Math Portfolio

Type 1

Stellar numbers

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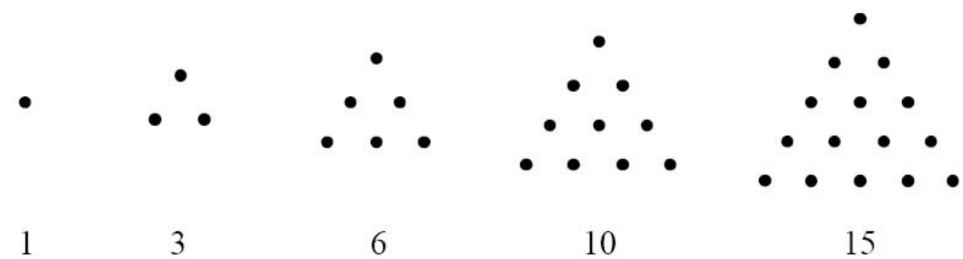
Class: IB08A

This assessment will investigate geometric shapes that lead to special numbers.
An example of this would be square numbers (1,4,9,16…), which can be represented by squares of side (1,2,3,4…).
First, triangles will be investigated.



The number of dots (1,3,6,10,15…) in each triangle is a triangular number, which will be labelled Tn, the triangular sequence of n. One can see that these triangles are equilateral triangles, where the nth triangle has n number of dots on each side.
It can be observed that the increase in dots follows the pattern 1, 1+2, 1+2+3, 1+2+3+4, 1+2+3+…+n. Therefore one can derive other numbers for Tn.

n	Tn	difference d	difference d^2
1	1		
2	3	2	
3	6	3	1
4	10	4	1
5	15	5	1
6	20	6	1
7	27	7	1
8	36	8	1
9	45	9	1
10	55	10	1

Since one is dealing with a triangular shape it would be likely to investigate if the double amount of dots, a rectangular shape, gives a pattern. The sequence of the rectangle is called Rn.(2,6,12,20,30……)

Observing Rn and the table one realizes that ever since d^2 seems to be an arithmetic sequence, it is indicated that Tn itself could have a quadratic formula as its generator.Actually,one can that the square of n plus n gives Rn. Expressing this mathematically,

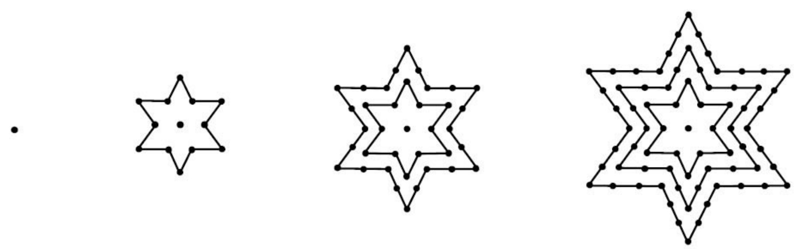
Rn $= n^2 + n$

Since Rn=2Tn one can substitute Rn in order to get Tn,

2Tn=$n^2 + n$
2Tn=n(n+1)

Tn=$\frac{n(n+1)}{2}$
Thus it seems like a general statement that represents the nth triangular number in terms of n.

After investigating the triangular shapes, more complex Stellar shapes will be investigated.

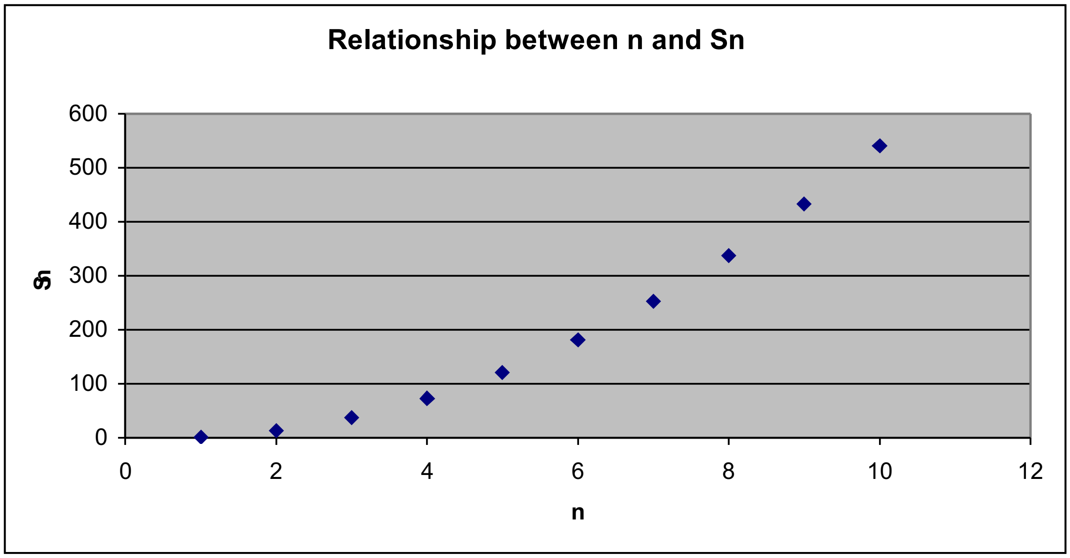


S_1 S_2 S_3 S_4

And so forth…
Each star has a number of vertices these will be labelled p. Every value of p leads to P-Stellar number, labelled Sn. In this case it is a 6-stellar number.

The sequence Sn in this case is:
(1,13,37,73,121,181…)

stellar n	nr of dots	difference d	difference d^2
1	1		
2	13	12	
3	37	24	12
4	73	36	12
5	121	48	12
6	181	60	12
7	253	72	12
8	337	84	12
9	433	96	12
10	541	108	12



The method that was used for the table heading in cellA1 and B1. And made an excel formula in A2 namely A1-B1. So the difference was obtained. For the graph A1 was plotted in relation to B1.

One can observe from the graph and the table that since difference of Sn values seem to be an arithmetic sequence then Sn is likely to be generated by a quadratic formula. Whith this knowledge following process is obtained.

Observing Sn one realizes that every value of Sn can be expressed as the difference of the difference times a number plus 1. So,

Sn=$d^2 \times x + 1$

From the table it can be assumed that d^2 is 12 consistently; a value for d^2 can be inserted,

Sn=$12 \times x + 1$

Now x is investigated, one observes that the values of x are of some kind of number sequence which will be called Cn. Cn will be investigated by exemplifying with the value of S7.

S7 = 253
S7= 252+1
S7= 12 × 21 +1

In order to receive a general statement this expression will be wrote in terms of n.

Sn = 12 × Cn+1

Calculating the values for Cn following values are obtained.

(0,1,3,6,10,15…)

Looking at this sequence one notices that it seems to be similar to the triangular sequence derived previously. In fact, observing and comparing the sequences one can see that the Cn corresponds to T(n-1).

Now I can substitute Cn with T(n-1)

Thus,

Sn= 12 × T(n-1)+1

Since,

$$Tn = \frac{n^2 + n}{2}$$

Sn will be,

$$Sn = 12 \times \frac{(n-1)^2 + (n-1)}{2} + 1$$

$$Sn = 12 \times \frac{(n^2 - 2n + 1) + (n-1)}{2} + 1$$

$$Sn = 12 \times (\frac{n^2 - 2n + 1 + n - 1}{2}) + 1$$

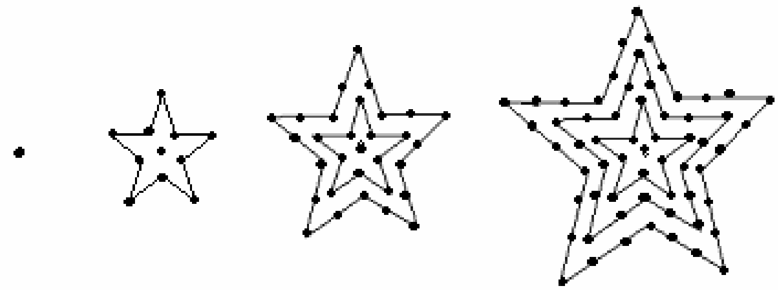
$$Sn = 6(n^2 - n) + 1$$

$$Sn = 6n(n-1) + 1$$

Looking at the general statement of the star it can be noticed that the coefficient 6 is equal to the number of vertices in the stellar shape. Assuming 6 represents p, a general statement in terms of p and n that generates the sequence of p-stellar numbers for any value of p at stage Sn can be derived. Expressing this mathematically,

Sn=pn(n-1)+1

Testing this hypothesis requires an investigation of the vertices-value p. First, a star with five vertices is investigated.



When p =5 the star has a five stellar number with the following values for Sn.

S1=1
S2=11
S3=31
S4=61

stellar n	nr of dots	difference d	difference d^2
1	1		
2	11	10	
3	31	20	10
4	61	30	10
5	101	40	10
6	151	50	10
7	211	60	10
8	281	70	10
9	361	80	10
10	451	90	10

From the table one can see that for the previously derived statement, $Sn = d^2 \times x + 1, d^2 = 10$.

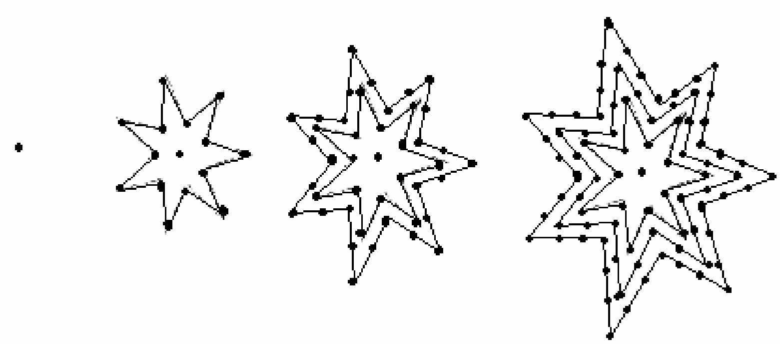
Investigating x one can again observe that it is again a sequence Cn (0,3,6,10,15…). Thus x=Tn-1 again.
Investigating this for the value S7,

S7 = 211
S7= 210+1
S7= 5×7(7-1) +1

To find the general statement for the 5-steller number, 7 is replaced by n.

Sn=5n(n-1)+1

Second, a star with seven vertices is investigated. When p =7 the star has a seven stellar number with the following values for Sn.



S1= 1
S2= 15
S3= 43
S4= 85

From the table one can see that for the previously derived statement, $Sn = d^2 \times x + 1, d^2 = 10$.
Investigating x one can again observe that it is again a sequence Cn (0,3,6,10,15…). Thus x=Tn-1 once again.
Investigating this for the value S7,

S7 = 295
S7= 294+1
S7= 7×7(7-1) +1

To find the general statement for the 7-stellar number, 7 is replaced by n.

Sn=7n(n-1)+1

To find the general statement for the 7-stellar numbers, 7 is replaced by n.

Sn=7n(n-1)+1

Testing other values for p one can see that tthey seem to follow the pattern.

P	The first four values of Pn	Difference of Pn numbers	Sn
3	1,7,19,37...	6,12,18...	3n(n-1)+1
4	1,9,25,49...	8,16,24...	4n(n-1)+1
5	1,11,31,61...	10,20,30...	5n(n-1)+1
6	1,13,37,73...	12,24,36...	6n(n-1)+1
7	1,15,43,85...	14,28,42...	7n(n-1)+1
8	1,17,49,97...	16,32,48...	8n(n-1)+1
9	1,19,55,109...	18,36,54...	9n(n-1)+1
10	1,21,61,212...	20,40,60...	10n(n-1)+1

*The values were calculated using a TI 84 calculator.

Observing this table it seems like the hypothesis Sn=pn(n-1)+1 is appliable.
At this point one could test the formula in many new situations, however, taking a more analytical approach will perhaps help to prove this general statement.

Investigating the 6-stellar number it was found that the stars follow the sequence Sn at the initial condition that .It was observed that Sn could be written in terms of Cn(another sequence) times the coefficient d^2+1. Since d^2 was 12 for all investigated values, it was found that the sequence Cn is equal to T(n-1); having derived Tn previously, Cn could be substituted in terms of n. Thus, we could derive that Sn= 6n(n-1)+1.
At this point it was noticed that 6 is the number of vertices in the Star so we investigated the Hypothesis Sn=pn(n-1)+1.After investigating the values 3-10 for p it seems like the general statement is valid . Thus, the formula seems to work for all positive integers. However, when p=2 the question arouses whether a shape with 2 vertices is a stellar shape? Investigating this one finds that the values obtained for Sn when p=2 are (1,5,13,25,41,61…). In fact, one can say that the shape could actually be a Diamond. Ever since the triangular sequence is included in the stellar sequence as T_{n-1} one may argue that a stellar shape with 2 vertices can be existent, but this remains an open question. Another value of p that aroused my attention is 1 one finds that the values obtained for Sn when p=1 are (1,3,6,10,15…) in fact one observes that this sequence is equal to Tn. Which would suggest that a stellar shape with the value p=1 is a triangle, one could at this point assume that a stellar shape with p=1 is a triangle. However, whether a triangle can be defined as a stellar shape with 1 vertex remains an open question.
Furthermore, investigating negative values, one can observe that for the general statement Sn=pn(n-1)+1

If p=negative $\rightarrow$ Sn=negative
If n=negative $\rightarrow$ Sn=negative
If p=negative and n=negative $\rightarrow$ Sn=positive

It seems that theoretically any rational number can be inserted for n and p in this formula; however, the shape then is questionable. If a number of negative dots and vertices can form a geometrical shape is a complex issue, which can not be investigated at this point. Also, the shapes when p= 2 and p= 1 if these vertices lead to actual Stellar shapes is open, but it seems likely since the sequence of Tn is included in the sequence Sn.

Thus, the conclusion can be drawn that when the conditions,

n>0
p>2

therefore if p is a fixed positive integer,

$$\sum_{n=1}^{\text{infinite}} n(n-1)+1$$

(not that infinite is written out instead of ∞**,** this was due to technical problems)

The formula Sn=pn(n-1)+1 can be applied to derive a certain Stellar shape.
When these conditions do not apply a clear pattern of a sequence can still be observed; however, if all the values lead to definite Stellar shapes remains an open question.