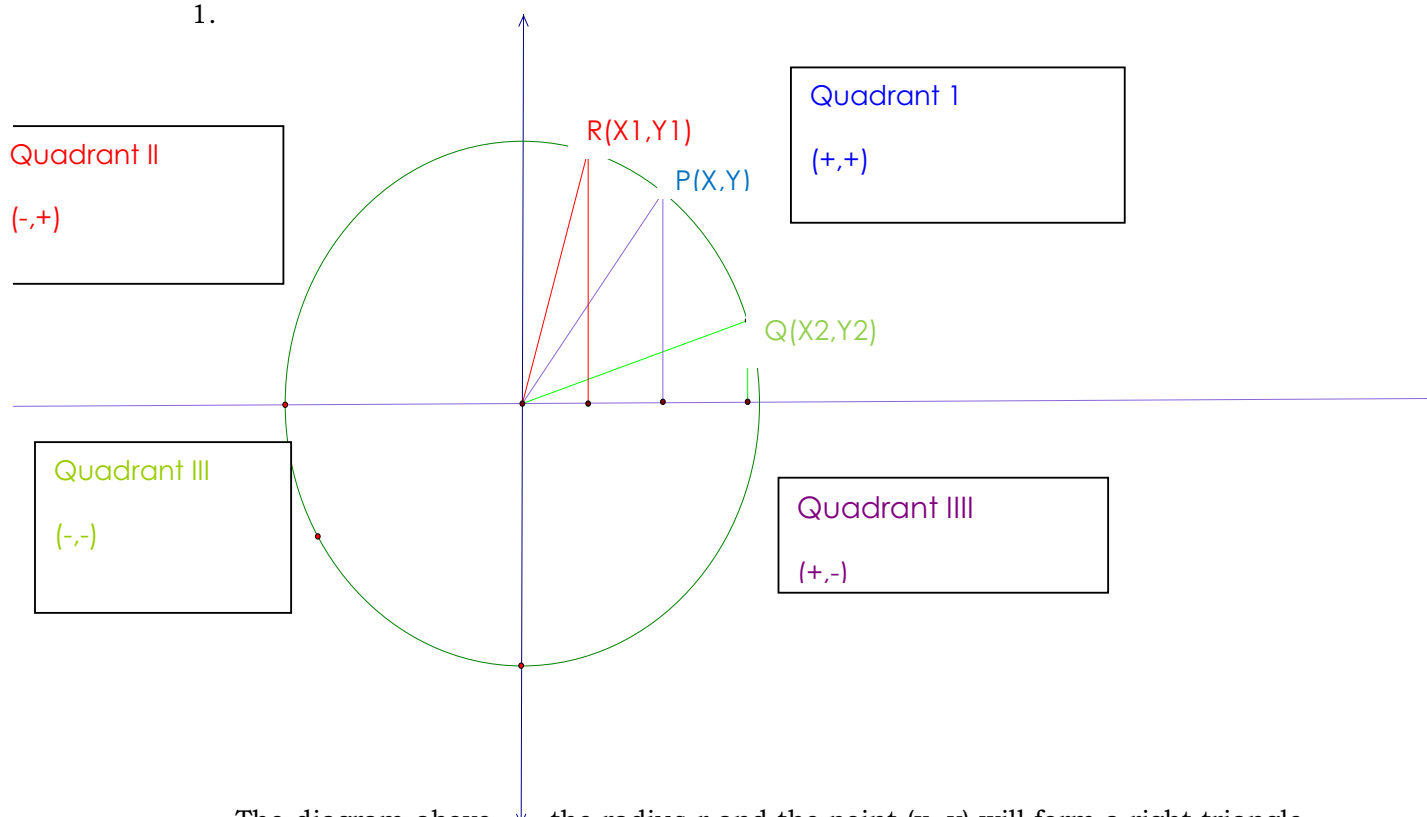


Math Honors 1: Trigonometry Investigation Task

This investigation will present an analysis on initial problem by setting patterns and establishing mathematical relationships between the parameters in the problem. In this specific investigation, I will find to see the relationship between radius R and point X and Y in a coordinate plane. The center of the circle will be (0, 0) or the origin and the radius R will be unknown. Point P with the coordinate (x,y) will always be on the circumference of the circle, and will always be perpendicular from the X axis to the point.

Part A: Circle Trigonometry

1.



The diagram above, the radius r and the point (x, y) will form a right triangle. Therefore we can state that the equation to find the relationship can be described as Pythagorean Theorem. Wherever the coordinates (x, y) lies on the circle it will always form right triangle.

Therefore we can use the equation distance between the two points.

$$d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$$

After we can state that the endpoints of the radius are set in the origin meaning that one end point will always set in the origin (0, 0). So we can simplify our equation further more.

$$r^2 = (0 - x)^2 + (0 - y)^2$$

Then after squaring this will result in Pythagorean Theorem

$$r^2 = x^2 + y^2$$

The diagram above with point R and Q is just to back up or example the idea of relationship use of right triangle to use the equation of Pythagorean Theorem. As a result, due to Pythagorean Theorem, r squared will always be positive. And by using the relation equation the x and y value can be positive and negative thus 0. Negative and positive values meaning that they can be in any exact quadrants but one endpoint being at origin creating a right triangle. So we can see that the radius can't ever be zero or negative it should always be positive or in other words real number. By using theta, if it goes counter clock wise it will become positive while going in the direction of clockwise it becomes negative. Further more, based on our knowledge on Math Honor 1, we always know that hypotenuse in right triangles are the longest of among three sides. And thus since R is the hypotenuse, as explained before x and y can be any number as long as it is less than the radius. Therefore leading radius R always positive and real number. Thus as logical expression the circle's radius can't be represented as a negative value. Plus Radius is fixed value.

Since we are working on counter two revolutions- counter clockwise, clock wise on the circle, positive angles are measured in a counter clock wise direction and negative angles are measure in clockwise direction. First, for counter clock wise direction where the theta is positive the 0° is on the positive x axis. Then theta will be the only constraint which is $0^\circ \leq \theta \leq 360^\circ$.

Used by Excel 2007, the table of values of theta in comparing in theta's angle and its value during sin, cos and tan. The table consists of Sin θ , Cos θ , and tan θ for 42 values of θ . Range of $-360^\circ \leq \theta \leq 360^\circ$. By the table we can see the range of sin, cos, and tan- $-1 \leq \sin \theta \leq 1$, $-1 \leq \cos \theta \leq 1$ Tan θ doesn't have definite range.

Table of **values of θ within the range of $-360^\circ \leq \theta \leq 360^\circ$**

| θ | $\sin \theta$ | $\cos \theta$ | $\tan \theta$ |
|----------|---------------|---------------|---------------|
| -360 | 2.4503E-16 | 1 | 2.4503E-16 |
| -340 | 0.342020143 | 0.939692621 | 0.363970234 |
| -320 | 0.64278761 | 0.766044443 | 0.839099631 |
| -300 | 0.866025404 | 0.5 | 1.732050808 |
| -280 | 0.984807753 | 0.173648178 | 5.67128182 |
| -270 | 1 | -1.83772E-16 | -5.44152E+15 |
| -260 | 0.984807753 | -0.173648178 | -5.67128182 |

| | | | |
|------|--------------|--------------|--------------|
| -240 | 0.866025404 | -0.5 | -1.732050808 |
| -220 | 0.64278761 | -0.766044443 | -0.839099631 |
| -200 | 0.342020143 | -0.939692621 | -0.363970234 |
| -180 | -1.22515E-16 | -1 | 1.22515E-16 |
| -160 | -0.342020143 | -0.939692621 | 0.363970234 |
| -140 | -0.64278761 | -0.766044443 | 0.839099631 |
| -120 | -0.866025404 | -0.5 | 1.732050808 |
| -100 | -0.984807753 | -0.173648178 | 5.67128182 |
| -90 | -1 | 6.12574E-17 | -1.63246E+16 |
| -80 | -0.984807753 | 0.173648178 | -5.67128182 |
| -60 | -0.866025404 | 0.5 | -1.732050808 |
| -40 | -0.64278761 | 0.766044443 | -0.839099631 |
| -20 | -0.342020143 | 0.939692621 | -0.363970234 |
| 0 | 0 | 1 | 0 |
| 20 | 0.342020143 | 0.939692621 | 0.363970234 |
| 40 | 0.64278761 | 0.766044443 | 0.839099631 |
| 60 | 0.866025404 | 0.5 | 1.732050808 |
| 80 | 0.984807753 | 0.173648178 | 5.67128182 |
| 90 | 1 | 6.12574E-17 | 1.63246E+16 |
| 100 | 0.984807753 | -0.173648178 | -5.67128182 |
| 120 | 0.866025404 | -0.5 | -1.732050808 |
| 140 | 0.64278761 | -0.766044443 | -0.839099631 |
| 160 | 0.342020143 | -0.939692621 | -0.363970234 |
| 180 | 1.22515E-16 | -1 | -1.22515E-16 |
| 200 | -0.342020143 | -0.939692621 | 0.363970234 |
| 220 | -0.64278761 | -0.766044443 | 0.839099631 |
| 240 | -0.866025404 | -0.5 | 1.732050808 |

| | | | |
|-----|--------------|--------------|--------------|
| 260 | -0.984807753 | -0.173648178 | 5.67128182 |
| 270 | -1 | -1.83772E-16 | 5.44152E+15 |
| 280 | -0.984807753 | 0.173648178 | -5.67128182 |
| 300 | -0.866025404 | 0.5 | -1.732050808 |
| 320 | -0.64278761 | 0.766044443 | -0.839099631 |
| 340 | -0.342020143 | 0.939692621 | -0.363970234 |
| 360 | -2.4503E-16 | 1 | -2.4503E-16 |

The number with E is error made by MS Excel calculations.

Highlighted means 0 and number with E will be just defined as undefined values.

This later was verified by TI-83 plus calculator.

Once again, upon the analysis of the above table of values, the values of $\sin\theta$ are found to have a specific range; no smaller than -1 and no bigger than 1 ($-1 \leq \sin\theta \leq 1$). The values of $\cos\theta$ have a specific range also; no smaller than -1 and no bigger than 1 ($-1 \leq \cos\theta \leq 1$). On the other hand, the values of $\tan\theta$ don't have definite range.

Four Quadrants-

Sine of theta in counter clockwise form

The value of Sin θ of range of $0 < \theta < 90$ quadrant 1, $90 < \theta < 180$ quadrant 2 are positive and $180 < \theta < 270$ quadrant 3, $270 < \theta < 360$ quadrant 4 are negative.

Sine of theta in clockwise form

The value of Sin θ of range of $0 < \theta < -90$ quadrant 4, $-90 < \theta < -180$ quadrant 3 are negative and $-180 < \theta < -270$ quadrant 2, $-270 < \theta < -360$ quadrant 1 are positive.

Cosine of theta in counter-clockwise form

The value of Cosine of quadrant 1 $0 < \theta < 90$, quadrant 4 $270 < \theta < 360$ are positive and quadrant 2 $90 < \theta < 180$, quadrant 3 which is $180 < \theta < 270$ are negative.

Cosine of theta in clockwise

The value of Cosine of quadrant 4 $0 < \theta < -90$, quadrant 1 $-270 < \theta < -360$ are positive and quadrant 3 $-90 < \theta < -180$, quadrant 2 which is $-180 < \theta < -270$ are negative.

Tangent of theta in counter clockwise form

The value of Tangent θ of quadrant 1 $0 < \theta < 90$ and quadrant 3 which is $180 < \theta < 270$ are positive range and quadrant 2 $90 < \theta < 180$, quadrant 4 $270 < \theta < 360$ are negative range.

Tangent of theta in clockwise form

The value of Tangent θ of quadrant 4 $0 < \theta < -90$ and quadrant 2 which is $-180 < \theta < -270$ are negative range and quadrant 3 $-90 < \theta < -180$, quadrant 1 $-270 < \theta < -360$ are positive range.

Axes:

The positive x axis of $-360, 0, 360$ degree the value of $\cos \theta = 1$, $\sin \theta = 0$, and $\tan \theta = 0$.

The positive y axis of -270 and 90 degree the value of $\sin \theta = 1$, $\cos \theta = 0$ and $\tan \theta$ is undefined.

The negative x axis of -180 and 180 degree the value of $\cos \theta = -1$, $\sin \theta = 0$, and $\tan \theta = 0$.

The negative y axis of $-90, 270$ degree the value of $\sin \theta = -1$, $\cos \theta = 0$ and $\tan \theta = \text{undefined}$.

To prove my angles, each sign in each quadrants, I would put any random values for theta for each right values of quadrants due to their ranges in counter clockwise and clockwise.

Counter clockwise

Just for trial, we will put a random angle for quadrant 1 which the range is $0 < \theta < 90$, in there we would put any number between 0 and 90 degrees. Such as 52, to verify the conjecture, the value of sin, cos and tan turned out to be positive. We have to remember that before using the TI-83 calculator, we should turn the mode to degree mode.

```
sin(52)
.7880107536
cos(52)
.6156614753
tan(52)
1.279941632
■
```

Now for quadrant 2, when we put a random angle from quadrant 2, the range of $90 < \theta < 180$, 174° in able to verify for the conjecture, the value of sin turn out to be positive while the values of cos and tan turn out to be negative.

```
sin(174)
    .1045284633
cos(174)
    -.9945218954
tan(174)
    -.1051042353
■
```

Next, when we put a random angle from quadrant 4, the range of $180 < \theta < 270$, I would put 196 degree, the value of tan turn out to be positive while the values of sin and cos turn out to be negative.

```
sin(196)
    -.2756373558
cos(196)
    -.9612616959
tan(196)
    .2867453858
```

When we put a random angle from quadrant 4, the range of $270 < \theta < 360$, 336° in trial to verify the conjecture, the value of cos turn out to be positive while the values of sin and tan turn out to be negative.

```
sin(336)
    -.4067366431
cos(336)
    .9135454576
tan(336)
    -.4452286853
■
```

Now going for clockwise

When we put a random angle from quadrant 1, the range of $-360 < \theta < -270$, -278° in trial to verify the conjecture, the values of sin, cos and tan turn out to be positive.

```
sin(-278)
    .9902680687
cos(-278)
    .139173101
tan(-278)
    7.115369722
■
```

When we put a random angle from quadrant 2, the range of $-270 < \theta < -180$, -204° in trial to verify the conjecture, the value of sin turn out to be positive while the values of cos and tan turn out to be negative.

```
sin(-204)
    .4067366431
cos(-204)
    -.9135454576
tan(-204)
    -.4452286853
■
```

When we put a random angle from quadrant 3, the range of $-180 < \theta < -90$, -164° in trial to verify the conjecture, the value of tan turn out to be positive while the values of sin and cos turn out to be negative.

```
sin(-164)
    -.2756373558
cos(-164)
    -.9612616959
tan(-164)
    .2867453858
■
```

When we put a random angle from quadrant 4, the range of $-90 < \theta < 0$, -7° in trial to verify the conjecture, the value of cos turns out to be positive and the values of sin and tan turn out to be negative.

```
sin(-7)
    -.1218693434
cos(-7)
    .9925461516
tan(-7)
    -.1227845609
```

Sin θ = opposite/ hypotenuse = O/H = y/r

Cos θ = adjacent/ hypotenuse = A/H = x/r

Tan θ = opposite/ adjacent = O/A = y/x

Since r should always be positive as what I said in the top. Also by this equation, this is also the another step to find out if it is positive or negative.

Quadrant 1—(+,+) sin θ = y/r = +

cos θ = x/r = +

tan θ = y/x = +

Quadrant I

$$\sin \theta = \frac{y}{r}$$

The value of y equals a positive number in the quadrant 1 and the value of r equals a positive number as mentioned beforehand. Likewise, when the value of y is divided by the value of r, a positive number is divided by a positive number resulting to a positive number.

$$\cos \theta = \frac{x}{r}$$

The value of x equals a positive number in the quadrant 1 and the value of r equals a positive number as mentioned beforehand. Likewise, when the value of x is divided by the value of r, a positive number is divided by a positive number resulting to a positive number.

$$\tan \theta = \frac{y}{x}$$

The value of y and the value of x equal to positive numbers respectively in quadrant 1. When the value of y is divided by the value of x, a positive number is divided by a positive number resulting to a positive number.

Quadrant 2—(-,+) $\sin \theta = y/r = -$

$$\cos \theta = x/r = -1$$

$$\tan \theta = y/x = -$$

Quadrant II

$$\sin \theta = \frac{y}{r}$$

The value of y equals a positive number in the quadrant 2 and the value of r equals a positive number as mentioned beforehand. Likewise, when the value of y is divided by the value of r, a positive number is divided by a positive number resulting to a positive number.

$$\cos \theta = \frac{x}{r}$$

The value of x equals a negative number in the quadrant 2 and the value of r equals a positive number as mentioned beforehand. Likewise, when the value of x is divided by the value of r, a negative number is divided by a positive number resulting to a negative number.

$$\tan \theta = \frac{y}{x}$$

The value of y equals a positive number and the value of x equals to a negative number in quadrant 2. When the value of y is divided by the value of x, a positive number is divided by a negative number resulting to a negative number.

Quadrant 3—(-,-) $\sin \theta = y/r = -$

$$\cos \theta = x/r = -$$

Quadrant III

$$\sin \theta = \frac{y}{r}$$

The value of y equals a negative number in the quadrant 3 and the value of r equals a positive number as mentioned beforehand. Likewise, when the value of y is divided by the value of r, a negative number is divided by a positive number resulting to a negative number.

$$\cos \theta = \frac{x}{r}$$

The value of x equals a negative number in the quadrant 3 and the value of r equals a positive number as mentioned beforehand. Likewise, when the value of x is divided by the value of r, a negative number is divided by a positive number resulting to a negative number.

$$\tan \theta = \frac{y}{x}$$

The value of y and the value of x equal to a negative number respectively in quadrant 3. When the value of y is divided by the value of x, a negative number is divided by a negative number resulting to a positive number.

Quadrant IV

$$\sin \theta = \frac{y}{r}$$

The value of y equals a negative number in the quadrant 4 and the value of r equals a positive number as mentioned beforehand. Likewise, when the value of y is divided by the value of r, a negative number is divided by a positive number resulting to a negative number.

$$\cos \theta = \frac{x}{r}$$

The value of x equals a positive number in the quadrant 4 and the value of r equals a positive number as mentioned beforehand. Likewise, when the value of x is divided by the value of r, a positive number is divided by a positive number resulting to a positive number.

$$\tan \theta = \frac{y}{x}$$

The value of y equals a negative number and the value of x equals a positive number in quadrant 4. When the value of y is divided by the value of x, a negative number is divided by a positive number resulting to a negative number.

Axis: The signs of the coordinates on the positive x axis are (positive,0); the value of x equals a positive number and the value of y equals 0.

Positive x axis--- (+, 0) $\sin \theta = y/r = +$

$$\cos \theta = x/r = 0$$

$$\tan \theta = y/x = 0$$

Negative x axis---(-,0) $\sin \theta = y/r = -$

$$\cos \theta = x/r = 0$$

$$\tan \theta = y/x = 0$$

Positive y axis---(0,+) $\sin \theta = y/r = +$

$$\cos \theta = x/r = 0$$

$$\tan \theta = y/x = \text{undefined}$$

Negative y axis---(0,-) $\sin \theta = y/r = -$

$$\cos \theta = x/r = 0$$

$$\tan \theta = y/x = \text{undefined}$$

$$\cos \theta = \frac{x}{r}$$

The value of x equals a positive number on the positive x axis and the value of r equals a positive number as mentioned beforehand. Therefore, when the value of x is divided by the value of r, a positive number is divided by a positive number resulting to a positive number.

$$\sin \theta = \frac{y}{r}$$

The value of y equals 0 on the positive x axis and the value of r equals a positive number as mentioned beforehand. Therefore, when the value of y is divided by the value of r, 0 is divided by a positive number resulting to 0.

$$\tan \theta = \frac{y}{x}$$

The value of y equals 0 and the value of x equals a positive number on the positive x axis. Therefore, when the value of y is divided by the value of x, 0 is divided by a positive number, resulting to 0.

The signs of the coordinates on the negative x axis are (negative,0); the value of x equals a negative number and the value of y equals 0.

$$\cos \theta = \frac{x}{r}$$

The value of x equals a negative number on the negative x axis and the value of r equals a positive number as mentioned beforehand. Therefore, when the value of x is divided by the value of r, a negative number is divided by a positive number resulting to a negative number.

$$\sin \theta = \frac{y}{r}$$

The value of y equals 0 on the negative x axis and the value of r equals a positive number as mentioned beforehand. Therefore, when the value of y is divided by the value of r, 0 is divided by a positive number resulting to 0.

$$\tan \theta = \frac{y}{x}$$

The value of y equals 0 and the value of x equals a positive number on the negative x axis. Therefore, when the value of y is divided by the value of x, 0 is divided by a positive number, resulting to 0.

The signs of the coordinates on the positive y axis are (0,positive); the value of x equals 0 and the value of y equals a positive number.

$$\sin \theta = \frac{y}{r}$$

The value of y equals a positive number on the positive y axis and the value of r equals a positive number as mentioned beforehand. Therefore, when the value of y is divided by the value of r, a positive number is divided by a positive number resulting to a positive number.

$$\cos \theta = \frac{x}{r}$$

The value of x equals 0 on the positive y axis and the value of r equals a positive number as mentioned beforehand. Therefore, when the value of x is divided by the value of r, 0 is divided by a positive number resulting to 0.

$$\tan \theta = \frac{y}{x}$$

The value of y equals a positive number and the value of x equals 0 on the positive y axis. Therefore, when the value of y is divided by the value of x, a positive number is divided by 0, resulting to undefined value.

The signs of the coordinates on the negative y axis are (0, negative); the value of x equals 0 and the value of y equals a negative number.

$$\sin \theta = \frac{y}{r}$$

The value of y equals a negative number on the negative y axis and the value of r equals a positive number as mentioned beforehand. Therefore, when the value of y is divided by the value of r, a negative number is divided by a positive number resulting to a negative number.

$$\cos \theta = \frac{x}{r}$$

The value of x equals 0 on the negative y axis and the value of r equals a positive number as mentioned beforehand. Therefore, when the value of x is divided by the value of r, 0 is divided by a positive number resulting to 0.

$$\tan \theta = \frac{y}{x}$$

The value of y equals a negative number and the value of x equals 0 on the negative y axis. Therefore, when the value of y is divided by the value of x, a positive number is divided by 0, resulting to undefined value.

The conjecture informally by considering further examples of $\sin \theta$, $\cos \theta$, and $\tan \theta$ that are not in the table of values, in quadrant 1 that have not been mentioned in the table of values are positive, all in Quadrant 2 only have the sine value as positive, all in Quadrant 3 have only the tangent value as positive, and in Quadrant 4 only the cosine value is positive.

$$\sin \theta = \text{opposite} / \text{hypotenuse} = y/r$$

$$\cos \theta = \text{adjacent} / \text{hypotenuse} = x/r$$

$$\tan \theta = \text{opposite} / \text{adjacent} = y/x$$

Since r should always be positive as what I said in the top.

Getting the sine value for theta, the length of the side opposite to theta is divided by the length of the hypotenuse or opposite side of theta divided by hypotenuse which is r. Thus, in this investigation, $\sin = \frac{y}{r}$. Similarly, getting the cosine of any value of θ , wherein the length of the adjacent side to θ is divided by the length of the hypotenuse, would be $\cos = \frac{x}{r}$. Attaining the tangent, in which the length of the opposite side is divided by the length of the side adjacent to θ , would have the equation $\tan = \frac{y}{x}$.

Now by step by step, the conjecture will be described in each step.

First, in first quadrant where $\sin = \frac{y}{r}$ (0° to 90° , as well as -270° to -360°) all values of y are positive, radius which is always positive or in other words real number, dividing the y value or opposite of that will always give positive quotient.

Since cosine is also x/r or adjacent divided by hypotenuse R. It will become positive again which the same situation of $\sin \theta$.

In first Quadrant Tangent opposite/hypotenuse will also be positive in quadrant 1 ($\tan = \frac{y}{x}$).

In Quadrant 2, the y value is negative or meaning that $\sin \theta$ is negative, but radius and x remaining positive. Influencing the sine and tangent value of θ . However by the y value being negative it will not only affect $\sin \theta$ but also $\tan \theta = y/x$, as a negative number divided by a positive results in a negative answer.

As it can be seen in the diagram, all x and y values in Quadrant 3 are negative, though the r value remains positive as it is a radius of a circle, which cannot be negative. As all three formulas for calculating the sine, cosine and tangent values involve x and y, all are affected. It is given that $\sin = \frac{y}{r}$ and $\cos = \frac{x}{r}$. Because the negative values of x and y are being divided by the positive value of r, the result will always be negative.

However, in the tangent formula, wherein $\tan = \frac{y}{x}$, a negative value is being divided by another negative value, which will in turn give us a positive quotient.

In Quadrant 4, as can be referred to in the diagram on the right, only the x value is negative, whilst the values for y and r are positive. This in turn affects the formulas for attaining the cosine and tangent of θ , as they are the formulas which involve the x value. In $\cos = \frac{x}{r}$ and $\tan = \frac{y}{x}$, the negative x value is either being divided by or is dividing a positive value, which would result in a negative quotient. However, because $\sin = \frac{y}{r}$ and both y and r values are in this case positive, the sine of θ in Quadrant 4 will always be positive.

Part B: Trigonometric Identities

r

Upon the analysis of the table of values from Part A, we can conjecture the relationship among $\sin \theta$, $\cos \theta$, and $\tan \theta$ for any angle θ .

| θ | $\sin \theta$ | $\cos \theta$ | $\tan \theta$ | $\sin \theta / \cos \theta$ |
|----------|---------------|---------------|---------------|-----------------------------|
| -360 | 2.4503E-16 | 1 | 2.4503E-16 | 2.4503E-16 |
| -340 | 0.34202014 | 0.939693 | 0.36397023 | 0.36397023 |
| -320 | 0.64278761 | 0.766044 | 0.83909963 | 0.83909963 |
| -300 | 0.8660254 | 0.5 | 1.7320508 | 1.7320508 |

| | | | | | |
|------|----------------|--------------|----------------|-----------------|--|
| -280 | 0.9848077 5 | 0.17364 8 | 5.6712818 2 | 5.67128182 | |
| -270 | 1 | -1.8E-16 | undefined | 5.4415E+15 | |
| -260 | 0.9848077 5 | - 0.17365 | 5.6712818 2 | 5.67128182 | |
| -240 | 0.8660254 | -0.5 | 1.7320508 | 1.73205081 | |
| -220 | 0.6427876 1 | - 0.76604 | 0.8390996 3 | 0.83909963 | |
| -360 | 2.4503E- 16 | 1 | 2.4503E- 16 | 2.4503E-16 | |
| -200 | 0.3420201 4 | - 0.93969 | 0.3639702 3 | 0.36397023 | |
| -180 | -1.225E-16 | -1 | 1.2251E- 16 | 1.22515E- 16 | |
| -160 | - 0.3420201 | - 0.93969 | 0.3639702 3 | 0.36397023 4 | |
| -140 | - 0.6427876 | - 0.76604 | 0.8390996 3 | 0.83909963 1 | |
| -120 | - 0.8660254 | -0.5 | 1.7320508 1 | 1.73205080 8 | |
| -100 | - 0.9848078 | - 0.17365 | 5.6712818 2 | 5.67128182 | |
| -90 | -1 | 6.13E- 17 | undefined | 1.6325E+16 | |
| -80 | - 0.9848078 | 0.17364 8 | 5.6712818 2 | 5.67128182 | |
| -60 | - 0.8660254 | 0.5 | 1.7320508 | 1.73205081 | |
| -40 | - 0.6427876 | 0.76604 4 | 0.8390996 3 | 0.83909963 1 | |
| -20 | - 0.3420201 | 0.93969 3 | 0.3639702 3 | 0.36397023 4 | |
| 0 | 0 | 1 | 0 | 0 | |
| 20 | 0.3420201 4 | 0.93969 3 | 0.3639702 3 | 0.36397023 4 | |
| 40 | 0.6427876 1 | 0.76604 4 | 0.8390996 3 | 0.83909963 1 | |
| 60 | 0.8660254 | 0.5 | 1.7320508 1 | 1.73205080 8 | |
| 80 | 0.9848077 5 | 0.17364 8 | 5.6712818 2 | 5.67128182 | |
| 90 | 1 | 6.13E- 17 | undefined | 1.63246E+1 6 | |
| 100 | 0.9848077 5 | - 0.17365 | 5.6712818 2 | 5.67128182 | |
| 120 | 0.8660254 | -0.5 | | | |

| | | | | | | |
|------------|------------|----------|------------|-------------|--|--|
| | | | 1.7320508 | 1.73205081 | | |
| 140 | 0.6427876 | - | | | | |
| | 1 | 0.76604 | 0.8390996 | 0.83909963 | | |
| 160 | 0.3420201 | - | | | | |
| | 4 | 0.93969 | 0.3639702 | 0.36397023 | | |
| 180 | 1.2251E-16 | -1 | -1.225E-16 | -1.2251E-16 | | |
| 200 | - | - | 0.3639702 | 0.36397023 | | |
| | 0.3420201 | 0.93969 | 3 | 4 | | |
| 220 | - | - | 0.8390996 | 0.83909963 | | |
| | 0.6427876 | 0.76604 | 3 | 1 | | |
| 240 | - | -0.5 | 1.7320508 | 1.73205080 | | |
| | 0.8660254 | | 1 | 8 | | |
| 260 | - | - | 5.6712818 | 5.67128182 | | |
| | 0.9848078 | 0.17365 | 2 | | | |
| 270 | -1 | -1.8E-16 | undefined | 5.44152E+1 | | |
| | | | | 5 | | |
| 280 | - | 0.17364 | | | | |
| | 0.9848078 | 8 | 5.6712818 | 5.67128182 | | |
| 300 | - | 0.5 | | | | |
| | 0.8660254 | | 1.7320508 | 1.73205081 | | |
| 320 | - | 0.76604 | | | | |
| | 0.6427876 | 4 | 0.8390996 | 0.83909963 | | |
| 340 | - | 0.93969 | | | | |
| | 0.3420201 | 3 | 0.3639702 | 0.36397023 | | |
| 360 | -2.45E-16 | 1 | -2.45E-16 | -2.4503E-16 | | |

As can be clearly seen in the table above, the values for $\tan\theta$ and $\frac{\sin\theta}{\cos\theta}$ are identical.

Further to prove this, I used TI 83 calculator for better explanation such as random numbers.

First for quadrant 1, I will choose a value or a degree for tangent theta between 0-90 degrees and -360 to -270. Which are 76 and -298 degrees.

```
tan(76)
.6867476894
sin(76)/cos(76)
.6867476894
```

```
tan(-298)
.4846620011
sin(-298)/cos(-298)
.4846620011
```

For other revolutions

For quadrant two the range between 90 to 180 and -270 to -180

With the example of 127 and -222

| | |
|--------------------------------|----------------------------------|
| <code>tan(127)</code> | <code>tan(-222)</code> |
| 4.185891832 | 1.755816573 |
| <code>sin(127)/cos(127)</code> | <code>sin(-222)/cos(-222)</code> |
| 4.185891832 | 1.755816573 |

For quadrant 3, the range of tan theta will be 180-270 and -180 to -90.

With the example of 199 and -101

| | |
|--------------------------------|----------------------------------|
| <code>tan(199)</code> | <code>tan(-101)</code> |
| 1.869700512 | 5.144554016 |
| <code>sin(199)/cos(199)</code> | <code>sin(-101)/cos(-101)</code> |
| 1.869700512 | 5.144554016 |

Finall for quadrant four with the range of 270 to 360 and -90 to 0

With the example of 354 and -4 degrees.

| | |
|--------------------------------|------------------------------|
| <code>tan(354)</code> | <code>tan(-4)</code> |
| -.1051042353 | -.0699268119 |
| <code>sin(354)/cos(354)</code> | <code>sin(-4)/cos(-4)</code> |
| -.1051042353 | -.0699268119 |

Therefore, through this further examples we again can see that the conjecture between $\tan\theta$ and $\frac{\sin\theta}{\cos\theta}$ **are identical.**

a) $\cos\theta = x/r$

$\sin\theta = y/r$

$\tan\theta = y/x$

$\sin \theta / \cos \theta = (y/r) / (x/r)$ so y/r divided by x/r equals y/r times r/x . Which r gets simplified equaling y/x . So if we see \tan it will become y/x which means $\sin \theta / \cos \theta$ equals to $\tan \theta$.

Equation:

$$\frac{\sin \theta}{\cos \theta} = \tan \theta$$

$$\frac{\frac{y}{r}}{\frac{x}{r}} = \frac{y}{x}$$

$$\left(\frac{y}{r} \right) \left(\frac{r}{x} \right) = \frac{y}{x}$$

$$\frac{y}{x} \frac{r}{r} = \frac{y}{x}$$

$$\frac{y}{x} = \frac{y}{x}$$

b) Expressing $\cos \theta$, $\tan \theta$ and $\sin \theta$ in terms of x , y and r

then for x value it will be $x = \cos \theta (r)$, y value $y = \sin \theta (r)$ and \tan equaling $\tan \theta = y/x$ equaling $\sin \theta (r)$ divided by $\cos \theta (r)$ then as what I said in the above, the r gets simplified just leaving.

$$y = \sin \theta (r)$$

$\tan \theta = y/x$ equaling $\sin \theta (r)$ divided by $\cos \theta (r)$ then as what I said in the above, the r gets simplified just leaving

$\sin^2 \theta + \cos^2 \theta$, the relationship between $\sin^2 \theta$ and $\cos^2 \theta$ is whenever I add them $\sin^2 \theta + \cos^2 \theta$ this becomes one and even though whatever angle we put for θ the sum $\sin^2 \theta + \cos^2 \theta$ will always be 1.

| θ | $\sin^2 \theta$ | $\cos^2 \theta$ | $\sin^2 \theta + \cos^2 \theta$ |
|----------|-----------------|-----------------|---------------------------------|
| -360 | 6.00395E-32 | 1 | 1 |

| | | | |
|------|-------------|----------|---|
| -340 | 0.116977778 | 0.883022 | 1 |
| -320 | 0.413175911 | 0.586824 | 1 |
| -300 | 0.75 | 0.25 | 1 |
| -280 | 0.96984631 | 0.030154 | 1 |
| -270 | 1 | 3.38E-32 | 1 |
| -260 | 0.96984631 | 0.030154 | 1 |
| -240 | 0.75 | 0.25 | 1 |
| -220 | 0.413175911 | 0.586824 | 1 |
| -360 | 6.00395E-32 | 1 | 1 |
| -200 | 0.116977778 | 0.883022 | 1 |
| -180 | 1.50099E-32 | 1 | 1 |
| -160 | 0.116977778 | 0.883022 | 1 |
| -140 | 0.413175911 | 0.586824 | 1 |
| -120 | 0.75 | 0.25 | 1 |
| -100 | 0.96984631 | 0.030154 | 1 |
| -90 | 1 | 3.75E-33 | 1 |
| -80 | 0.96984631 | 0.030154 | 1 |
| -60 | 0.75 | 0.25 | 1 |
| -40 | 0.413175911 | 0.586824 | 1 |
| -20 | 0.116977778 | 0.883022 | 1 |
| 0 | 0 | 1 | 1 |
| 20 | 0.116977778 | 0.883022 | 1 |
| 40 | 0.413175911 | 0.586824 | 1 |
| 60 | 0.75 | 0.25 | 1 |
| 80 | 0.96984631 | 0.030154 | 1 |
| 90 | 1 | 3.75E-33 | 1 |
| 100 | 0.96984631 | 0.030154 | 1 |
| 120 | 0.75 | 0.25 | 1 |
| 140 | 0.413175911 | 0.586824 | 1 |
| 160 | 0.116977778 | 0.883022 | 1 |
| 180 | 1.50099E-32 | 1 | 1 |
| 200 | 0.116977778 | 0.883022 | 1 |
| 220 | 0.413175911 | 0.586824 | 1 |
| 240 | 0.75 | 0.25 | 1 |
| 260 | 0.96984631 | 0.030154 | 1 |
| 270 | 1 | 3.38E-32 | 1 |
| 280 | 0.96984631 | 0.030154 | 1 |
| 300 | 0.75 | 0.25 | 1 |
| 320 | 0.413175911 | 0.586824 | 1 |
| 340 | 0.116977778 | 0.883022 | 1 |
| 360 | 6.00395E-32 | 1 | 1 |

In Part A when expressing $\sin \theta$, $\cos \theta$ and $\tan \theta$ in terms of x , y and r . Which $\sin \theta = y/r$, $\cos \theta = x/r$ and $\cos^2 \theta = x^2/r^2$ so when squaring cosine theta and squaring sine theta $\sin^2 \theta$ will equal y^2/r^2 after this when adding $\sin^2 \theta + \cos^2 \theta$ this will equal $x^2/r^2 + y^2/r^2$ which the sum is $x^2 + y^2/r^2$ then since $x^2 + y^2$ is r^2 we will substitute r^2 in the equation of $x^2 + y^2/r^2$ giving r^2/r^2 simplifying to 1. As a result $x^2 + y^2 = r^2$ so in number 1 Part A

$$\sin^2 \theta + \cos^2 \theta = 1.$$

The value of $\sin \theta$ and $\cos \theta$ in the first quadrant $0 \leq \theta \leq 90$. The value of sine theta equals the value of y divided by the value of r and the value of cosine theta equals the value of x divided by the value of r . Therefore, the square of the value of sine theta equals the square of the value of y divided by the square of the value of r and the square of the value of cosine theta equals the square of the value of x divided by the square of the value of r . When the square of the value of cosine theta is added to the square of the value of sine theta, the result of the square of the value of x divided by the square of the value of r is added to the result of the square of the value of y divided by the square of the value of r .

$$\sin \theta = \frac{y}{r}$$

$$\cos \theta = \frac{x}{r}$$

$$\sin^2 \theta = \frac{y^2}{r^2}$$

$$\cos^2 \theta = \frac{x^2}{r^2}$$

$$\cos^2 \theta + \sin^2 \theta$$

$$= \left(\frac{x^2}{r^2} \right) + \left(\frac{y^2}{r^2} \right)$$

$$= \frac{(x^2 + y^2)}{r^2}$$

$$= \frac{r^2}{r^2}$$

$$= 1$$

The values of $\sin\theta$ and $\cos\theta$ within the range of $0^\circ \leq \theta \leq 90^\circ$ are analyzed in order to conjecture another relationship between $\sin\theta$ and $\cos\theta$ for any angle θ .

A portion of Table 1 within the range of $0^\circ \leq \theta \leq 90^\circ$ showing the relationship between $\sin\theta$ and $\cos\theta$ for any angle θ

| θ | $\sin\theta$ | $\cos\theta$ |
|----------|--------------|--------------|
| 0 | | 1 |
| 10 | 0.17364818 | 0.984808 |
| 20 | 0.34202014 | 0.939693 |
| 30 | 0.5 | 0.866025 |
| 40 | 0.64278761 | 0.766044 |
| 50 | 0.76604444 | 0.642788 |
| 60 | 0.8660254 | 0.5 |
| 70 | 0.93969262 | 0.34202 |
| 80 | 0.98480775 | 0.173648 |
| 90 | 1 | 0 |

The colors matching, by twos are meaning that they share the same values. Based on this table we can see that there is relation between $\sin\theta$ and $\cos\theta$ from 0 to 90 degree. The example of $\sin 30$ equals to $\cos 60$ which the relation is that the theta is same. So the value of $\sin\theta$ is equal to $\cos(90-\theta)$ then next step if we determine $\sin x = \cos y$. Then $x+y$ will result 90 degree this tells us that it leads to complementary angle. As shown in the colored portions of the table above, $\sin(0)$ equals $\cos(90)$, $\sin(10)$ equals $\cos(80)$, $\sin(20)$ equals $\cos(70)$, $\sin(30)$ equals $\cos(60)$, $\sin(40)$ equals $\cos(50)$, $\sin(50)$ equals $\cos(40)$, $\sin(60)$ equals $\cos(30)$, $\sin(70)$ equals $\cos(20)$, $\sin(80)$ equals $\cos(10)$, and $\sin(90)$ equals $\cos(0)$.

Therefore, when the angle of sine and the angle of cosine are summed up, it is equal to 90; complementary.

$$\sin x = \cos y$$

$$x + y = 90$$

Thus, the value of sine theta is equal to the value of cosine 90 minus theta.

$$\sin \theta = \cos(90 - \theta)$$

Again to verify the conjecture, two random angles that are not already in the table of values are tested from the first quadrant in the range of $0^\circ \leq \theta \leq 90^\circ$.

When 36 is to represent the value of x and 54 is to represent the value of y in the conjecture $\sin x = \cos y$, $\sin(36)$ would equal to $\sin(54)$ Again this should add up to 90 degrees.

$$\sin x = \cos y$$

$$\sin(36) = \cos(54)$$

$$\sin \theta = \cos(90 - \theta)$$

$$\sin(36) = \cos(90 - 36)$$

$$0.5878 = 0.5878$$

When 13 is to represent the value of x and 77 is to represent the value of y in the conjecture $\sin x = \cos y$, $\sin(13)$ would equal to $\cos(77)$. Again this should add up to 90 degrees.

$$\sin x = \cos y$$

$$\sin(13) = \cos(77)$$

$$\sin \theta = \cos(90 - \theta)$$

$$\sin(13) = \cos(90 - 13)$$

$$0.2250 = 0.2250$$

When 54 is to represent the value of x and 36 is to represent the value of y in the conjecture $\sin x = \cos y$, $\sin(54)$ would equal to $\sin(36)$. Again this should add up to 90 degrees.

$$\sin x = \cos y$$

$$\sin(54) = \cos(36)$$

$$\sin \theta = \cos(90 - \theta)$$

$$\sin(54) = \cos(90 - 54)$$

$$0.8090 = 0.8090$$

When 77 is to represent the value of x and 13 is to represent the value of y in the conjecture $\sin x = \cos y$, $\sin(77)$ would equal to $\sin(13)$. Again this should add up to 90 degrees.

$$\sin x = \cos y$$

$$\sin(77) = \cos(13)$$

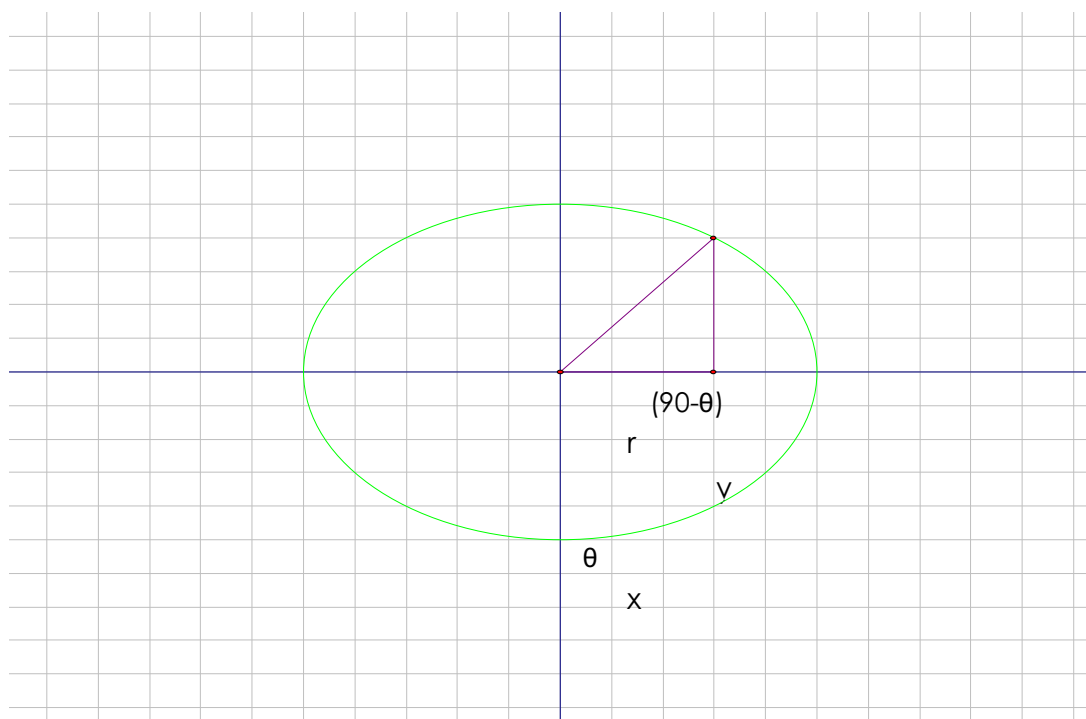
$$\sin \theta = \cos(90 - \theta)$$

$$\sin(77) = \cos(90 - 77)$$

$$0.9744=0.9744$$

Considering a triangle in the first quadrant with angle θ , the measure of the other acute angle in the triangle in terms of θ is $(90-\theta)$. The conjecture $\sin \theta = \cos(90-\theta)$ is to be proved by using the values of sine and cosine in terms of x, y, r of the angle $(90-\theta)$.

The origin and a radius of r units drawn to some point in the four quadrant of the circle forming a right triangle with its sides x, y , and r and its acute angles θ and $(90-\theta)$

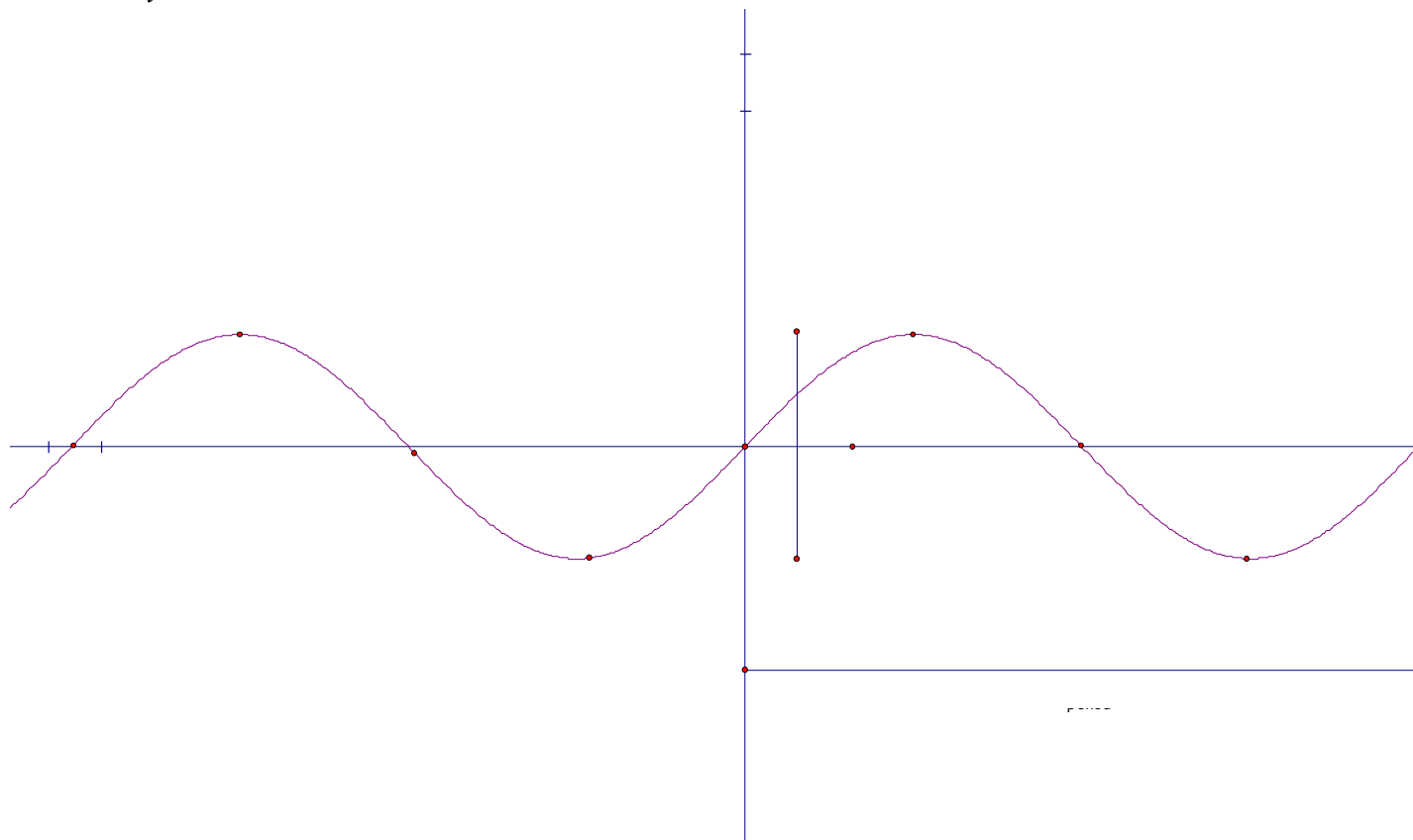


With a triangle in the graph above labeled with x, y and r as its sides, the value of $\cos \theta$ equals to the adjacent side over the radius which means x/r . The value of $\cos \theta$ is also proved to equal the value of $\sin(90-\theta)$ which equals to the opposite side over the radius meaning x over r also. As $\cos \theta = \frac{x}{r}$ and $\sin(90-\theta) = \frac{x}{r}$, therefore, $\cos \theta = \sin(90-\theta)$.

So, the value of $\sin \theta$ equals to the opposite side over the radius meaning y over r . This equals to the value of $\cos(90-\theta)$ which means the adjacent side over the radius resulting y over r also. As $\sin \theta = \frac{y}{r}$ and $\cos(90-\theta) = \frac{y}{r}$, therefore, $\cos(90-\theta) = \sin \theta$.

Part C: Sine and Cosine Graphs

$$y = \sin \theta$$



The domains $-2\pi \leq \theta \leq 2\pi$ or $-6.28 \leq \theta \leq 6.28$ is used in the domains of $360^\circ \leq \theta \leq 360^\circ$ is expressed in radians as $2\pi \approx 6.28$.

The terms maxima, minima, amplitude, period and frequency are described in the graphs of $y = \sin \theta$ and $y = \cos \theta$ for the domains $-2\pi \leq \theta \leq 2\pi$.

The $\sin \theta$ graph passes through the points $(-2\pi, 0)$, $(-\frac{3\pi}{2}, 1)$, $(-\pi, 0)$, $(-\frac{\pi}{2}, -1)$, $(0, 0)$, $(\frac{\pi}{2}, 1)$, $(\pi, 0)$, $(\frac{3\pi}{2}, -1)$, and $(2\pi, 0)$ with 9 coordinates as seen in the graph above.

Upon the analysis of the pattern of the graph,

As the value of θ increases from -2π to $-\frac{3\pi}{2}$, $\sin \theta$ goes from 0 to 1.

As the value of θ increases from $-\frac{3\pi}{2}$ to $-\pi$, $\sin\theta$ goes from 1 to 0.

The value of θ increases from $-\pi$ to $-\frac{\pi}{2}$, $\sin\theta$ goes from -1 to 0.

The value of θ increases from 0 to $\frac{\pi}{2}$, $\sin\theta$ goes from 0 to 1.

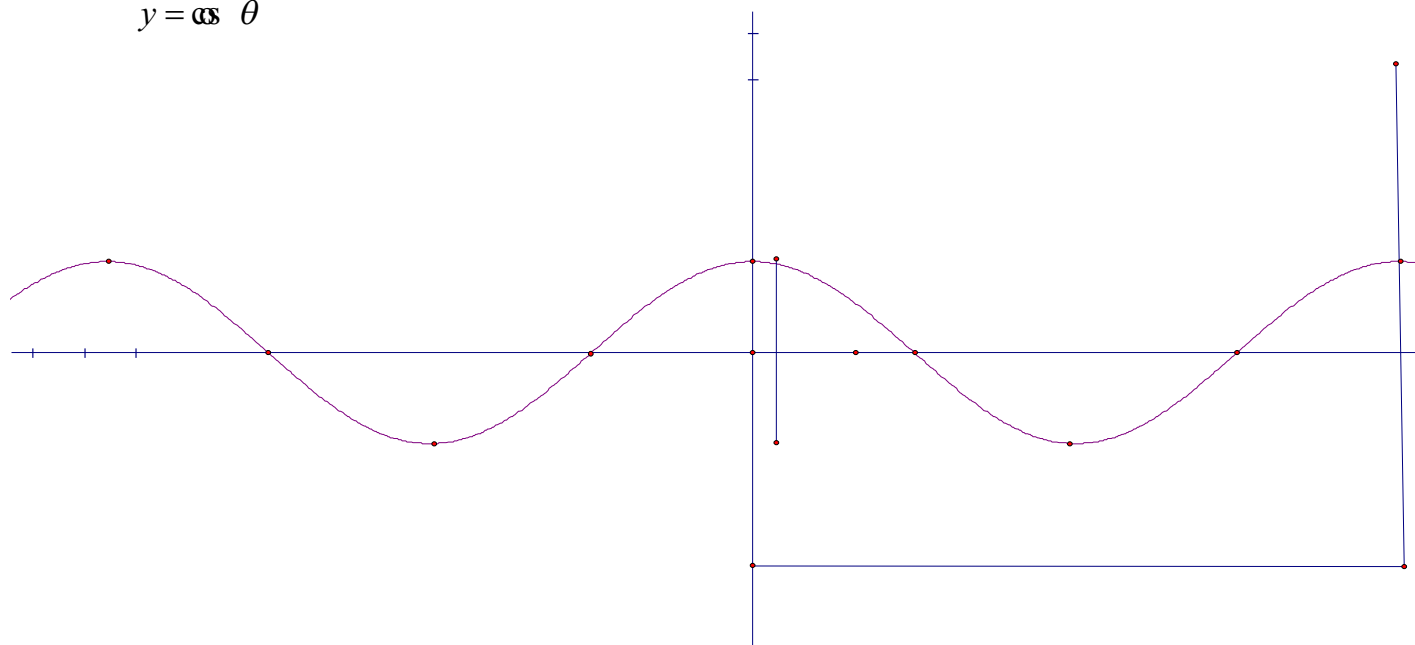
The value of θ increases from $\frac{\pi}{2}$ to π , $\sin\theta$ goes from 1 to 0.

The value of θ increases from π to $\frac{3\pi}{2}$, $\sin\theta$ goes from 0 to -1.

The value of θ increases from $\frac{3\pi}{2}$ to 2π , $\sin\theta$ goes from -1 to 0.

The maxima, the highest point in the y value, of the $y = \sin \theta$ graph is 1 and the minima, the lowest point in the y value, of the $y = \sin \theta$ graph is -1. The amplitude is the distance of the maximum y value to the middle y value, which in this graph shows $\frac{[1 - (-1)]}{2}$, resulting to 1. The period refers to x values, how long it takes for the pattern to begin again. The period in the $y = \sin \theta$ graph is 2π which approximately 6.28 when expressed to radian are. The frequency is how many cycles or pattern the graph have gone through. When expressed to radian the frequency is the reciprocal of the period meaning $\frac{1}{2\pi}$ which is approximately 0.1592.

$$y = \sin \theta$$



In this $\cos\theta$ graph the line passes through the points $(-2\pi, 1)$, $(-\frac{3\pi}{2}, 0)$, $(-\pi, -1)$, $(-\frac{\pi}{2}, 0)$, $(0, 1)$, $(\frac{\pi}{2}, 0)$, $(\pi, -1)$, $(\frac{3\pi}{2}, 0)$, and $(2\pi, 1)$ again with 9 coordinates.

Upon the analysis of the pattern of the graph,

As the value of θ increases from -2π to $-\frac{3\pi}{2}$, $\cos\theta$ goes from 1 to 0.

As the value of θ increases from $-\frac{3\pi}{2}$ to $-\pi$, $\cos\theta$ goes from 0 to -1.

As the value of θ increases from $-\pi$ to $-\frac{\pi}{2}$, $\cos\theta$ goes from -1 to 0.

As the value of θ increases from 0 to $\frac{\pi}{2}$, $\cos\theta$ goes from 1 to 0.

As the value of θ increases from $\frac{\pi}{2}$ to π , $\cos\theta$ goes from 0 to -1.

As the value of θ increases from π to $\frac{3\pi}{2}$, $\cos\theta$ goes from -1 to 0.

As the value of θ increases from $\frac{3\pi}{2}$ to 2π , $\cos\theta$ goes from 0 to 1.

The maxima which are the maximum value for y or the highest point in y value, of this the maxima, the highest point in the y value $y = \cos\theta$ graph is 1 and the minima, the lowest point in the y value, of the $y = \cos\theta$ graph is -1.

For the $y = \cos\theta$ graph is 1 and the minima, the lowest point in the y value, of the $y = \cos\theta$ graph is -1.

The amplitude, the distance of the maximum y value to the middle y value, in the $y = \cos\theta$ is $\frac{[1 - (-1)]}{2}$, resulting to 1.

The relationship between the uses of x is that to see the pattern repeating.

The period in the $y = \cos\theta$ graph is 2π which approximately 6.28 when expressed to radian are. The frequency refers to how many cycles a graph have gone through. When

expressed to radian the frequency is the reciprocal of the period meaning $\frac{1}{2\pi}$ which is approximately 0.1592.

When referring to maxima, minima, period, amplitude and frequency, the $\sin \theta$ graph and the $\cos \theta$ graph are exactly the same.

The comparison of the points the graphs $y = \sin \theta$ and $y = \cos \theta$ pass through the 9 coordinates.

| x | sinx | cosx |
|------------------|----------------------|-----------------------|
| 0 | 0 | 1 |
| $\frac{\pi}{6}$ | $\frac{1}{2}$ | $\frac{\sqrt{3}}{2}$ |
| $\frac{\pi}{4}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{2}}{2}$ |
| $\frac{\pi}{3}$ | $\frac{\sqrt{3}}{2}$ | $\frac{1}{2}$ |
| $\frac{\pi}{2}$ | 1 | 0 |
| $\frac{3\pi}{4}$ | $\frac{\sqrt{2}}{2}$ | $-\frac{\sqrt{2}}{2}$ |
| π | 0 | -1 |
| $\frac{3\pi}{2}$ | -1 | 0 |
| 2π | 0 | 1 |

$$y=3\cos(\theta)$$

$$y= (-3)\cos\theta$$

$$y=3\cos(-\theta)$$

The maxima of this graph $y=3\cos\theta$ is 3 and the minima is -3.

The amplitude in the $y=3\cos\theta$ is $\frac{3 - (-3)}{2}$ resulting to 3. The period in the $y=3\cos\theta$ is 2π which is approximately 6.28 when expressed to radian. Expressing it to radian the frequency is approximately 0.1592.

$$y=3\cos(-\theta)$$

Reflected through the y-axis from the $y=3\cos\theta$ graph so it is coincident with the $y=3\cos\theta$ graph; therefore, the values of maxima, minima, amplitude, period and frequency are the same as the $y=3\cos\theta$ graph.

$$y= (-3) \cos\theta$$

Reflected through the x axis from the $y=3\cos\theta$ graph; therefore, the values of maxima, minima, amplitude, period and frequency are the same as the $y=3\cos\theta$ graph.

$$y=2\sin\theta$$

$$y=-2\sin\theta$$

$$y=2\sin(-\theta)$$

Period:

The period in the $y=\sin\theta$ is 2π which is approximately 6.28 when expressed to radian. Then the frequency will be 0.1592 in terms of radian.

$$y=2\sin(-\theta)$$

Reflected through the y axis from the $y=2\sin\theta$ graph so it is coincident with the $y=2\sin\theta$ graph; therefore, the values of maxima, minima, amplitude, period and frequency are the same as the $y=2\sin\theta$ graph.

$$y=-2\sin\theta$$

Reflected through the x axis from the $y=2\sin\theta$ graph; therefore, the values of maxima, minima, amplitude, period and frequency are the same as the $y=2\sin\theta$ graph.

$$y=2\sin\theta$$

The maxima of the $y=2\sin\theta$ graph is 2 and the minima of the $y=2\sin\theta$ graph is -2. The amplitude in the $y=2\sin\theta$ is $\frac{2 - (-2)}{2}$ resulting to 2.

The 3 graphs above, After analyzing the 3 $a\sin\theta$, we can see that the values of a in $y=a\sin\theta$ becomes the ranges the graph to be $-a\leq\sin\theta\leq a$ instead of $-1\leq\sin\theta\leq 1$.

Also, the amplitude of $y=$ we can understand that the values of a in $y=a\sin\theta$ sets the range of the graph to be $-a\leq\sin\theta\leq a$ instead of $-1\leq\sin\theta\leq 1$. Also, the amplitude of $y=a\sin x$ is the largest value of y and which given by $|a|$.

Therefore, the amplitude of $y=a\sin x$ and $y=a\cos x$ will be the largest value of y and will be given by amplitude= $|a|$ and sets the range of the graph by giving the values of both maxima and the minima. (The curvy scribble looking line.)

$$y = \sqrt{2} \sin \theta$$

$$y = \sqrt{2} \sin(-\theta)$$

$$y = -\sqrt{2} \sin \theta$$

$$y = \sqrt{2} \sin(-\theta)$$

The graph is reflected across the y axis from the $y = \sqrt{2} \sin \theta$ graph so it is coincident with the $y = \sqrt{2} \sin \theta$ graph;

Therefore, the values of maxima, minima, amplitude, period and frequency are the same as the $y = \sqrt{2} \sin \theta$ graph.

The frequency is approximately 0.1592 when expressed to radian.

$$y = -\sqrt{2} \sin \theta$$

The graph is reflected across the x axis from the $y = \sqrt{2} \sin \theta$ graph; therefore, the values of maxima, minima, amplitude,

Period and Frequency are the same as in the $y = \sqrt{2} \sin \theta$ graph which is 0.1592.

The conjecture is verified as the value of $\alpha, \sqrt{2}$ giving the amplitude, maxima and minima in the graph,

thus setting the range of $-\sqrt{2} \leq \sqrt{2} \sin \theta \leq \sqrt{2}$.

$$y = \sqrt{2} \sin \theta$$

The maxima of the $y = \sqrt{2} \sin \theta$ graph is $\sqrt{2}$ and the minima of the $y = \sqrt{2} \sin \theta$ graph is $-\sqrt{2}$.

The amplitude in the $y = \sqrt{2} \sin \theta$ graph is calculated by $\frac{|\sqrt{2} - (-\sqrt{2})|}{2}$ resulting to $\sqrt{2}$.

The period in the $y = \sqrt{2} \sin \theta$ graph is 2π which approximately 6.28 when expressed to radian are.

$$y = -\sqrt{3} \cos(\theta)$$

$$y = \sqrt{3} \cos(-\theta)$$

$$y = \sqrt{3} \cos(\theta)$$

$$y = -\sqrt{3} \cos(\theta)$$

The $y = -\sqrt{3} \cos(\theta)$ graph is reflected across the x axis from the $y = \sqrt{3} \cos \theta$ graph; therefore, the values of maxima, minima, amplitude, period and frequency are the same as the $y = \sqrt{3} \cos \theta$ graph.

$$y = \sqrt{3} \cos(-\theta)$$

The $y = \sqrt{3} \cos(-\theta)$ graph is reflected across the y axis from the $y = \sqrt{3} \cos \theta$ graph so it is coincident with the $y = \sqrt{3} \cos \theta$ graph; therefore, the values of maxima, minima, amplitude, period and frequency are the same as the $y = \sqrt{3} \cos \theta$ graph.

$$y = \sqrt{3} \cos(\theta)$$

The maxima of the $y = \sqrt{3} \cos \theta$ graph is $\sqrt{3}$ and the minima of the $y = \sqrt{3} \cos \theta$ graph is $-\sqrt{3}$.

The amplitude in the $y = \sqrt{3} \cos \theta$ graph is calculated by $\frac{|\sqrt{3} - (-\sqrt{3})|}{2}$ resulting to $\sqrt{3}$.

The period in the $y = \sqrt{3} \cos \theta$ graph is 2π which approximately 6.28 when expressed to radian are.

The frequency is approximately 0.1592 when expressed to radian.

Therefore, the conjecture will be the value of a, $\sqrt{3}$ gives the amplitude, maxima and minima in the graph, thus setting the range of $-\sqrt{3} \leq \sqrt{3} \cos \theta \leq \sqrt{3}$.

$y = \sin b\theta$ and $y = \cos b\theta$ graphs for different values of b using the domains of $-2\pi \leq \theta \leq 2\pi$ are formed to see any constraints on the values of b.

$$y = \sin(3\theta)$$

$$y = \sin\left(\frac{1}{3}\theta\right)$$



$$y = \sin\left(\frac{1}{3}\theta\right)$$

The maxima of the $y = \sin\left(\frac{1}{3}\theta\right)$ graph is 1 and the minima of the $y = \sin\left(\frac{1}{3}\theta\right)$ graph is -1.

The amplitude in the $y = \sin\left(\frac{1}{3}\theta\right)$ graph is calculated by $\frac{1 - (-1)}{2}$, resulting to 1. The

period in the $y = \sin\left(\frac{1}{3}\theta\right)$ graph is 6π which approximately 18.8496 when expressed to radian are. The frequency is approximately 0.5236 when expressed to radian.

$$y = \sin(3\theta)$$

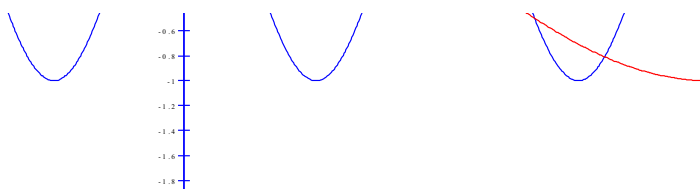
The maxima of the $y = \sin(3\theta)$ graph is 1 and the minima of the $y = \sin(3\theta)$ graph is -1.

The amplitude in the $y = \sin(3\theta)$ graph is $\frac{1 - (-1)}{2}$ resulting to 1.

The period in the $y = \sin(3\theta)$ graph is $\frac{2\pi}{3}$ which is approximately 2.0944 when expressed to radian. The frequency is approximately 0.4775 when expressed to radian.

$$y = \cos(2\theta)$$

$$y = \cos\left(\frac{1}{2}\theta\right)$$



$$y = \cos\left(\frac{1}{2}\theta\right)$$

The maxima of the $y = \cos\left(\frac{1}{2}\theta\right)$ graph is 1 and the minima of the $y = \cos\left(\frac{1}{2}\theta\right)$ graph is -1. The amplitude in the $y = \cos\left(\frac{1}{2}\theta\right)$ graph is calculated by $\frac{1 - (-1)}{2}$, resulting to 1. The period in the $y = \cos\left(\frac{1}{2}\theta\right)$ graph is 4π which approximately 12.5664 when expressed to radian are. The frequency is approximately 0.7854 when expressed to radian.

$$y = \cos(2\theta)$$

The maxima of the $y = \cos(2\theta)$ graph is 1 and the minima of the $y = \cos(2\theta)$ graph is -1. The amplitude in the $y = \cos(2\theta)$ graph is $\frac{1 - (-1)}{2}$, resulting to 1.

The period in the $y = \cos(2\theta)$ graph is π which is approximately 3.1416 when expressed to radian.

The frequency is approximately 0.3183 when expressed to radian.

When we observe some specific features of the above graphs, we can see how the value of b affects graphs of $y = \sin bx$ and $y = \cos bx$. The period of both $y = \sin bx$ and $y = \cos bx$ graphs is $\frac{2\pi}{b}$. The value of b in $y = \sin(3\theta)$ is 3 so it is $\frac{2\pi}{3}$, which is how long the cycle takes to repeat while the value of b in $y = \sin(\frac{1}{2}\theta)$ is $\frac{1}{2}$ so it is $\frac{2\pi}{\frac{1}{2}}$ and 4π when calculated, which is how long the cycle takes to repeat.

$$-2\pi \leq b\theta \leq 2\pi$$

If b is positive, the above inequality is $0 \leq \theta \leq \frac{2\pi}{b}$. When $0 < b < 1$, the period of $y = \sin bx$ is greater than 2π and represents a horizontal stretching of the graph of $y = \sin x$. Similarly, if $b > 1$, the period of $y = \sin bx$ is less than 2π and represents a horizontal compression of the graph of $y = \sin x$. When b is negative, the above inequality becomes $\frac{2\pi}{b} \leq \theta \leq 0$.

For either a positive or negative value of b , one cycle (period) of the graph of $y = \sin b\theta$ and $y = \cos b\theta$ are obtained respectively on an interval of $\frac{2\pi}{|b|}$. Then, the frequency, the reciprocal of the period, is $\frac{|b|}{2\pi}$.

The conjecture is now verified by considering further examples of b .

$$y = \sin(\sqrt{3}\theta)$$

$$y = \sin(\sqrt{\frac{1}{3}}\theta)$$

The maxima of the $y = \sin(\sqrt{3}\theta)$ graph is 1 and the minima of the $y = \sin(3\theta)$ graph is -1. The amplitude in the $y = \sin(\sqrt{3}\theta)$ graph is calculated by $\frac{1 - (-1)}{2}$ resulting to 1. The period in the $y = \sin(\sqrt{3}\theta)$ graph is $\frac{2\pi}{\sqrt{3}}$ which approximately 3.6276 when expressed to radian are. The frequency is $\frac{\sqrt{3}}{2\pi}$ which is approximately 0.2757 when expressed to radian.

The maxima of the $y = \sin(\sqrt{\frac{1}{3}}\theta)$ graph is 1 and the minima of the $y = \sin(\sqrt{\frac{1}{3}}\theta)$ graph is -1. The amplitude in the $y = \sin(\sqrt{\frac{1}{3}}\theta)$ graph is calculated by $\frac{1 - (-1)}{2}$ resulting to 1. The period in the $y = \sin(\sqrt{\frac{1}{3}}\theta)$ graph is $\frac{2\pi}{\sqrt{\frac{1}{3}}}$ which approximately 10.8828 when expressed to radian are. The frequency is $\frac{\sqrt{\frac{1}{3}}}{2\pi}$ which is approximately 0.9069 when expressed to radian.

$$y = \sin(\sqrt{2}\theta)$$

$$y = \sin(-\sqrt{\frac{1}{2}}\theta)$$

The maxima of the $y = \sin(\sqrt{2}\theta)$ graph is 1 and the minima of the $y = \sin(\sqrt{2}\theta)$ graph is -1. The amplitude in the $y = \sin(\sqrt{2}\theta)$ graph is calculated by $\frac{1 - (-1)}{2}$ resulting to 1.

The period in the $y = \sin(\sqrt{2}\theta)$ graph is $\frac{2\pi}{\sqrt{2}}$ which approximately 4.4429 when expressed to radian are.

The frequency is $\frac{\sqrt{2}}{2\pi}$ which is approximately 0.5642 when expressed to radian.

The maxima of the $y = \sin(-\sqrt{\frac{1}{2}}\theta)$ graph is 1 and the minima of the $y = \sin(-\sqrt{\frac{1}{2}}\theta)$ graph is -1.

The amplitude in the $y = \sin(-\sqrt{\frac{1}{2}}\theta)$ graph is calculated by $\frac{1 - (-1)}{2}$ resulting to 1.

The period in the $y = \sin(-\sqrt{\frac{1}{2}}\theta)$ graph is $\frac{2\pi}{-\sqrt{\frac{1}{2}}}$ which is approximately $|-8.888|$ when expressed to radian. The frequency is $\frac{-\sqrt{\frac{1}{2}}}{2\pi}$ which is $|-0.125|$ when expressed to radian.

Conclusion: