Math portfolio: Modeling a functional building

The task is to design a roof structure for the given building. The building has a rectangular base 150 meters long and 72 meters wide. The height of the building should not exceed 75% of its width for stability or be less than half the width for aesthetic purpose. The minimum height of a room in a public building is 2.5 meters.

The height of the structure ranges from 36m to 54 (72×75%) m as per the specification.

At first I will model a curved roof structure using the minimum height of the structure that is 36 meters.

From the diagram given, the curve roof structure seems to be a parabola hence I will use a general equation of parabola that is

$y= ax^2 + bx +c$ -----------(1)

Now the width of the structure is 72 meters and the height is 36 meters.

Let the coordinate of the left bottom corner of the base is (0,0)

Then the coordinate of the right bottom corner will be (72,0) and the coordinate of the vertex of the parabola will be (36,36)

Since the above three points lies in the parabola, we will get 3 equation by substituting these coordinate in equation (1)

C=0 --------(2)

5184a + 72b = 0 -------(3)

1296a + 36b = 36 ------------(4)

Solving:

5184a + 72b = 0

5184a =-72b

$a = -\frac{72b}{5184}$

$a = -\frac{b}{72}$

Substituting $a = -\frac{b}{72}$ to (4)

$1296(-\frac{b}{72}) + 36b = 36$

-18b + 36b = 36

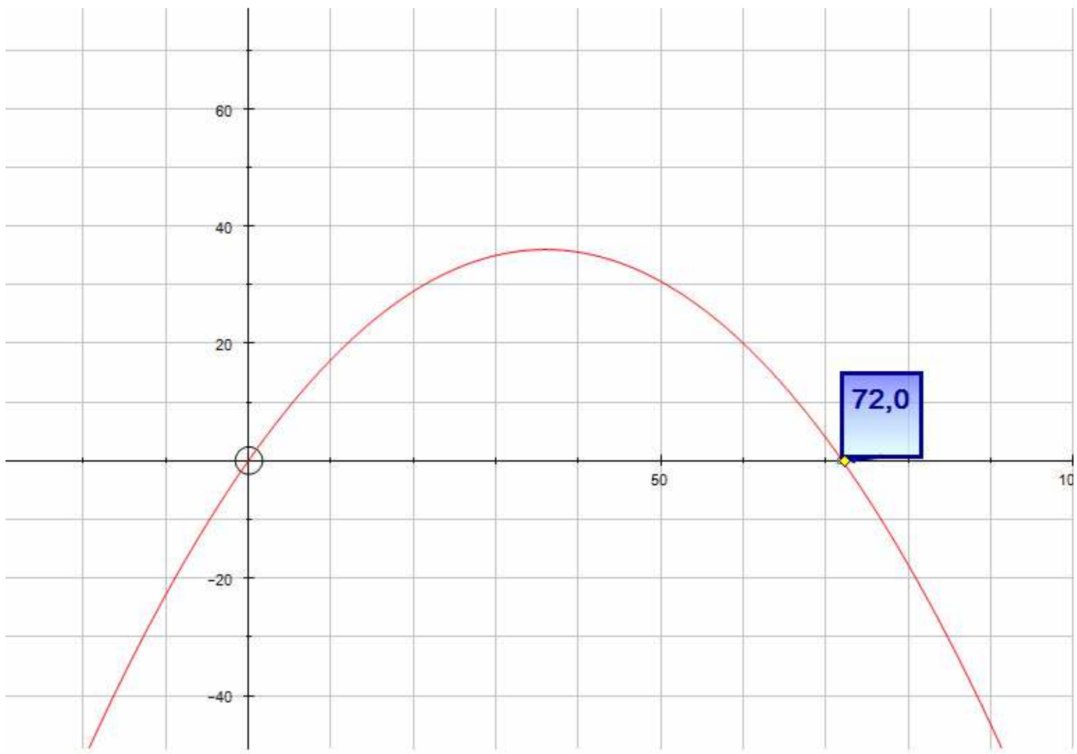
b = 2

Therefore: a $= -\frac{1}{36}$, b = 2 , c = 0

So the equation for the curved roof structure of 36 meters height will be

$y = -\frac{1}{36}x^2 + 2x$ -----(5)

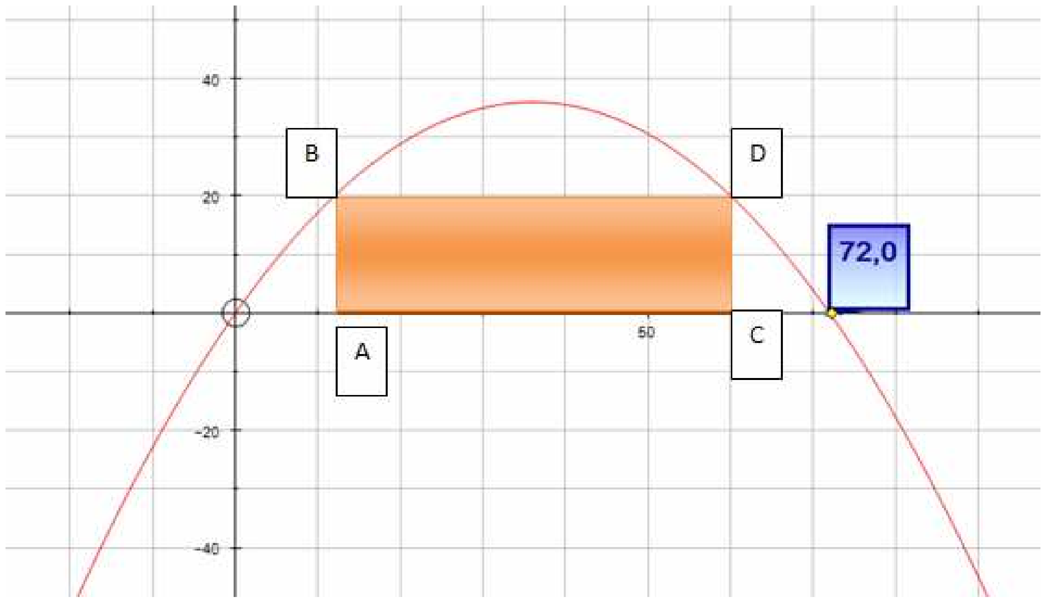
The given equation is graphed as shown below.



Now we need to find the dimensions of the largest possible cuboids which would fit inside the above curved roof structure.

I know that the length of the cuboid is 150 meters. So I have to find out the height and the width of the cuboid.

The diagram below shows a cuboid which is fitted inside the curve roof structure.



Let ABCD be the largest possible cuboid which can be fitted inside this curved roof structure

I will let “2V” be the width of the cuboid and “H” be the height.

The parabola is symmetrical structure, so do the cuboid, so I have taken the width as “2V” that is the width “V” is on the left side and “V” is on the right side of the axis of symmetry of the parabola.

The whole width of the structure is 72 meters.

Now AE=ED=V

OE=36 meters

So OA=36-V

Coordinate of A=(36-V,0)

Similarly, coordinate of D=(36+V,0)

As the upper corner of the cuboid B and C are lying on the curved roof structure so

Coordinate of B = (36-V, y(36-V))

Coordinate of C= (36+V, y(36+V))

y(36-V) is the height of the cuboid "H"

Substituting (36-V) into equation (5)

$y(36-V) = -\frac{1}{36}(36-V)^2 + 2(36-V)$

$= -\frac{1}{36}[(36-V)^2 - 72(36-V)]$

$= -\frac{1}{36}(36-V)(36-V-72)$

$=\frac{1}{36}(36-V)(36+V)$

$y(36-V)= \frac{1}{36}(1296-V^2)$ --------------(6)

The volume of the cuboid = length × width × height

Volume = 150 × 2v × $\frac{1}{36}(1296-V^2)$

Volume = $\frac{150}{18}[(1296V-V^3)]$------------(7)

To get the value of "v" for which cuboid has maximum volume, the above equation (7) should be differentiated with regard to "v" and then equated to zero

$\frac{dv}{d"v"} = (1296V-V^3) = 0$------------(8)

So $3v^2 = 1296$

V= ±20.78

This value can be maxima or minima hence I will differentiate the equation (8) again and check whether the result is positive or negative.

$\frac{d^2y}{dx^2} = (-6V) = -124.68$

As the above value is negative, hence the value of "v"20.78 is for largest cuboid.

Therefore width of the cuboid = 2v = 41.56 meters

Using equation (6)

$y(36-V)= \frac{1}{36}(1296-(20.78)^2)=24$

Hence the height = 24meters

Volume=150 $\times$ 41.56 $\times$ 24=149616 m^3

So the dimensions of the cuboid are

Height = 24m , Width=41.56m, Length=150m

I have developed the equation for the curve roof structure when the height of the structure was 36 m

Now I will develop a general equation for the curved roof structure where the height of the structure is "h"

Now the coordinate of the left bottom corner of the base is (0,0), the coordinate of the right bottom corner is (72,0) and the coordinate of the top corner will be (36,h)

Since the above 3 point lies in the parabola, we will get 3 equations.

C=0 -------(9)

5184a + 72b = 0 -------(10)

1296a + 36b = h ------------(11)

Solving:

5184a + 72b = 0

5184a =-72b

$a = -\frac{72b}{5184}$

$a = -\frac{b}{72}$

Substituting $a = -\frac{b}{72}$ to (4)

$1296(-\frac{b}{72}) + 36b = h$

-18b + 36b = h

18b = h

$b=\frac{h}{18}$

Substituting $b=\frac{h}{18}$ to $a = -\frac{b}{72}$

$a= -\frac{h}{1296}$, $b=\frac{h}{18}$

Therefore: $a = -\frac{h}{1296}$, $b = \frac{h}{18}$

So the equation will be

$y = -\frac{h}{1296}x^2 + \frac{h}{18}x$ --------(12)

This is the equation for the curved roof structure of height "h"

I will find the dimension of the cuboid of maximum volume in this curved roof structure.

Coordinate of A=(36-V,0)

Similarly, coordinate of D=(36+V,0)

As the upper corner of the cuboid B and C are lying on the curved roof structure so

Coordinate of B = (36-V, y(36-V))

Coordinate of C= (36+V, y(36+V))

y(36-V) is the height of the cuboid "h"

Substituting (36-V) into equation (12)

$Y(36-v) = -\frac{h}{1296}(36-v)^2 + \frac{h}{18}(36-v)$

$= -\frac{h}{1296}[(36-v)^2-72(36-v)]$

$= -\frac{h}{1296}[(36-w)^2- 2592+72v]$

$= -\frac{h}{1296}[1296-72v+v^2-2592+72v]$

$h = \frac{h}{1296}[1296 - v^2]$ ------(13)

Volume=width × height × length

$=2v \times [\frac{h}{1296}(1296 - v^2)] \times 150$

$= 150[\frac{h}{648}(1296v – v^3)]$

To get "v" for the maximum volume of the cuboid, the above equation should be differentiated with regard to "v" and then equate to zero

$\frac{dy}{dx}=150[\frac{h}{648}(1296v – v^3)]=0$

Therefore $1296-3v^2=0$

$-3V^2=-1296$

$V=\pm 20.78$m

This value can be maxima or minima hence I will differentiate the equation (8) again and check whether the result is positive or negative.

$\frac{d^2y}{dx^2}=(-6V)=-124.68$

As the above value is negative, hence the value of “v”20.78 is for largest cuboid.

Therefore width of the cuboid = 2v = 41.56 meters

The value is same as in the case of 36 meters height structure hence I can say that the width of the largest cuboid does not change in the structure’s height.

Using the equation (13)to find out the height:

$h = \frac{h}{1296}[1296 - (20.78)^2]$

h =0.67h

The above relation is used to find the dimensions and the volume of the cuboid for different heights of the structure.

The length of the cuboid= 150 meters

The width of the cuboid =41.56meters

The width and the length of the cuboid do not change with the change in the structure’s height.

Height of the cuboid is the only variable which affects the volume of the cuboid.

The table below shows the height and the volume of the cuboid for different height of the structure.

Here the height of the structure varies from 50% of 72 to 75% of 72 meters that is from 36 to 54 meters.

M2 fx =SUM(D2*G2*J2)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	height of the structure			width of the cuboid			height of the cuboid			depth of the cuboid			volume of the cuboid	
2	36			41.56			24.12			150			150364.08	
3	37			41.56			24.79			150			154540.86	
4	38			41.56			25.46			150			158717.64	
5	39			41.56			26.13			150			162894.42	
6	40			41.56			26.8			150			167071.2	
7	41			41.56			27.47			150			171247.98	
8	42			41.56			28.14			150			175424.76	
9	43			41.56			28.81			150			179601.54	
10	44			41.56			29.48			150			183778.32	
11	45			41.56			30.15			150			187955.1	
12	46			41.56			30.82			150			192131.88	
13	47			41.56			31.49			150			196308.66	
14	48			41.56			32.16			150			200485.44	
15	49			41.56			32.83			150			204662.22	
16	50			41.56			33.5			150			208839	
17	51			41.56			34.17			150			213015.78	
18	52			41.56			34.84			150			217192.56	
19	53			41.56			35.51			150			221369.34	
20	54			41.56			36.18			150			225546.12	

The above table shows that the height of cuboid increase with the increase in height of the structure. The volume of the structure of the cuboid increases with the increase in the height of the cuboid hence we can say that the volume of the cuboid increase with the increase in the height of the structure.

Now I calculate the ratio of the volume of the wasted space to the volume of the office block for each height mentioned above.

Volume of the waste space= volume of the structure – volume of the office block

Volume of the office block= height × length × width

=0.67H × 150 × 41.56

= 4176.78H

The volume of the structure= length × area of ABCD

= 150× area of ABCD

In order to find the area of ABCD, I then integrate the equation (12)from 0 to 72m

$\int_0^{72}\left(\frac{-h}{1296}x^2+\frac{h}{18}x\right)dx$

$\int_0^{72} -H\,[\frac{1}{1296}(x^2) + \frac{h}{18}x]\,dx$

$=-H\,[\frac{x^3}{3888} - \frac{x^2}{36}]_0^{72}$

$=-H\,[(\frac{72^3}{3888} - \frac{72^3}{36})-0]$

= 48H

Volume of the structure = 150×48H=7200H

Using this I will be able to calculate the ratio of wasted space to the volume of the office blocks for the structure height ranging from 36m to 54m.

fx =SUM(S2/M2)

M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
volume of the cuboid			volume of the structure			volume of the wasted space			ratio of the wasted space			height of the structure	
150364.08			259200			108835.9			0.723816			36	
154540.86			266400			111859.1			0.723816			37	
158717.64			273600			114882.4			0.723816			38	
162894.42			280800			117905.6			0.723816			39	
167071.2			288000			120928.8			0.723816			40	
171247.98			295200			123952			0.723816			41	
175424.76			302400			126975.2			0.723816			42	
179601.54			309600			129998.5			0.723816			43	
183778.32			316800			133021.7			0.723816			44	
187955.1			324000			136044.9			0.723816			45	
192131.88			331200			139068.1			0.723816			46	
196308.66			338400			142091.3			0.723816			47	
200485.44			345600			145114.6			0.723816			48	
204662.22			352800			148137.8			0.723816			49	
208839			360000			151161			0.723816			50	
213015.78			367200			154184.2			0.723816			51	
217192.56			374400			157207.4			0.723816			52	
221369.34			381600			160230.7			0.723816			53	
225546.12			388800			163253.9			0.723816			54	

The above table shows that the ratio of the wasted space to the office block is the same for different height of the structure.

I will calculate the total maximum office floor area in the block for different heights with the given specification.

Minimum height of a floor = 2.5meters

We know that h=0.67h

Area of the floor =width × height

=41.56×150

=6234m^2

Total maximum floor area=number of floors × 6234m^2

height of the structure	height of the cuboid	maximum number of floor	total maximum office floor area
36	24.12	9	56106
37	24.79	9	56106
38	25.46	10	62340
39	26.13	10	62340
40	26.8	10	62340
41	27.47	10	62340
42	28.14	11	68574
43	28.81	11	68574
44	29.48	11	68574
45	30.15	12	74808
46	30.82	12	74808
47	31.49	12	74808
48	32.16	12	74808
49	32.83	13	81042
50	33.5	13	81042
51	34.17	13	81042
52	34.84	13	81042
53	35.51	14	87276
54	36.18	14	87276

The above table shows that the number of floors that can be constructed in a building increase with the increase in the height of the structure. The total maximum office floor area increase with the increase with the number of floors that can be constructed

Now I will do the whole investigation when the façade is placed on the longer side of the base.

So the width of the base of the rectangular building = 150m

And the length of the rectangular building = 72m

Here height of the structure varies from 50%of 150m to 75% of 150mwhich is from 75m to 112.5m

Let the coordinates of the left bottom corner of the base is (0, 0)

Then the coordinate of the left bottom corner be (150, 0),and the coordinate of the top corner be (75, 75)

Since the above 3 points lies in the parabola, we will get 3 equations from equation(1)

C=0

5625a + 75b= 75

$\rightarrow$75a + b =1 ----(14)

22500a + 150b = 0

$\rightarrow$ 150a + b = 0 ----(15)

Solving:

(15)-(14):

75a = -1

a =$-\frac{1}{75}$

Substituted a =$-\frac{1}{75}$ to (14):

75($-\frac{1}{75}$) + b =1

-1 +b =1

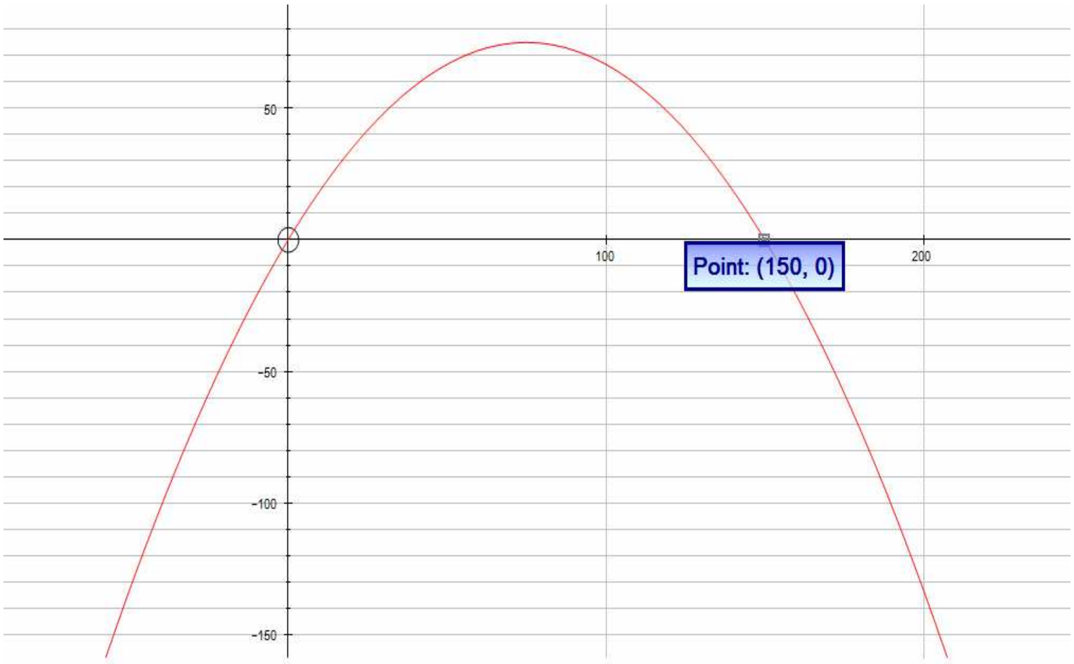
b = 2

We get a =$-\frac{1}{75}$, b=2, c= 0

So the equation for the curved roof structure of 75m height will be

Y=$-\frac{1}{75}x^2$ + 2x ------(16)

The above equation is graphed below.



Now I will develop a general equation for this curved roof structure when the height of the structure is "h"

Now the left and the right bottom corner will remain the same hence we have the same equations as earlier:

150a + b =0 ---------(17)

The coordinate for the top will be (75, h)

5625a + 75b = h ----------(18)

Solving the equation:

b = -150a

Substitute b = -150a to equation (18)

5625a + 75(-150a) = h -----------(19)

5625a – 11250a = h

a = $-\frac{h}{5625}$

Substituted a = $-\frac{h}{5625}$ to (17):

$150(-\frac{h}{5625}) + b = 0$

$b = \frac{2h}{75}$

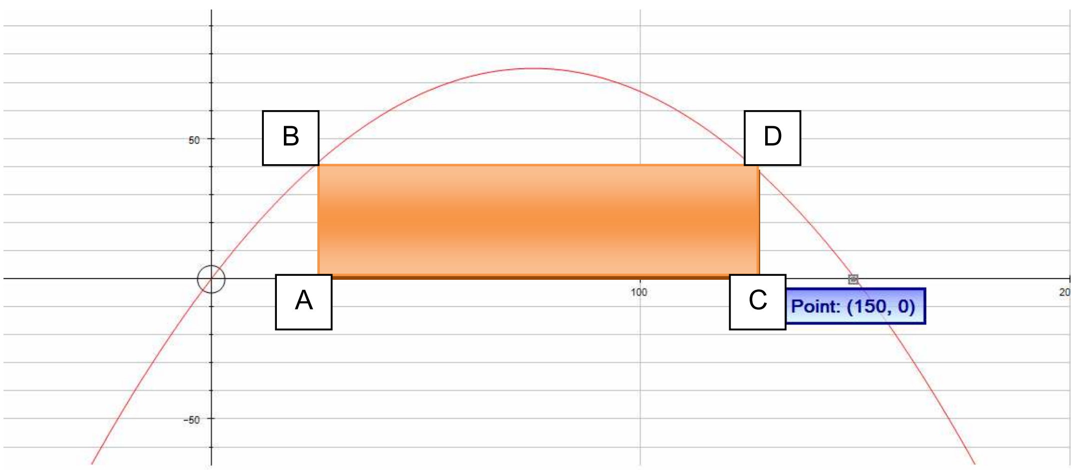
so the equation of the roof curve structure will be:

$y = -\frac{h}{5625}x^2 + \frac{2h}{75}x$

$= \frac{h}{75}[\frac{-1}{75}x^2 + 2x]$ ------(20)

Next I will find out the dimensions of the largest cuboid which can be fitted inside the curved roof structure of height 75 meters

The graph of the cuboid fitted inside the cruved



Coordinates of A = (75-v, 0)

Similarly, coordinate of D =(75+v, 0)

As the upper corner of the cuboid B and C are lying on the curved roof structure so

Coordinate of B= (75-v, y(75-v))

Coordinate of C=(75+v, y(75+v))

Y(75+v) is the height of the cuboid and lei it be "h"

Using equation (16), and coordinate of "B"

$Y(75\text{-}v) = -\frac{1}{75}(75\text{-}v)^2 + 2(75\text{-}v)$

$= -\frac{1}{75}[(75\text{-}v)^2\text{-}150(75\text{-}w)]$

$= -\frac{1}{75}(5625\text{-}150v\text{-}v^2\text{-}11250\text{+}150v)$

$= \frac{1}{75}(5625 - v^2)$ -----(21)

Volume of the cuboid = length × width × height

Volume = 72× 2v × $\frac{1}{75}(5625 - v^2)$

$= \frac{144}{75}(5625v - v^3)$ ----------(22)

To get the maximum volume of the cuboid, the above equation should be differentiate with regard to "v" and then equate to zero.

$\frac{dy}{dx} = \frac{144}{75}(5625v - v^3) = 0$

$\frac{144}{75}(5625v - 3v^2) = 0$ -------------(23)

$10800 - \frac{144}{25}v^2 = 0$

$V = \pm 43.3$

This value can be maxima or minima hence I will again differentiate the equation (23) and check for the result whether is positive or negative.

$\frac{d^2\text{"V"}}{dv^2} = (-6v) = -259.80$

As the above value is negative, hence the value of "v" is maximum.

Therefore width of the cuboid = 2v = 86.6m

Using equation (21)

$Y(75\text{-}v) = \frac{1}{75}(5625\text{-}43.3^2)$

= 50

Hence the height h= 50m

Volume= 72×86.6×50=311760m^3

So the dimensions of the cuboid are

Height = 50m , width = 86.6m, length= 72m

Now I will find out the dimension of the cuboid of maximum volume in the curved roof structure of height"h"

Coordinate of A = (75-v, 0)

Coordinate of D = (75+v, 0)

Coordinate of B = (75-v, y(75-v))

Coordinate of C= (75+v, y(75+v))

Y(75-v) is the height of the largest cuboid

Using equation (20) and coordinate of B

$Y(75-v)=\frac{h}{75}[\frac{-1}{75}(75-v)^2 + 2(75-v)]$

$=-\frac{h}{5625}((75-v)^2 -150(75 - v))$

$= -\frac{h}{5625}(5625-150v +v^2 -11250 +150v)$

$= -\frac{h}{5625}(v^2 -5625)$

$= \frac{h}{5625}(5625 - v^2)$ ------------------(24)

Volume = height × width × length

$=\frac{h}{5625}(5625 - v^2) \times 2v \times 72$

$=72[\frac{2h}{5625}(5625v - v^3)$

To get the maximum volume of the cuboid, the above equation should be differentiate with regard to "v" and then equate to zero.

$\frac{dy}{dx}= 72[\frac{2h}{5625}(5625v - v^3) = 0$

$=5625-3v^2 =0$

$V=\pm 43.3m$

Width = 2v = 86.6

This value is the same as in case of 75m height structure hence I can say that the width of the largest cuboid does not change with the change in the structure's height.

Using equation (24)

H= $\frac{h}{5625}(5625 - 43.3^2)$

=0.67h

Therefore the change of the width of the façade does not affect the relation between the height of the structure and the height of the cuboid of maximum volume.

The above relation is used to find the dimensions and the volume of the cuboid for different height of the structure.

The width and the length of the cuboid do not change with the change in the structure's height.

The height of the cuboid is the only variable which affect the volume of the cuboid.

The table below shoes the height and the volume of the cuboid for different height of the structure.

Here the height of the structure varies from 50% of 150 to 75% of 150 meters that is from 75 to 112.5 meters.

height of the structure	width of the cuboid	height of the cuboid	length of the cuboid	volume of the cuboid
75	86.6	50.25	72	313318.8
76	86.6	50.92	72	317496.4
77	86.6	51.59	72	321674
78	86.6	52.26	72	325851.6
79	86.6	52.93	72	330029.1
80	86.6	53.6	72	334206.7
81	86.6	54.27	72	338384.3
82	86.6	54.94	72	342561.9
83	86.6	55.61	72	346739.5
84	86.6	56.28	72	350917.1
85	86.6	56.95	72	355094.6
86	86.6	57.62	72	359272.2
87	86.6	58.29	72	363449.8
88	86.6	58.96	72	367627.4
89	86.6	59.63	72	371805
90	86.6	60.3	72	375982.6
91	86.6	60.97	72	380160.1
92	86.6	61.64	72	384337.7
93	86.6	62.31	72	388515.3
94	86.6	62.98	72	392692.9
95	86.6	63.65	72	396870.5
96	86.6	64.32	72	401048.1
97	86.6	64.99	72	405225.6
98	86.6	65.66	72	409403.2
99	86.6	66.33	72	413580.8
100	86.6	67	72	417758.4
101	86.6	67.67	72	421936
102	86.6	68.34	72	426113.6
103	86.6	69.01	72	430291.2
104	86.6	69.68	72	434468.7
105	86.6	70.35	72	438646.3
106	86.6	71.02	72	442823.9
107	86.6	71.69	72	447001.5
108	86.6	72.36	72	451179.1
109	86.6	73.03	72	455356.7
110	86.6	73.7	72	459534.2
111	86.6	74.37	72	463711.8
112	86.6	75.04	72	467889.4
112.5	86.6	75.375	72	469978.2

The above table shows that the height of the cuboid increase with the increase in the height of the structure. The volume of the cuboid increases with the increase in the height of the cuboid hence we can say that the volume of the cuboid increases with the increase in the height of the structure.

Now I going to calculated the ratio of the volume of the waste space to the volume of the office block for each height mentioned above.

Volume of the wasted space= volume of the structure – volume of the office block

Volume of the office block= width × height × length

86.6× 0.67h × 72 = 4177.58h

The volume of the structure= length × the area of the structure

To find the area of the structure, I will integrate the equation (20) from 0 to 150

$$\int_0^{150} \frac{h}{75}[\frac{-1}{75}x^2 + 2x]$$

$$\rightarrow \int_0^{150} -h[\frac{x^2}{5625} - \frac{2x}{75}] \, dx$$

$$A=\int_0^{150} -h[\frac{x^2}{5625} - \frac{2x}{75}] \, dx$$

$$A=-h\left[\frac{x^3}{16875} - \frac{2x^2}{150}\right]_0^{150}$$

$$A=-h(\frac{(150)^3}{16875} - \frac{2(150)^2}{150}) - 0$$

A= 100H

Volume of the structure= 72×100H=7200h

Using this, I will calculate the ratio of wasted space to the volume of the office blocks for the structure height ranging from 75m to 112.5m

volume of the cuboid	volume of the structure	volume of the wasted space	ratio of the wasted space	height of the structure
313318.8	540000	226681.2	0.723484	75
317496.4	547200	229703.6	0.723484	76
321674	554400	232726	0.723484	77
325851.6	561600	235748.4	0.723484	78
330029.1	568800	238770.9	0.723484	79
334206.7	576000	241793.3	0.723484	80
338384.3	583200	244815.7	0.723484	81
342561.9	590400	247838.1	0.723484	82
346739.5	597600	250860.5	0.723484	83
350917.1	604800	253882.9	0.723484	84
355094.6	612000	256905.4	0.723484	85
359272.2	619200	259927.8	0.723484	86
363449.8	626400	262950.2	0.723484	87
367627.4	633600	265972.6	0.723484	88
371805	640800	268995	0.723484	89
375982.6	648000	272017.4	0.723484	90
380160.1	655200	275039.9	0.723484	91
384337.7	662400	278062.3	0.723484	92
388515.3	669600	281084.7	0.723484	93
392692.9	676800	284107.1	0.723484	94
396870.5	684000	287129.5	0.723484	95
401048.1	691200	290151.9	0.723484	96
405225.6	698400	293174.4	0.723484	97
409403.2	705600	296196.8	0.723484	98
413580.8	712800	299219.2	0.723484	99
417758.4	720000	302241.6	0.723484	100
421936	727200	305264	0.723484	101
426113.6	734400	308286.4	0.723484	102
430291.2	741600	311308.8	0.723484	103
434468.7	748800	314331.3	0.723484	104
438646.3	756000	317353.7	0.723484	105
442823.9	763200	320376.1	0.723484	106
447001.5	770400	323398.5	0.723484	107
451179.1	777600	326420.9	0.723484	108
455356.7	784800	329443.3	0.723484	109
459534.2	792000	332465.8	0.723484	110
463711.8	799200	335488.2	0.723484	111
467889.4	806400	338510.6	0.723484	112
469978.2	810000	340021.8	0.723484	112.5

The above table shows that the ratio of the wasted space to the office block is same for different height of the structure.

I will calculate the total maximum office floor area in the block for different height within the given specifications.

Minimum height of a floor = 2.5m

We know that h = 0.67h

Area of the floor = width × length = 72 × 86.6=6235m^2

Total maximum floor area = number of floors × 6235m^2

Using the above information I will calculate the total maximum office floor area and the maximum number of floors that can be constructed in the building of different height.

height of the structure	height of the cuboid	maximum number of floors	total maximum office floor area
75	50.25	20	124700
76	50.92	20	124700
77	51.59	20	124700
78	52.26	20	124700
79	52.93	21	130935
80	53.6	21	130935
81	54.27	21	130935
82	54.94	21	130935
83	55.61	22	137170
84	56.28	22	137170
85	56.95	22	137170
86	57.62	23	143405
87	58.29	23	143405
88	58.96	23	143405
89	59.63	23	143405
90	60.3	24	149640
91	60.97	24	149640
92	61.64	24	149640
93	62.31	24	149640
94	62.98	25	155875
95	63.65	25	155875
96	64.32	25	155875
97	64.99	25	155875
98	65.66	26	162110
99	66.33	26	162110
100	67	26	162110
101	67.67	27	168345
102	68.34	27	168345
103	69.01	27	168345
104	69.68	27	168345
105	70.35	28	174580
106	71.02	28	174580
107	71.69	28	174580
108	72.36	28	174580
109	73.03	29	180815
110	73.7	29	180815
111	74.37	29	180815
112	75.04	30	187050
112.5	75.375	30	187050

The above table shows that the number of floors that can be constructed in a building increase with the increase in the height of the structure. The total maximum office floor area increase with the increase in the number of floors

We can see that the maximum number of floors that can be accommodated was 14 when the structure was 36 meters, whereas it is 30 when the height of the structure is 75meters.

Now if I have to maximize the office space then I have to have multiple cuboid instead of single cuboid. The cuboid are to be designed in such a way that the lowest cuboid whose base lies on the x axis has the maximum width and its upper edges are touching the parabola. Similarly the next cuboid whose base lies on the previous cuboid's upper edge and its upper edges are touching the parabola also, and so on.

I will calculate the maximized office space for the curved roof structure of height 36m and length 150m

As the height of the cuboid are fixed as 2.5m,the y coordinate of the upper edge will be 2.5 for the lowest cuboid. The x coordinate can be found out by suing equation (5) of the curved roof structure.

$y = -\frac{1}{36}x^2 + 2x$

$2.5= -\frac{1}{36}x^2 + 2x$

$-\frac{1}{36}x^2 + 2x -2.5=0$

$x^2-72x+90=0$

x=1.27, 70.73

Width of the cuboids base= 70.73-1.27=69.46m

Volume of the cuboid=150×69.46×2.5=26046m^3

The next cuboid's base will be on the upper edge of the previous cuboid. The height of this cuboid will be 2.5+2.5=5, therefore use the same method as before

$y = -\frac{1}{36}x^2 + 2x$

$5= -\frac{1}{36}x^2 + 2x$

$-\frac{1}{36}x^2 + 2x -5=0$

x^2-72x+180=0

x=2.593, 69.406

Width of the cuboids base= 69.406-2.593=66.81m

Volume of the cuboid=150×66.81×2.5=25055m^3

The height of this cuboid will be 2.5+2.5+2.5=7.5

$y = -\frac{1}{36}x^2 + 2x$

$7.5= -\frac{1}{36}x^2 + 2x$

$-\frac{1}{36}x^2 + 2x -7.5=0$

x^2-72x+270=0

x=3.97, 68.03

Width of the cuboids base= 68.03-3.97=64.06m

Volume of the cuboid=150×64.06×2.5=24022.5m^3

The height of this cuboid will be 2.5+2.5+2.5+2.5=10

$y = -\frac{1}{36}x^2 + 2x$

$10= -\frac{1}{36}x^2 + 2x$

$-\frac{1}{36}x^2 + 2x -10=0$

x^2-72x+360=0

x=5.41, 66.59

Width of the cuboids base= 66.59-5.41=61.18m

Volume of the cuboid=150×61.18×2.5=22942.5m^3

The height of this cuboid will be 2.5+2.5+2.5+2.5+2.5=12.5

$y = -\frac{1}{36}x^2 + 2x$

$12.5 = -\frac{1}{36}x^2 + 2x$

$-\frac{1}{36}x^2 + 2x - 12.5 = 0$

$x^2 - 72x + 450 = 0$

x=6.91, 65.09

Width of the cuboids base= 65.09-6.91=58.18m

Volume of the cuboid=150×58.18×2.5=21817.5m^3

The height of this cuboid will be 2.5+2.5+2.5+2.5+2.5+2.5=15

$y = -\frac{1}{36}x^2 + 2x$

$15 = -\frac{1}{36}x^2 + 2x$

$-\frac{1}{36}x^2 + 2x - 15 = 0$

$x^2 - 72x + 540 = 0$

x=8.5, 63.5

Width of the cuboids base= 63.5-8.5=55m

Volume of the cuboid=150×55×2.5=20625m^3

The height of this cuboid will be 2.5+2.5+2.5+2.5+2.5+2.5+2.5=17.5

$y = -\frac{1}{36}x^2 + 2x$

$17.5 = -\frac{1}{36}x^2 + 2x$

$-\frac{1}{36}x^2 + 2x - 17.5 = 0$

$x^2 - 72x + 630 = 0$

x=10.19, 61.81

Width of the cuboids base= 61.81-10.19=51.62m

Volume of the cuboid=150×51.62×2.5=19357.5m^3

The height of this cuboid will be 2.5+2.5+2.5+2.5+2.5+2.5+2.5+2.5=20

$y = -\frac{1}{36}x^2 + 2x$

$20= -\frac{1}{36}x^2 + 2x$

$-\frac{1}{36}x^2 + 2x -20=0$

$x^2-72x+720=0$

x=12, 60

Width of the cuboids base= 60-12=48m

Volume of the cuboid=150×48×2.5=18000m^3

The height of this cuboid will be 2.5+2.5+2.5+2.5+2.5+2.5+2.5+2.5+2.5=22.5

$y = -\frac{1}{36}x^2 + 2x$

$22.5= -\frac{1}{36}x^2 + 2x$

$-\frac{1}{36}x^2 + 2x -22.5=0$

$x^2-72x+810=0$

x=13.95, 58.05

Width of the cuboids base= 58.05-13.95=44.1m

Volume of the cuboid=150×44.1×2.5=16537.5m^3

The height of this cuboid will be 2.5+2.5+2.5+2.5+2.5+2.5+2.5+2.5+2.5+2.5=25

$y = -\frac{1}{36}x^2 + 2x$

$25= -\frac{1}{36}x^2 + 2x$

$-\frac{1}{36}x^2 + 2x - 25 = 0$

$x^2 - 72x + 900 = 0$

x=16.1, 55.9

Width of the cuboids base= 55.9-16.1=39.8m

Volume of the cuboid=150×39.8×2.5=14925m^3

The height of this cuboid will be 2.5+2.5+2.5+2.5+2.5+2.5+2.5+2.5+2.5+2.5+2.5=27.5

$y = -\frac{1}{36}x^2 + 2x$

$27.5 = -\frac{1}{36}x^2 + 2x$

$-\frac{1}{36}x^2 + 2x - 27.5 = 0$

$x^2 - 72x + 990 = 0$

x=18.51, 53.49

Width of the cuboids base= 53.49-18.51=34.98m

Volume of the cuboid=150×34.98×2.5=13117.5m^3

The height of this cuboid will be 2.5+2.5+2.5+2.5+2.5+2.5+2.5+2.5+2.5+2.5+2.5+2.5=30

$y = -\frac{1}{36}x^2 + 2x$

$30 = -\frac{1}{36}x^2 + 2x$

$-\frac{1}{36}x^2 + 2x - 30 = 0$

$x^2 - 72x + 1080 = 0$

x=21.30, 50.70

Width of the cuboids base= 50.7-21.3=29.4m

Volume of the cuboid=150×29.4×2.5=11025m^3

The height of this cuboid will be
2.5+2.5+2.5+2.5+2.5+2.5+2.5+2.5+2.5+2.5+2.5+2.5+2.5=32.5

$y = -\frac{1}{36}x^2 + 2x$

$32.5 = -\frac{1}{36}x^2 + 2x$

$-\frac{1}{36}x^2 + 2x - 32.5 = 0$

$x^2 - 72x + 1170 = 0$

x=24.78, 47.22

Width of the cuboids base= 47.22-24.78=22.44m

Volume of the cuboid=150×22.44×2.5=8415m^3

The height of this cuboid will be
2.5+2.5+2.5+2.5+2.5+2.5+2.5+2.5+2.5+2.5+2.5+2.5+2.5+2.5=35

$y = -\frac{1}{36}x^2 + 2x$

$35 = -\frac{1}{36}x^2 + 2x$

$-\frac{1}{36}x^2 + 2x - 35 = 0$

$x^2 - 72x + 1260 = 0$

x=30, 42

Width of the cuboids base= 42-30=12m

Volume of the cuboid=150×12×2.5=4500m^3

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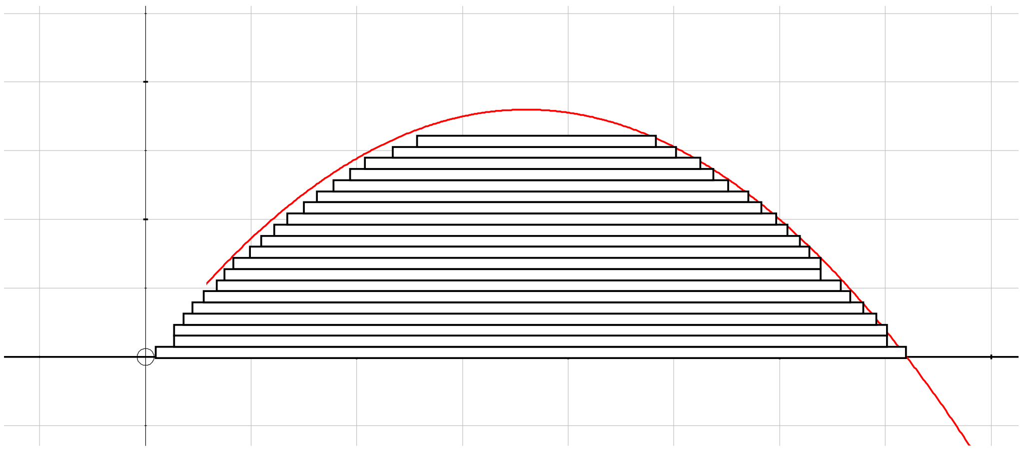

Total volume of the cuboids are=246386m^3

For 36m height, the volume of the structure= 7200h = 7200×36=259200m^3

Wasted space= 259200-246386=12814m^3

So the ratio of the volume of the wasted space to the office block is $\frac{12819}{246386} = 0.05$

The ratio of the volume of the wasted space to the office block is 0.72 in case of single cubiod arrangement whereas there is only 0.05 in the case of multiple cuboids. From the result we can say that for using multiple cuboid is more economical!!