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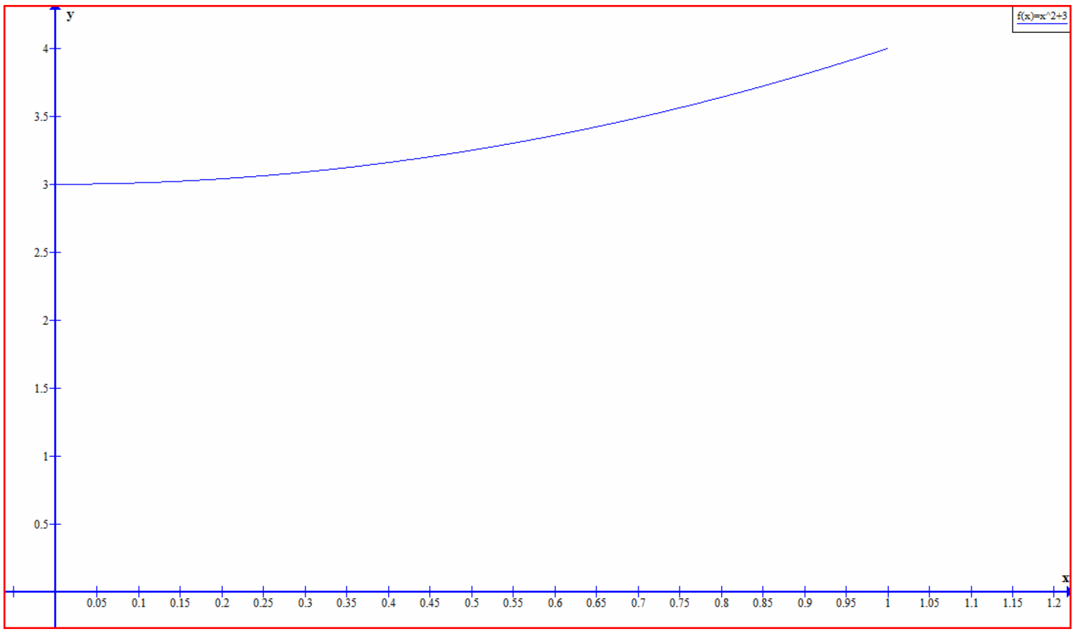
In order for one to understand, the process and subsequent work one must understand the purpose of this assignment, which is, to find a rule to approximate the area under the curve. Considering the following: **g(x) = x^2+3 (Ex 1)**

Ex. 1



(Ex 2) allows one to see what area we are trying to find. The area is represented is represented by the green diagonals. This example is still from intervals **x=0 to x=1**.

Ex. 2

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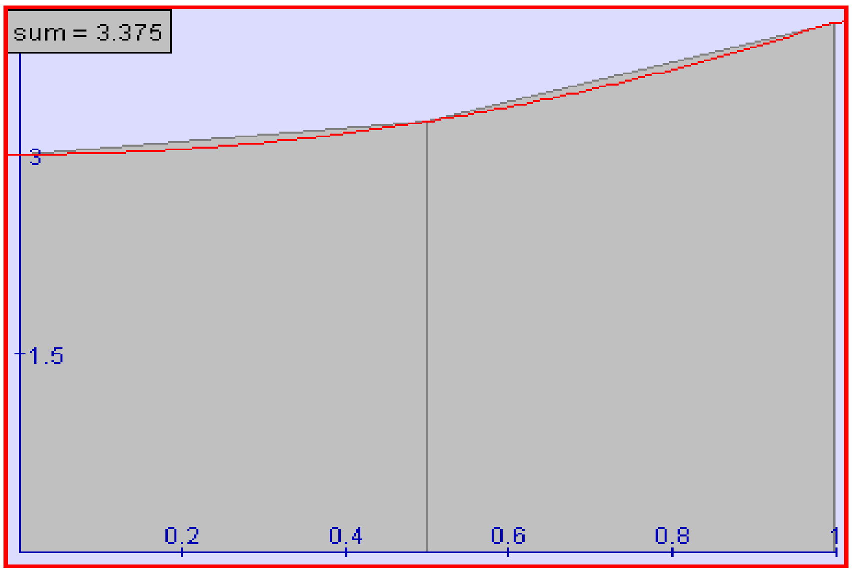
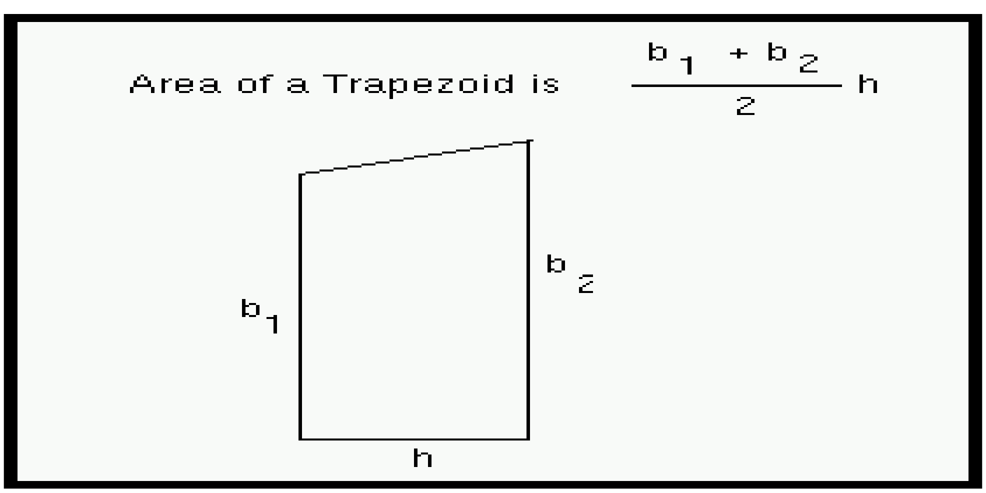
(Ex 3) shows us the estimation of the area by adding two trapezoids underneath the curve. One can observe that the area is not exact but still a good approximation. Using technology, I have found that the sums of the areas of both trapezoids are equal to 3.375. The next step will show one how to calculate this without the use of technology.

Ex. 3



To calculate the area one must know the formula for area of a trapezoid. The area of a trap is equal to the **height (H)** multiplied by the **first base(b_1)** added to the **second base(b_2)** divided by **2** represented by the formula **(Ex 4):** $A_{Trp} = \frac{h(b_1 + b_2)}{2}$

Ex. 4



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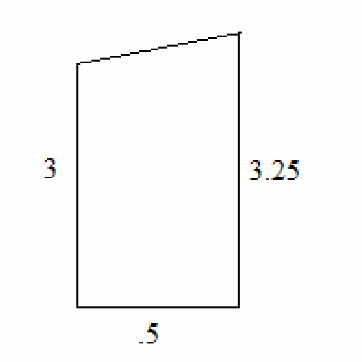
As shown in **(Ex 5) & (Ex 6)** I will replace those variables for numbers that correspond to the two trapezoids seen in **Ex 3.**

Ex. 5

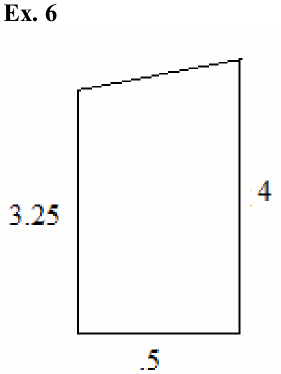


As seen above I allowed:
b_2= 3.25
b_1= 3
H = .5
Once you substitute these numbers into the equation simple arithmetic will allow one to come to an answer

$$A_{Trp} = \frac{.5(3.25+3)}{2}$$

Thus, the area of the first trapezoid would be **1.5625.**

Ex. 6



Above I allowed:
b_2= 4
b_1= 3.25
H = .5

After substituting the variables for values, the equation should look like:

$$A_{Trp} = \frac{.5(4+3.25)}{2}$$

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Thus, the Area of second trapezoid would be **1.81.**

Since the area under the curve is equal to $A_{trap1}+A_{trap2}$: **1.5625+ 1.81**

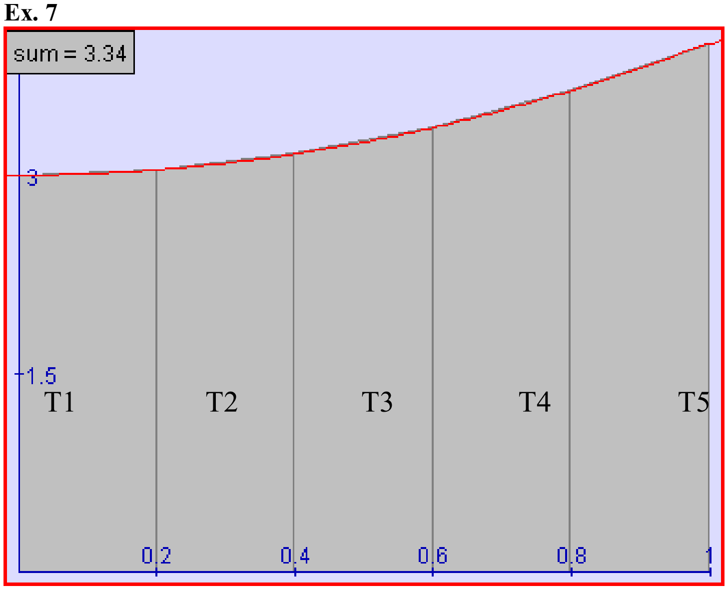
Then.... $A_{Trp} = \mathbf{3.37}$

Based on two trapezoids and on the interval x=0 and x=1.

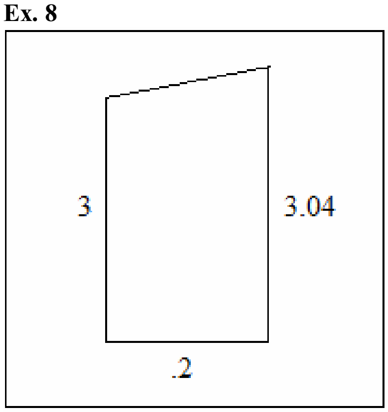
This approximation is the same as the approximation generated by the computer so one can assume this is correct but if one looks closely at **(Ex 3)** they will see that this approximation is only a rough estimate and in order to find more accurate results one must add more trapezoids.

(Ex 7) shows the same function with trapezoids increased from 2 to 5. This also decreased the x intervals from .5 to .2.

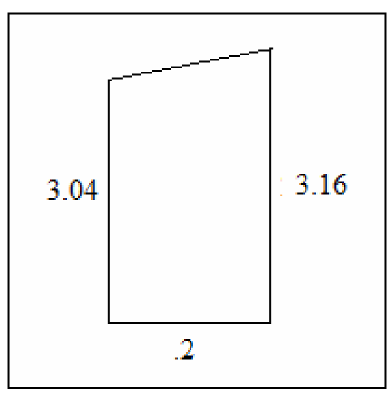
Ex. 7



Ex. 8



Ex. 9



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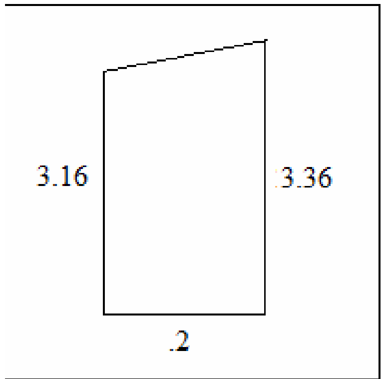
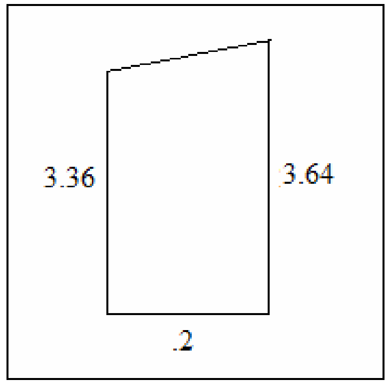
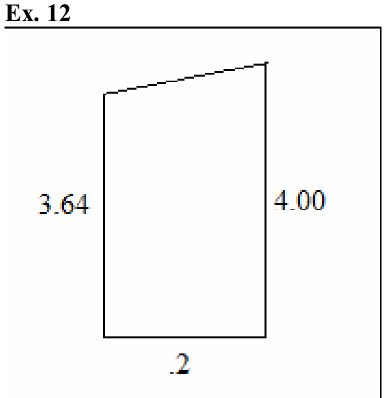
Ex. 10



Ex. 11



Ex. 12



Ex (8-12) illustrates each trapezoid from left to right and in order to find each area I have created a table in order to simplify the process- (Ex 13)

Ex. 13

Trapezoid	b_1	b_2	h	Area of trap
T1	3	3.04	0.2	**0.604**
T2	3.04	3.16	0.2	**0.620**
T3	3.16	3.36	0.2	**0.652**
T4	3.36	3.64	0.2	**0.700**
T5	3.64	4.00	0.2	**0.764**

The order is from left to right starting with the first and ending with the fifth. Adding all the areas….

0.604+0.620+0.652+0.700+0.764= $A_{trap=}$ 3.34.

One can assume that this is a more accurate approximation for the area under the curve because with each additional trapezoid one becomes closer to the actual value.

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Number of trapeziums	Total area
T1	3.5
T2	3.37
T4	3.34375
T5	3.34

As a result of creating a table, one can see a pattern more clearly. One learns know that the height will always depend on the interval and number of trapezoids being drawn. Therefore, the height equals $\frac{(b-a)}{n}$. N= number of trapezoids

Hence….

$$A_{Tap} = \left[\frac{\left[\left(\frac{b-a}{n}\right)b_1 + b_2\right]}{2}\right]$$

Then I also noticed that b_1 and b_2 for the first trapezoid would always be f_0 and $f_{\Delta x}$. Then that each successive trapezoid will contain the b_2 of the previous triangle for its b_1, and then the b_2 will be equal to $f_{2\Delta x}$; adding one to the constant in front of the Δx with each triangle. Based on the discovery the formula for the first trapezoid would be:

Also, b_1 and b_2 are equal to f_0 and $f_{\Delta x}$ and the next trapezoids b_2 will be that of the previous trapezoids b_1. b_2 is also equal to $f_{\Delta x}$.. This changes the formula:

$$A_{Tap} = \left[\frac{\left[\left(\frac{b-a}{n}\right)f_0 + f_{\Delta x}\right]}{2}\right]$$

This only applies to the area of one trapezoid; One needs the sum of the total trapezoids under the curve in order to find a accurate answer.

In order to simplify this:

$$\int_a^b f(x)dx = \left[\frac{\left[\left(\frac{b-a}{n}\right)f_a + f_{\Delta x}\right]}{2}\right] + .\bullet\bullet\bullet + \left[\frac{\left[\left(\frac{b-a}{n}\right)f_{n-1} + f_b\right]}{2}\right]$$

One can factor out the $\left(\frac{b-a}{n}\right)$ and 2 in order to have a more concise way of finding the solution. One is then left with:

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$$\int_a^b f(x)dx = \left(\frac{b-a}{n}\right) f_a + f_{\Delta x} + .\bullet\bullet\bullet + f_{n-1} + f_b$$

To make the above formula easier to use one can simplify to:

$$\int_a^b f(x)dx = b-a)\left(\frac{f_a - f_b}{2}\right)$$

Because one will use the base multiple times excluding the first and last $b_{1..}$. Which are f_a and f_b respectively. Just as done in u substitution the 2 can divided under either one and the answer will stay the same. Either general statement is correct but the first requires more information and be time consuming with a many trapeziums. While the second is easier to use but it could be somewhat less accurate.

I have considered the following using my general statement on the interval form x=1 to x=3

$y = (\frac{x}{2})^{\frac{2}{3}}$

$y = \frac{9x^2}{\sqrt{x^3 + 9}}$

$y = 4x^3 - 23x^2 + 40x - 18$

1)

$y = (\frac{x}{2})^{\frac{2}{3}}$

X	y	2g
1	0.629960525	-
1.25	0.731004435	1.462009
1.5	0.825481812	1.650964
1.75	0.914826428	1.829653
2	1	2
2.25	1.081687178	2.163374
2.5	1.160397208	2.320794
2.75	1.236521861	2.473044
3	1.310370697	-

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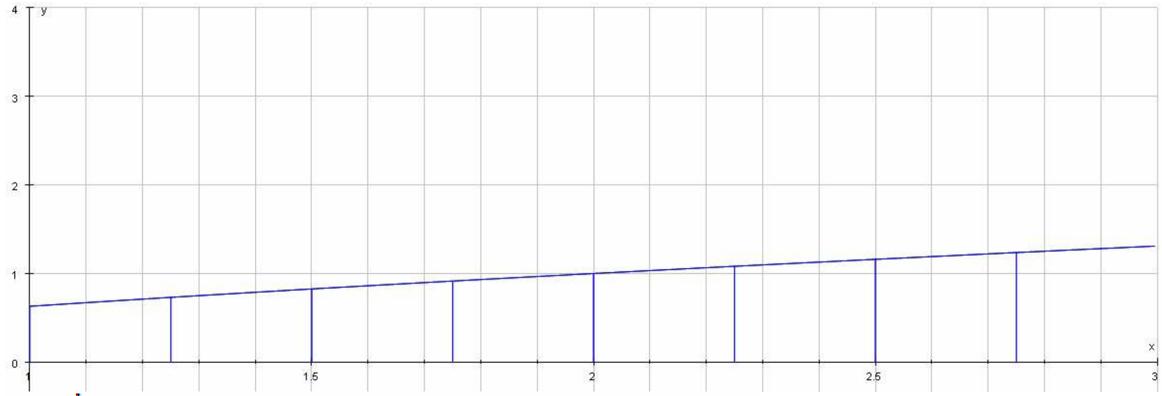

$$A_n = \frac{b-a}{2n}\left(g(x)_0 + 2g(x)_1 + 2g(x)_2 + \cdots + 2g(x)_{n-1} + g(x)_n\right)$$

$$A_8 = \frac{3-1}{2(8)}\left(g(1)_0 + 2g(1.25)_1 + 2g(1.5)_2 + 2g(1.75)_3 + 2g(2)_4 + 2g(2.25)_5 + 2g(2.5)_6 + 2g(2.75)_{8-1} + g(3)_8\right)$$

$$A_8 = \frac{2}{16}(0.629960525 + 1.462009 + 1.650964 + 1.829653 + 2 + 2.163374 + 2.320794 + 2.473044 + 1.310370697)$$

$$A_8 = \frac{2}{16}(15.84016906)$$

A_8=1.98

2)

$$y = \frac{9x^2}{\sqrt{x^3+9}}$$

x	y	2y
1	2.846	-
1.25	4.249	8.498
1.5	5.756	11.512
1.75	7.835	15.67
2	8.731	17.462
2.25	10.09	20.18
2.5	11.335	22.67
2.75	12.469	24.938
3	13.5	-

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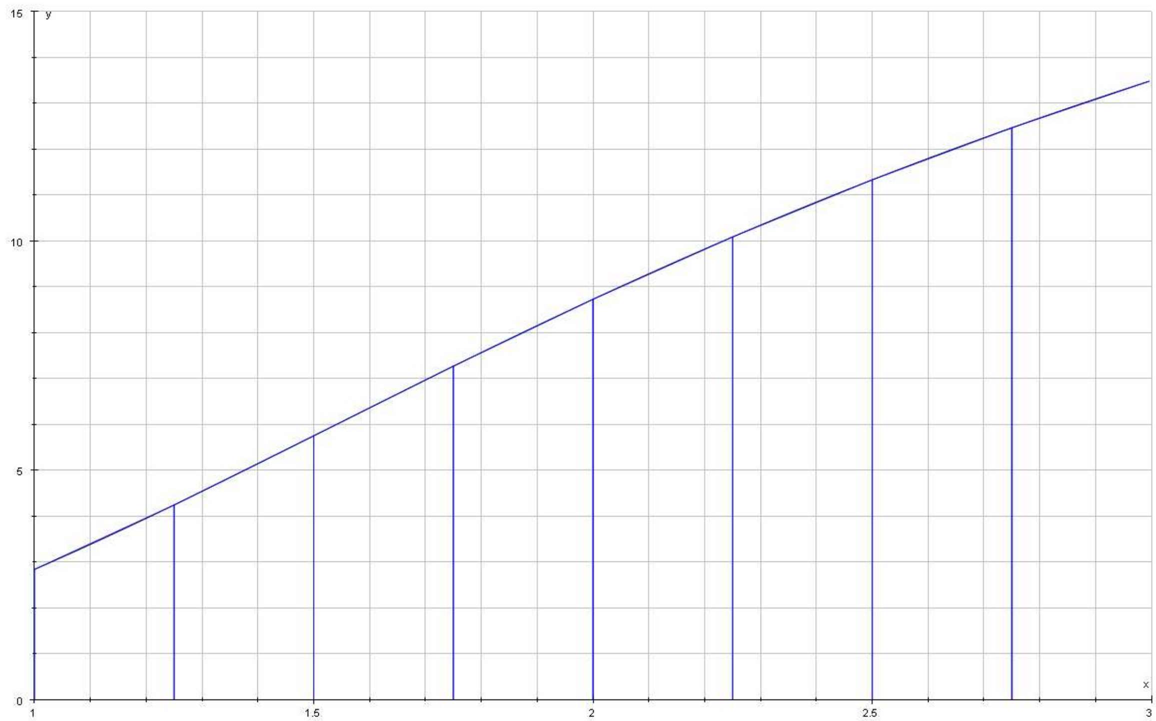

$$A_n = \frac{b-a}{2n}\left(g(x)_0 + 2g(x)_1 + 2g(x)_2 + \cdots + 2g(x)_{n-1} + g(x)_n\right)$$

$$A_8 = \frac{3-1}{2(8)}\left(g(1)_0 + 2g(1.25)_1 + 2g(1.5)_2 + 2g(1.75)_3 + 2g(2)_4 + 2g(2.25)_5 + 2g(2.5)_6 + 2g(2.75)_{8-1} + g(3)_8\right)$$

$$A_8 = \frac{2}{16}(2.846 + 8.498 + 11.512 + 15.67 + 17.462 + 20.18 + 22.67 + 24.938 + 13.5)$$

A_8=17.159

$y = 4x^3 - 23x^2 + 40x - 18$

x	y	2y
1	3	-
1.25	3.875	7.75
1.5	3.75	7.5
1.75	3	6
2	2	4
2.25	1.125	2.25
2.5	0.75	1.5
2.75	1.25	2.5
3	3	-

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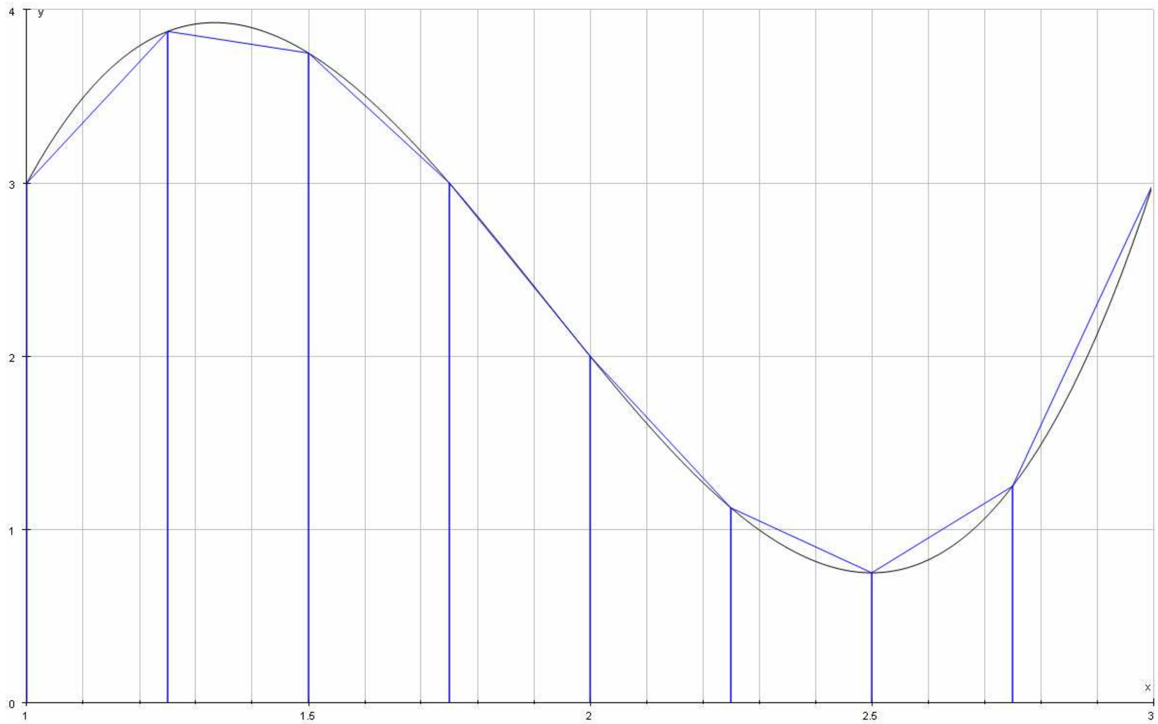

$$A_n = \frac{b-a}{2n}(g(x)_0 + 2g(x)_1 + 2g(x)_2 + \cdots + 2g(x)_{n-1} + g(x)_n)$$

$$A_8 = \frac{3-1}{2(8)}(g(1)_0 + 2g(1.25)_1 + 2g(1.5)_2 + 2g(1.75)_3 + 2g(2)_4 + 2g(2.25)_5 + 2g(2.5)_6 + 2g(2.75)_{8-1} + g(3)_8)$$

$$A_8 = \frac{3-1}{2(8)}(3 + 7.75 + 7.5 + 6 + 4 + 2.25 + 1.5 + 2.5 + 3)$$

A_8=4.6875

$$y = (\frac{x}{2})^{\frac{2}{3}}$$

$$\int_1^3 (\frac{x}{2})^{\frac{2}{3}} = 1.9807$$

$$y = \frac{9x^2}{\sqrt{x^3 + 9}}$$

$$\int_1^3 \frac{9x^2}{\sqrt{x^3 + 9}} = 17.026$$

$$y = 4x^3 - 23x^2 + 40x - 18$$

$$\int_1^3 4x^3 - 23x^2 + 40x - 18 = 4.667$$

I used my calculator to compare the trapeziums rule to the actual area. I pressed Y on my calculator and placed the function in my equation then graphed it. Next I pressed second function trace, entered the lower limit as one and the upper as 3, the calculator then calculated the area under the curve

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	Area		
Y=	Trapezium	Integration	Difference
$y = (\frac{x}{2})^{\frac{2}{3}}$	1.98	1.9807	0.0007
$y = \frac{9x^2}{\sqrt{x^3+9}}$	17.1595	17.026	0.1335
$y = 4x^3 - 23x^2 + 40x - 18$	4.6875	4.667	0.0205

The trapezium rule was very accurate in determining the area and one could find a closer approximation by adding more trapeziums under the curve.

Scope and Limitations:

The trap rule over estimates

$y = x^2$
Lower limit: 0
Upper limit: 3
N: 3

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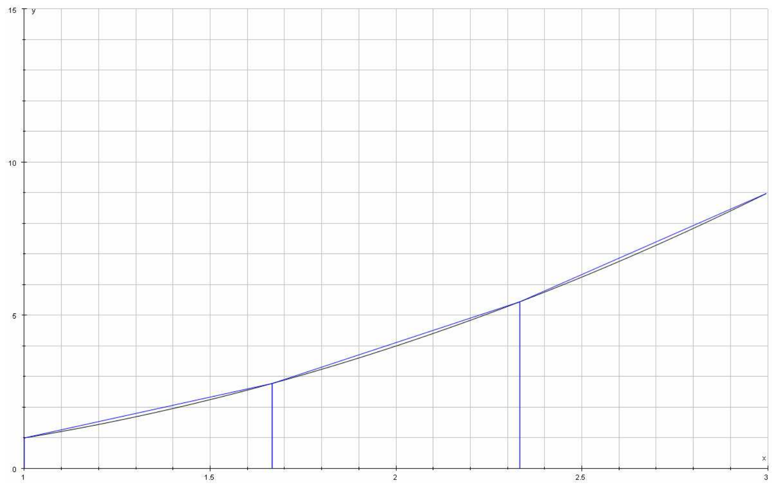
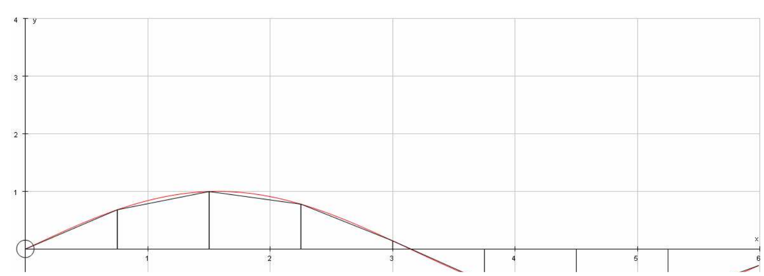


Sin curves are the only exception I found to the rule not working. The fluctuation between axes is too difficult for the rule to calculate. The graphs go from positive to negative and they areas negate each other.



The positive area and negative area-will negate each other. Thus if the formula doesn't change it will not work

$y = \sin(x)$

Lower limit: 0