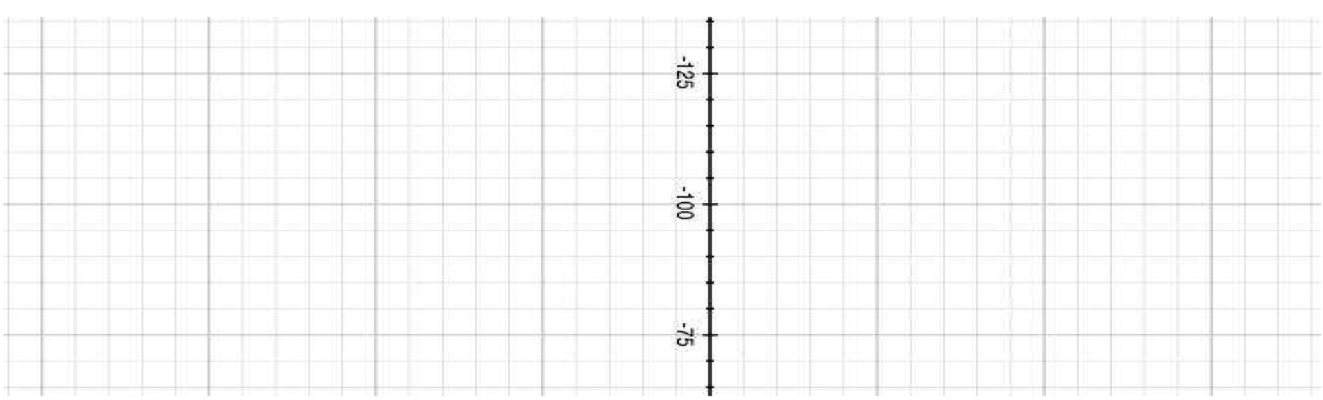
$$v(5) = 6.\frac{100}{729}5^5 + 5.-\frac{200}{81}.5^4 + 4.\frac{400}{27}5^3 + 3.-\frac{800}{27}5^2 + 2.0.5$$

$$v(5) = 41.15$$

When t=6

$$v(6) = 6.\frac{100}{729}6^5 + 5.-\frac{200}{81}.6^4 + 4.\frac{400}{27}6^3 + 3.-\frac{800}{27}6^2 + 2.0.6$$

$$v(6) = 0$$



Acceleration (a)

When t=0

$$a(0)=30.\frac{100}{729}0^4+20.-\frac{200}{81}.0^3+12.\frac{400}{27}0^2+6.-\frac{800}{27}0+2.0$$

$$a(0)=0$$

When t=1

$$a(1) = 30.\frac{100}{729}1^4 + 20.-\frac{200}{81}.1^3 + 12.\frac{400}{27}1^2 + 6.-\frac{800}{27}1 + 2.0$$

$$a(1) = -45.27$$

When t=2

$$a(2) = 30.\frac{100}{729}2^4 + 20.-\frac{200}{81}.2^3 + 12.\frac{400}{27}2^2 + 6.-\frac{800}{27}2 + 2.0$$

$$a(2) = 26.34$$

When t=3

$$a(3) = 30.\frac{100}{729}3^4 + 20.-\frac{200}{81}.3^3 + 12.\frac{400}{27}3^2 + 6.-\frac{800}{27}3 + 2.0$$

$$a(3) = 66.67$$

When t=4

$$a(4) = 30.\frac{100}{729}4^4 + 20.-\frac{200}{81}.4^3 + 12.\frac{400}{27}4^2 + 6.-\frac{800}{27}4 + 2.0$$

$$a(4) = 26.34$$

When t=5

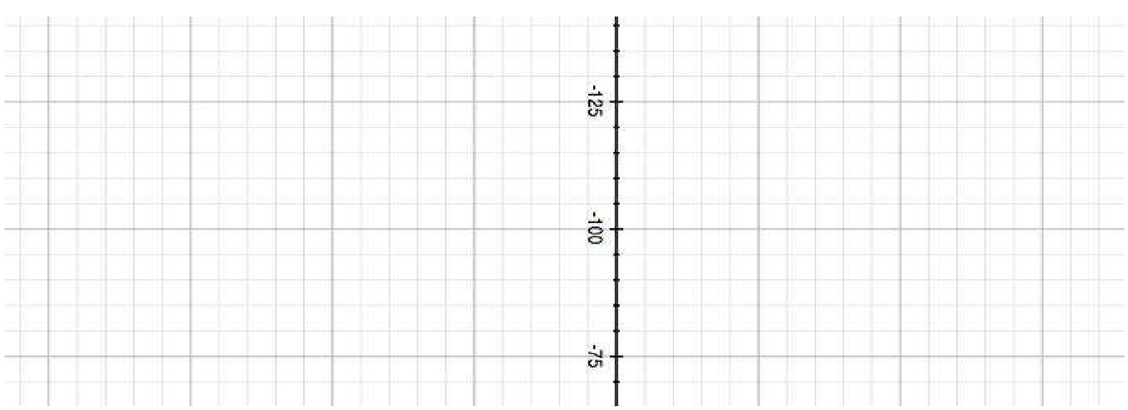
$$a(5) = 30.\frac{100}{729}5^4 + 20.-\frac{200}{81}.5^3 + 12.\frac{400}{27}5^2 + 6.-\frac{800}{27}5 + 2.0$$

$$a(5) = -45.27$$

When t=6

$$a(6) = 30.\frac{100}{729}6^4 + 20.-\frac{200}{81}.6^3 + 12.\frac{400}{27}6^2 + 6.-\frac{800}{27}6 + 2.0$$

$$a(6) = 0$$



Analysis

Therefore the values of displacement, velocity and acceleration as a function of time in the model created are:

Time	Displacement (s)	Velocity (v)	Acceleration (a)
0	0.00	0.00	0.00

1	-17.15	-41.15	-45.27
2	-70.23	-52.67	26.34
3	-100.00	0.00	66.67
4	-70.23	52.67	26.34
5	-17.15	41.15	-45.27
6	0.00	0.00	0.00

7 Conditions for ideal table

1. At t=0 , S=0 . This means the lift should be at the starting point.
2. At t=0 , v=0 as the lift is standing still at the starting position.
3. At t=3 the elevator should be at 0 m/s.
4. The maximum displacement should be of -100 and be at t = 3 to ensure uniform movement of elevator.
5. The velocity has to be 0 at t = 6 to ensure that the elevator will stop.
6. The acceleration has to be 0 at t = 6 minutes as to ensure that the elevator stops at the starting position.
7. At t=6 the displacement of the elevator should be 0, as it has reached back to the starting position

Therefore we see that all 7 conditions stated for redesign are satisfied , which allows the elevator to be used efficiently for mining purposes as :

- The company can now mine its products from 100 m instead of 80 m
- The elevator stops for loading the material at the bottom and for unloading at the top.
- The elevator has a uniform motion as it takes 3 min to travel both ways, hence there are no jerks or non uniform motions taking place.

Applying Model To Different Situations

The model can also be applied in other various situations. The model that I am going to create to apply to another situation will be as follows.

1. Elevator will have vertical motion for 6 s as in the above redesigned model.

2. As the model reaches the initial starting point it will start moving horizontally, for a distance of 50 m. This motion will be for 4 seconds.

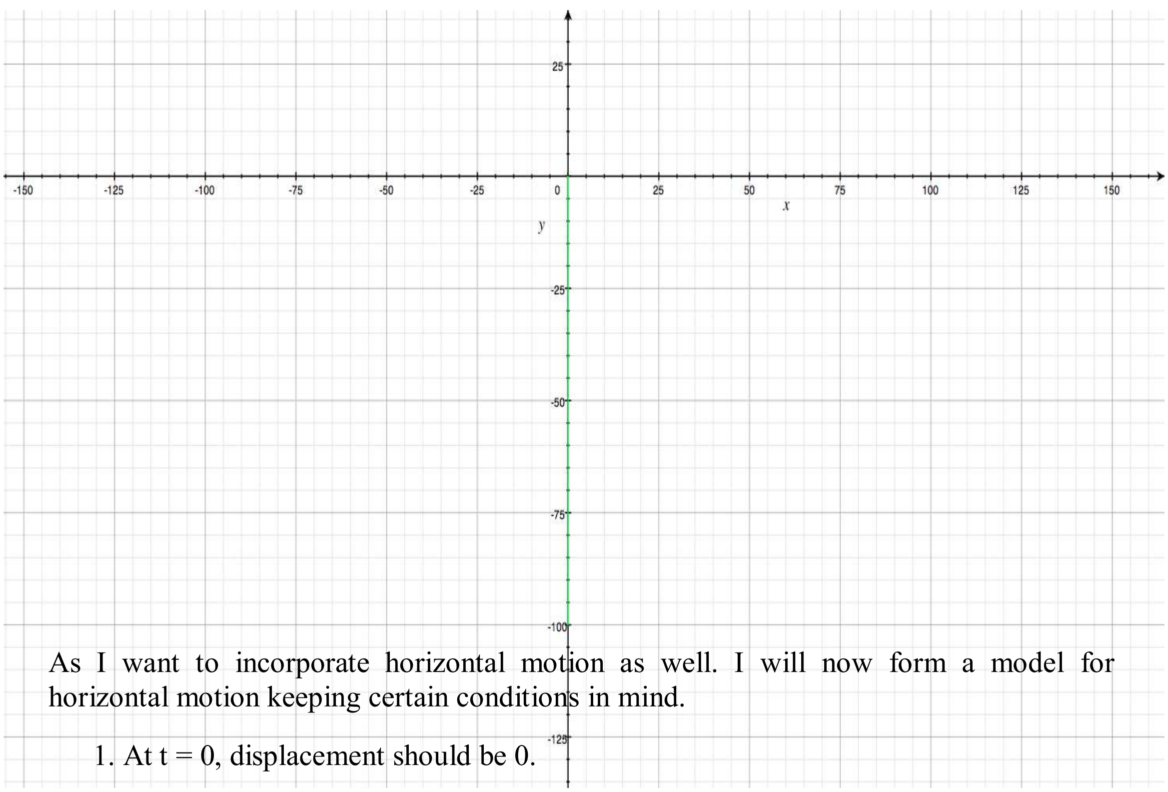
In order to form this model I will have to use parametric equations due to the dual motion in the “x” and “y” axis. These two motions are independent from each other but can be expressed in parametric form because this allows there to be independent motion in the “x” and “y” axis as a function of time.

For Vertical Motion, y will be expressed as a function of time and x will be a constant function.

For horizontal Motion ,x will be expressed as a function of time and y will be a constant function.

For the model I have designed vertical motion is of 0-6 seconds and its parametric equation will be

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{100}{729}t^6 + -\frac{200}{81}t^5 + \frac{400}{27}t^4 + -\frac{800}{27}t^3 \end{bmatrix}, t=0\ldots6$$



As I want to incorporate horizontal motion as well. I will now form a model for horizontal motion keeping certain conditions in mind.

1. At t = 0, displacement should be 0.

2. At t= 2, the elevator should reach its maximum displacement of 50 m.
3. At t= 4, the elevator should reach its initial point and hence displacement should be 0.
4. At t = 0, velocity should be 0.
5. At t= 2 the velocity of the elevator should be 0, so that the elevator stops.
6. At t=4 the velocity of the elevator should be 0, so that the elevator stops.
7. At t=4 the acceleration of the elevator should be 0, so that the elevator stops.

Therefore:

Ideal Table

Time	Displacement (s)	Velocity (v)	Acceleration (a)
1	0	0	
2	50	0	
3	0	0	0

Therefore we will use 6 degree equation as there are 7 variables needed for the 7 equations.

For

Displacement (s):

$$x(t) = at^6 + bt^5 + ct^4 + dt^3 + et^2 + ft + g$$

For

Velocity (v):

$$v(t) = 6at^5 + 5bt^4 + 4ct^3 + 3dt^2 + 2et + f$$

For

Acceleration (a):

$$x(t)'' = a(t) = 30at^4 + 20bt^3 + 12ct^2 + 6dt + 2e$$

I will now use these equations to find out the desired horizontal movement of the elevator.

Substituting condition 1. t=0, x=0

$$x(0) = a0^6 + b0^5 + c0^4 + d0^3 + e0^2 + f0 + g$$

$$0 = g$$

Substituting condition 2. t=2, x=50

$$x(2) = a2^6 + b2^5 + c2^4 + d2^3 + e2^2 + f2 + g$$

$$50 = 64a + 32b + 16c + 8d + 4e + 2f$$

Substituting condition 3. t=4, x=0

$$x(4) = a4^6 + b4^5 + c4^4 + d4^3 + e4^2 + f4 + g$$

$$0 = 4096a + 1024b + 256c + 64d + 16e + 4f$$

Substituting condition 4. t=0,v=0

$$v(0) = 6a0^5 + 5b0^4 + 4c0^3 + 3d0^2 + 2e0 + f$$

$$0 = f$$

Substituting condition 5. t=2, v=0

$$v(0) = 6a2^5 + 5b2^4 + 4c2^3 + 3d2^2 + 2e2 + f$$

$$0 = 192a + 80b + 32c + 12d + 4e + 0$$

Substituting condition 6. t=4, v=0

$$v(0) = 6a4^5 + 5b4^4 + 4c4^3 + 3d4^2 + 2e4 + f$$

$$0 = 6144a + 1280b + 256c + 48d + 8e + 0$$

Substituting condition 6. t=4, a=0

$$a(4) = 30a4^4 + 20b4^3 + 12c4^2 + 6d4 + 2e$$

$$0 = 7680a + 1280b + 192c + 24d + 2e$$

As f and g are 0 all the equations become 5 variables and I have solved them using technology.

Therefore:

Variables	Values
a	$\frac{-25}{32}$
b	$\frac{75}{8}$
c	$\frac{-75}{2}$
d	50
e	0

Therefore the general equations of the horizontal model are:

For

Displacement (s):

$$x(t)=\frac{-25}{32}t^6+\frac{75}{8}t^5+\frac{-75}{2}t^4+50t^3+0t^2+0t+0$$

For Velocity (v):

$$v(t)=\frac{-75}{16}t^5+\frac{375}{8}t^4-150t^3+150t^2+0t+0$$

For Acceleration (a):

$$a(t)=\frac{-375}{16}t^4+\frac{375}{2}t^3-450t^2+300t+0$$

By substituting the t=0, 1,2,3,4 into displacement, velocity and acceleration general equations, the value of the displacement, velocity and acceleration is found at different times.

Time	Displacement (s)	Velocity (v)	Acceleration (a)
0	0	0	0
1	$\frac{675}{32}$	$\frac{675}{16}$	$\frac{225}{16}$
2	50	0	-75
3	$\frac{675}{32}$	$-\frac{675}{16}$	$\frac{225}{16}$
4	0	0	0

Now the elevator is designed to have vertical motion and horizontal motion. For Dual motion in x and y , the equation will be represented in parametric form as independent equations.

Therefore horizontal motion:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{-25}{32}t^6 + \frac{75}{8}t^5 + \frac{-75}{2}t^4 + 50t^3 \\ 0 \end{bmatrix}, t = 0 \ldots 4$$

Therefore vertical motion:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{100}{729}t^6 + -\frac{200}{81}t^5 + \frac{400}{27}t^4 + -\frac{800}{27}t^3 \end{bmatrix}, t = 0 \ldots 6$$

Through this model we see that the lift can move horizontally as well as vertically. The 2 equations have been kept independent as this allows more flexibility in the movement of the Elevator. The horizontal motion of the lift can be used for unloading at 50 m displacement or for picking up people. This model has now become useful for the intended purpose and has also allowed a horizontal motion to be incorporated when needed. Therefore the elevator model is successfully formed.