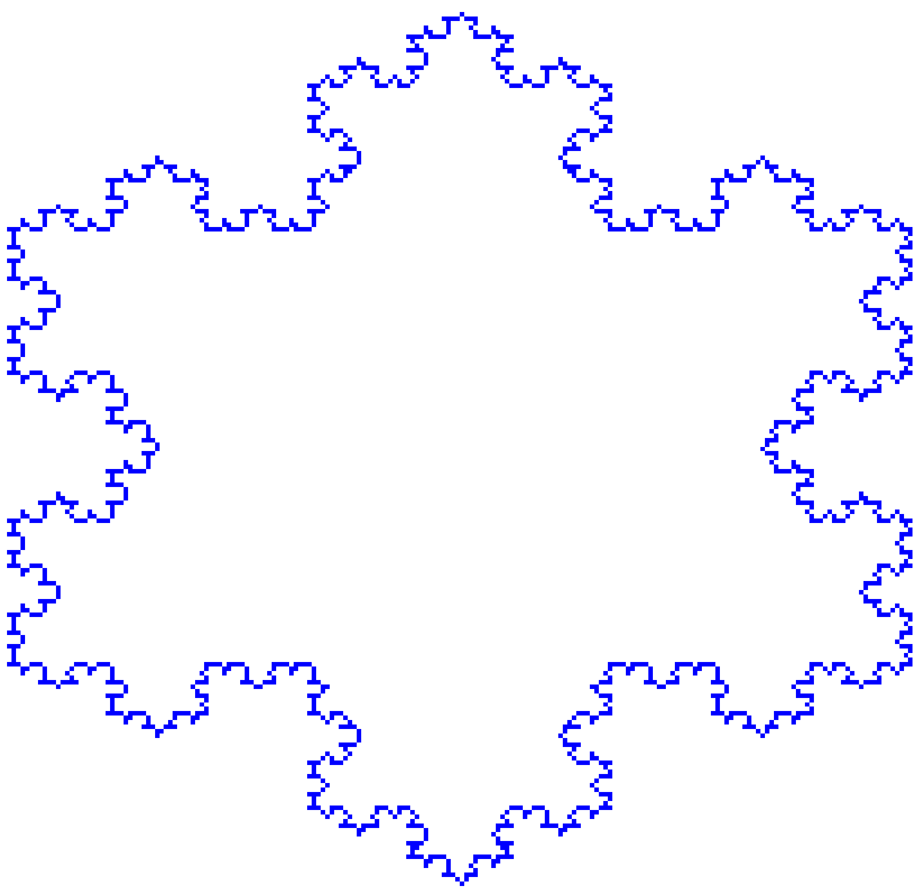
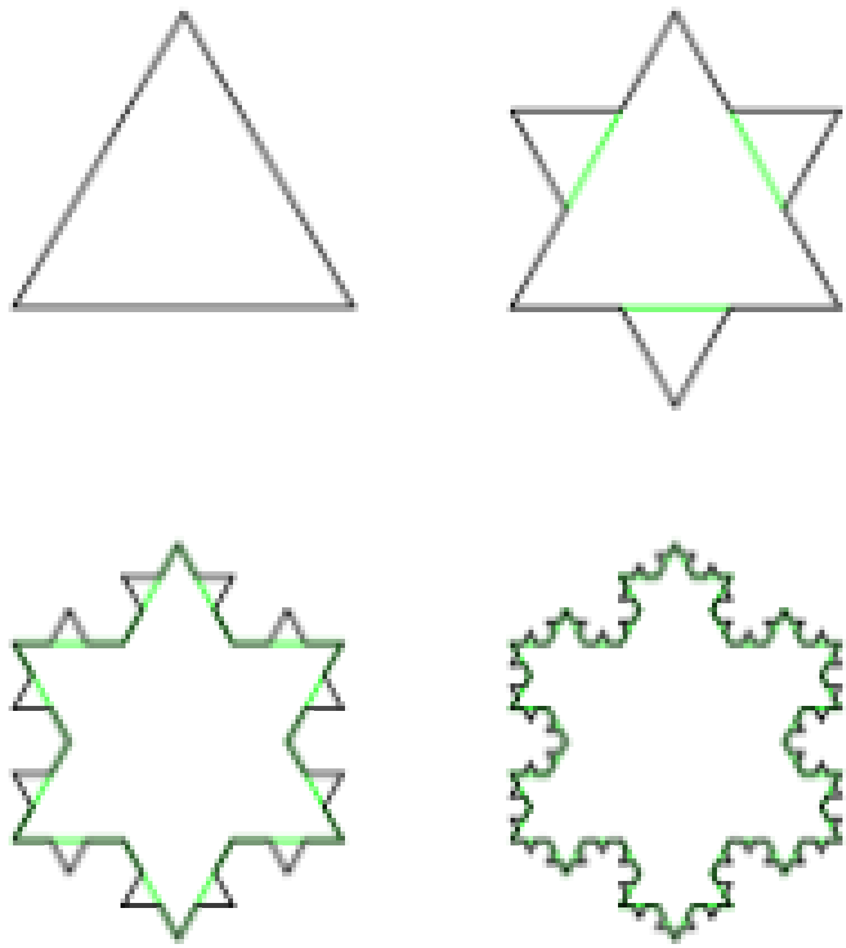
# The Koch Snowflake



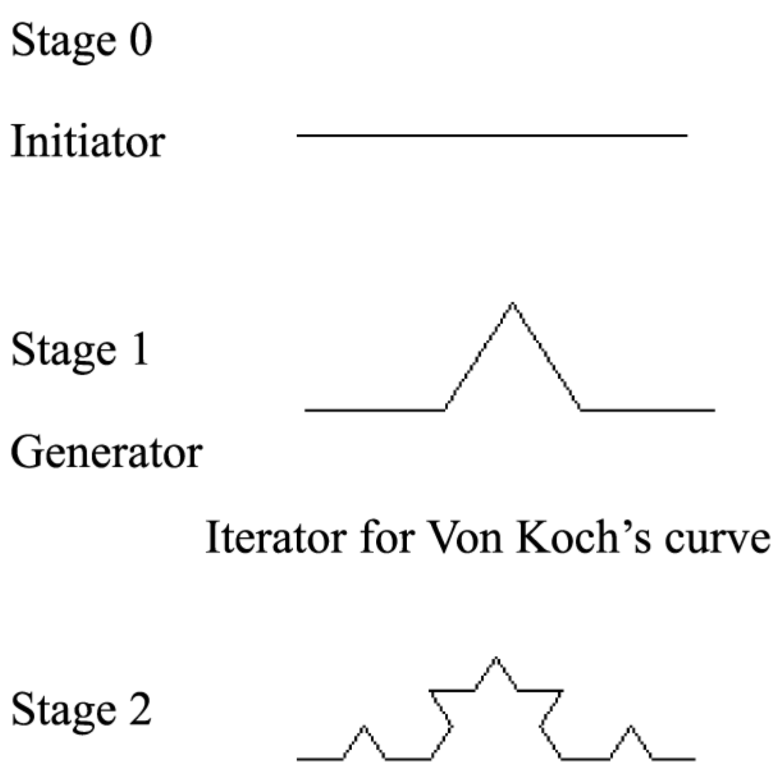
The Koch snowflake (also known as the Koch star and Koch island[1]) is a mathematical curve and one of the earliest fractal curves to have been described. It is based on the Koch curve, which appeared in a 1904 paper titled "On a continuous curve without tangents, constructible from elementary geometry" by the Swedish mathematician Helge von Koch.



In the above stages, certain notations are used for the nth term:

$N_n$: number of sides
$L_n$: length of a single side
$P_n$: the perimeter
$A_n$: the area of the snowflake

The Koch's snowflake curve, simply starts of an equilateral triangle which is when n=0. The triangle has 3 sides, which are 1 unit each. Then each of those sides is divided into 3 equal parts. Thus each of those 3 parts is 1/3 unit. On the middle part an equilateral triangle is drawn. And this continues in the following stages.




Iterator for Von Koch's curve

Below are the values for the first 4 diagrams, $n \in \{0, 1, 2, 3\}$.

| n | $N_n$ | $L_n$ | $P_n$ | $A_n$ |
|---|---|---|---|---|
| 0 | 3 | 1 | 3 | $\frac{\sqrt{3}}{4}$ |
| 1 | 12 | $\frac{1}{3}$ | 4 | 0.57735 |
| 2 | 48 | $\frac{1}{9}$ | $\frac{16}{3}$ | 0.64150 |
| 3 | 192 | $\frac{1}{27}$ | $\frac{64}{9}$ | 0.67001 |

(5 s.f)

**Number of sides**

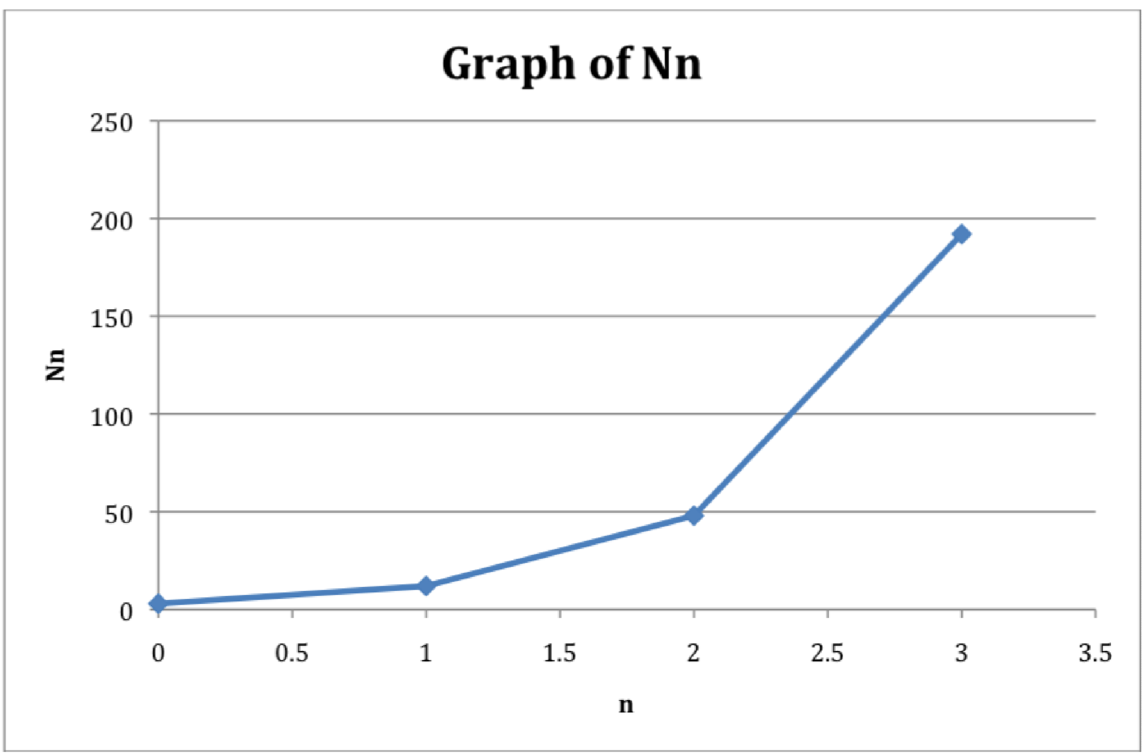
Initially there is an equilateral triangle at stage n=0, each of those sides is divided into 3 sides. And there is another equilateral triangle created at the center points. Thus from that I can conclude that each side becomes from sides.
as the diagrams are given to us, for the first 4 stages I counted the sides. There is a pattern present, and from that I

observed that the number of sides increases by a factor 4 of the previous stage.

| $n$ | $N_n$ |
|---|---|
| 0 | 3 |
| 1 | $N_0 \times 4 = 3 \times 4 = 12$ |
| 2 | $N_1 \times 4 = 3 \times 4^2 = 48$ |
| 3 | $N_2 \times 4 = 3 \times 4^3 = 192$ |

From the table above, I can determine the $n^{th}$ term as:

$N_n : 3\times4^n$



From the graph above, we can observe that the number of sides increases as n increases.

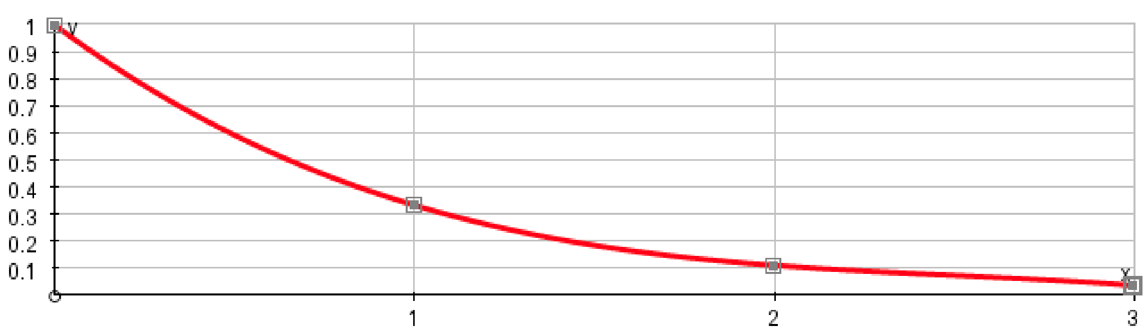
**Length of sides**

At stage n=0, there is an equilateral triangle which has 3 sides. Those 3 sides each are 1 unit. For the next stage where n=1, each side is divided into 3 parts. Thus the length of the new side is one-third the previous side.

| n | $L_n$ |
|---|---|
| 0 | 1 |
| 1 | $\frac{1}{3} \times 1 = \frac{1}{3}$ |
| 2 | $\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$ |
| 3 | $\frac{1}{3} \times \frac{1}{9} = \frac{1}{27}$ |

From the table above, I can determine the $n^{th}$ term as:

$L_n$: $\left(\frac{1}{3}\right)^n$



From the graph above, we can observe that the length of the side decreases gradually as the number of sides increase. They have a limiting value of 0 (tend to 0). X=0 is the asymptote.

**Perimeter**

The perimeter of the koch's snowflake can be found by multiplying the length of one side into the total number of sides. The perimeter of n=0 is 3 as 3 sides are multiplied into 1 unit.

The table below has the perimeter of the 1st four stages:

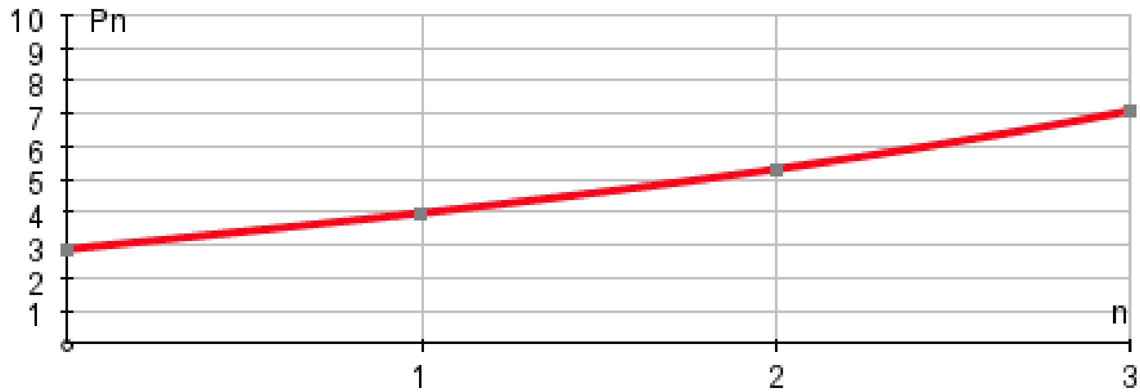
| n | Pn |
|---|---|
| 1 | $(3 \times 4^0) = 3$ |
| 2 | $(3 \times 4) \times \frac{1}{3} = 4$ |
| 3 | $(3 \times 4^2) \times \frac{1}{9} = \frac{16}{3}$ |
| 4 | $(3 \times 4^3) \times \frac{1}{27} = \frac{64}{9}$ |

From the above table, the nth term can be determined as

Pn : $\left(\frac{1}{3}\right) \times 3 \times 4^n$

Pn : $3 \times \left(\frac{4}{3}\right)^n$

Perimeter graph




Pn

n

From the graph above we can observe that we can see as the number of sides increase (n), the perimeter also increases.

**Area**

The area of the 1st stage at n=0 is $\frac{\sqrt{3}}{4}$, as the shape changes in the 2nd stage, the area increases to the area of the first triangle+ area of new triangles

- $A_1 = A_0$ + area of new triangles

There is an area scale factor present, which is $\frac{1}{9}$.

- $A_1 = A_0 + \text{new triangles} \times \frac{1}{9} \times A_0$

$A_1 = A_0\,(1 + \text{new triangles} \times \frac{1}{9})$

At the 2nd stage the area scale factor becomes

$\frac{1}{81} = (\frac{1}{9} \times \frac{1}{9})$.

- $A_2 = A_1$ + new area

The table below has the areas of the 1st four stages:

| n | $N_n$ | Area scale factor | Extra triangles |
|---|---|---|---|
| 0 | 3 | - | - |
| 1 | 12 | $\frac{1}{9}$ | 3 |
| 2 | 48 | $\left(\frac{1}{9}\right)^2$ | 12 |
| 3 | 192 | $\left(\frac{1}{9}\right)^3$ | 48 |

$A_0$: $\frac{\sqrt{3}}{4} \times 1^2 = \frac{\sqrt{3}}{4}$

: $A_0$ (1 + new triangles $\times \frac{1}{9}$)

: $\frac{\sqrt{3}}{4}(1 + 3 \times \frac{1}{9})$

$A_1$: $\frac{\sqrt{3}}{4}(1 + \frac{3}{9})$

Stage 2 has 12 new triangles and also the scale factor becomes square of the previous scale factor. So it is $\left(\frac{1}{9}\right)^2$

$A_2$: $A_1$+ new area

: $A_1 + (12 \times \frac{1}{81} \times \frac{\sqrt{3}}{4})$

: $\frac{\sqrt{3}}{4}(1 + \frac{1}{3}) + 12 \times \frac{1}{81} \times \frac{\sqrt{3}}{4}$

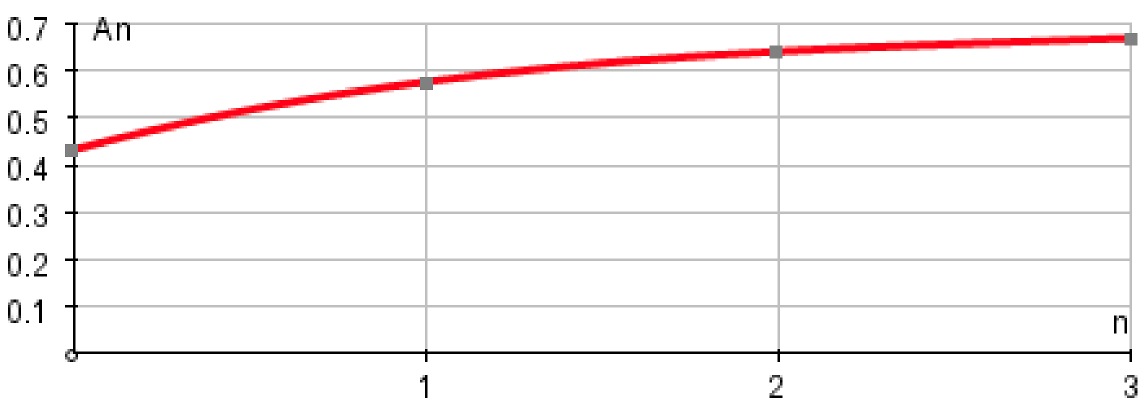
$A_2$: $\frac{\sqrt{3}}{4}(1 + \frac{1}{3} + \frac{12}{81})$

The formula can be generalized here:

$A_n$: $\frac{\sqrt{3}}{4}(1 + (\frac{3}{9} + \frac{4}{3(9)} + \frac{4^2}{3(9^2)} + \ldots \ldots + \frac{4^{n-1}}{3\left(9^{n-1}\right)})$

$$A_n = \frac{\sqrt{3}}{4}\left(1 + \sum_{r=1}^{n} \frac{4^{r-1}}{3\left\{9^{r-1}\right\}}\right)$$

**Area Graph**



The area graph above highlights that as the n increases the area also increases. It is directly proportional.

## Generalizations and predictions

As we know the nth term for Nn, Ln, Pn and An we can now predict the results for terms till n=6.

| n | Nn | Ln | Pn | An |
|---|---|---|---|---|
| 0 | 3 | 1 | 3 | 0.433012701 |
| 1 | 12 | $\frac{1}{3}$ | 4 | 0.577350269 |
| 2 | 48 | $\frac{1}{9}$ | $\frac{16}{3}$ | 0.641500298 |
| 3 | 192 | $\frac{1}{27}$ | $\frac{64}{9}$ | 0.670011422 |
| 4 | 768 | $\frac{1}{81}$ | $\frac{256}{27}$ | 0.68268303 |
| 5 | 3072 | $\frac{1}{243}$ | $\frac{1024}{81}$ | 0.68831486 |
| 6 | 12288 | $\frac{1}{729}$ | $\frac{4096}{243}$ | 0.6908179 |

| n | $N_n$ | $L_n$ | $P_n$ | $A_n$ |
|---|---|---|---|---|
| 4 | $N_3 \times 4 = 3 \times 4^4 = 768$ | $\frac{1}{3} \times \frac{1}{27} = \frac{1}{81}$ | $(3 \times 4^4) \times \frac{1}{81} = \frac{256}{27}$ | 0.68268000 |

$A_4$: $\frac{\sqrt{3}}{4}\left(1+\sum_{i=1}^{n}\frac{4^{4-1}}{3(9^{4-1})}\right)$

: $\frac{\sqrt{3}}{4}\left(1+\sum_{r=1}^{n}\frac{4^{3}}{3(9^{3})}\right)$

: 0.68268303
: 0.68268000 (5s.f.)

| n | Pn | An |
|---|---|---|
| 0 | 3 | 0.433012702 |
| 1 | 4 | 0.57735027 |
| 2 | 5.333333333 | 0.64150030 |
| 3 | 7.111111111 | 0.67001142 |
| 4 | 9.481481481 | 0.68268303 |
| 5 | 12.64197531 | 0.68831486 |
| 6 | 16.85596708 | 0.69081790 |
| 7 | 22.47462277 | 0.69193036 |
| 8 | 29.96616369 | 0.69242478 |
| 9 | 39.95488493 | 0.69264453 |
| 10 | 53.2731799 | 0.69274219 |
| 11 | 71.03090654 | 0.69278560 |
| 12 | 94.70787538 | 0.69280489 |
| 13 | 126.2771672 | 0.69281346 |
| 14 | 168.3695562 | 0.69281727 |
| 15 | 224.4927416 | 0.69281897 |
| 16 | 299.3236555 | 0.69281972 |
| 17 | 399.0982074 | 0.69282006 |
| 18 | 532.1309432 | 0.69282020 |
| 19 | 709.5079242 | 0.69282027 |
| 20 | 946.0105656 | 0.69282030 |

The table above has values of n from 0-20.
From the table we can observe that from the 17th term, the area of $A_{n+1}$ equals $A_n$ to 6 decimal places.

Also from the table, we can see that at n=15, the area is 0.69281897 and as you go ahead, the consecutive terms start to have very minute changes and the difference isn't a lot.

The $17^{th}$ term: 0.69282006

$18^{th}$ term:0. 69282020

The difference between the two only is 0.00000014, which is very minute. This shows that as the n increases and goes up to infinity the area increases very minutely (tends to 0).

Limits and Scope

The calculator and the ms excel provided acute values along with sufficient amount of significant figures.

The limitations could be that there is no possibility of finding the value for n= ∞

## Bibliography

http://en.wikipedia.org/wiki/Koch_snowflake - intro

http://library.thinkquest.org/26242/full/fm/images/51.gif - picture of snowflake
http://www.enotes.com/w/images/thumb/d/d9/KochFlake.svg/280px-KochFlake.svg.png - the four stages
http://www.emeraldinsight.com/content_images/fig/1560120405005.png - curves