ST. ROBERT CATHOLIC HIGH SCHOOL
PORTFOLIO ASSIGNMENT 1

OCTOBER 2009
LOGARITHM BASES SL TYPE I

This task consists of two parts. While both parts consider logarithms with different bases of the same argument, these parts are not necessarily directly related to each other.

PART 1 Exploring $\log_{m^n} m^k$

Determine the numerical values of the following sequences. Explain how you got these values. Justify your answers using technology.

$\log_2 8, \log_4 8, \log_8 8, \log_{16} 8, \log_{32} 8, \ldots$

$\log_3 81, \log_9 81, \log_{27} 81, \log_{81} 81, \ldots$

$\log_5 25, \log_{25} 25, \log_{125} 25, \log_{625} 25, \ldots$

$\log_2 8 = x$	$\log_4 8 = x$	$\log_8 8 = x$	$\log_{16} 8 = x$	$\log_{32} 8 = x$
$2^x = 8$	$4^x = 8$	$8^x = 8$	$16^x = 8$	$32^x = 8$
$2^x = 2^3$	$2^{2x} = 2^3$	$2^{3x} = 2^3$	$2^{4x} = 2^3$	$(2)^{5x} = 2^3$
$x = 3$	$2^x = 3$	$3^x = 3$	$4^x = 3$	$5^x = 3$
	$x = 3/2$	$x = 1$	$x = 3/4$	$x = 3/5$

Use of Technology

```
log(8)/log(2)
                3
log(8)/log(4)
              1.5
Ans▸Frac
              3/2
```

```
log(8)/log(8)
                1
log(8)/log(16)
              .75
Ans▸Frac
              3/4

log(8)/log(32)
               .6
Ans▸Frac
              3/5
```

$\log_3 81 = x$	$\log_9 81 = x$	$\log_{27} 81 = x$	$\log_{81} 81 = x$
$3^x = 81$	$9^x = 81$	$27^x = 81$	$81^x = 81$
$3^x = 3^4$	$3^{2x} = 3^4$	$3^{3x} = 3^4$	$3^{4x} = 3^4$
$x = 4$	$x = 2$	$x = 4/3$	$x = 1$

```
log(81)/log(3)
                4
log(81)/log(9)
                2
```

```
log(81)/log(27)
      1.333333333
Ans▸Frac
              4/3
log(81)/log(81)
                1
```

$\log_5 25 = x$	$\log_{25} 25 = x$	$\log_{125} 25 = x$	$\log_{625} 25 = x$
$5^x = 25$	$25^x = 25$	$125^x = 25$	$625^x = 25$
$5^x = 5^2$	$5^{2x} = 5^2$	$5^{3x} = 5^2$	$5^{4x} = 5^2$
$x = 2$	$x = 1$	$x = 2/3$	$x = 1/2$

```
log(25)/log(5)
                2
log(25)/log(25)
                1
```

```
log(25)/log(125)
      .6666666667
Ans▸Frac
              2/3

log(25)/log(625)
               .5
Ans▸Frac
              1/2
```

Write the next two terms in each of these sequences, in both logarithmic and numerical forms. Explain how you got these values.

The next two terms for the first sequence is based off this equation: $\log_{2^n} 8$ where n represents the term number. In this case the next two term numbers are 6 and 7.

$\log_{2^6} 8$ and $\log_{2^7} 8$

$= \log_{64} 8$ $= \log_{128} 8$

$\log_{64} 8 = x$	$\log_{128} 8 = x$
$64^x = 8$	$128^x = 8$
$2^{6x} = 2^3$	$2^{7x} = 2^3$
$6x = 3$	$7x = 3$
$x = 3/6 = 1/2$	$x = 3/7$

The next two terms for the second sequence is based off this equation: $\log_{3^n} 81$ where n represents the term number. In this case the next two term numbers are 5 and 6.

$\log_{3^5} 81$ and $\log_{3^6} 81$

$= \log_{243}81$ $\quad$ $= \log_{729}81$

$\log_{243}81 = x$
$243^x = 81$
$3^{5x} = 3^4$
$5x = 4$
$x = 4/5$

$\log_{729}81 = x$
$729^x = 81$
$3^{6x} = 3^4$
$6x = 4$
$x = 4/6 = 2/3$

The next two terms for the third sequence is based off this equation: $\log_{5^n} 25$ where n represents the term number. In this case the next two term numbers are 5 and 6.

$\log_{5^5} 25$ and $\log_{5^6} 25$

$= \log_{3125}25$ $\quad$ $= \log_{15625}25$

$\log_{3125}25 = x$
$3125^x = 25$
$5^{5x} = 5^2$
$5x = 2$
$x = 2/5$

$\log_{15625}25 = x$
$15625^x = 25$
$5^{6x} = 5^2$
$6x = 2$
$x = 2/6 = 1/3$

Write the nth term of each of these sequences, in logarithmic form and in the form $\frac{p}{q}$, where $p, q \in \mathbb{Z}$. Explain how you got these values.

The nth term of the first sequence is: $\log_{2^n} 2^3 = x$
In the form p/q:

$\log_{2^n} 2^3 = x$
$2^{nx} = 2^3$
$nx = 3$
$x = 3/n$

The nth term of the second sequence is: $\log_{3^n} 3^4 = x$
In the form p/q:

$\log_{3^n} 3^4 = x$
$3^{nx} = 3^4$
$nx = 4$
$x = 4/n$

The nth term of the third sequence is: $\log_{5^n} 5^2 = x$
In the form p/q:

$\log_{5^n} 5^2 = x$
$5^{nx} = 5^2$
$nx = 2$
$x = 2/n$

Now consider the general sequence, $\log_m m^k, \log_{m^2} m^k, \log_{m^3} m^k, \log_{m^4} m^k, \ldots$ where $k \in Z$.

Determine the values of the first five terms. Explain how you got these values.

$\log_m m^k = x$
$m^x = m^k$
$x = k$

$\log_{m^2} m^k = x$
$m^{2x} = m^k$
$2x = k$
$x = k/2$

$\log_{m^3} m^k = x$
$m^{3x} = m^k$
$3x = k$
$x = k/3$

$\log_{m^4} m^k = x$
$m^{4x} = m^k$
$4x = k$
$x = k/4$

$\log_{m^5} m^k = x$
$m^{5x} = m^k$
$5x = k$
$x = k/5$

Write the nth term of this sequence, in logarithmic form and in the form $\frac{p}{q}$, where $p, q \in Z$. Explain how you got this value.

The nth term of this sequence is:
$\log_{m^n} m^k = x$
$m^{nx} = m^k$
$nx = k$
$x = k/n$

What must be the relationship between the argument and first base if each term in the sequence is to have the form $\frac{p}{q}$, where $p, q \in Z$?

The relationship is that both the sequences have the form $\log_{m^q} m^p = x$ where m^q equals the base and m^p equals the resultant number of the logarithm. When both the base and resultant number are turned into the same base m to the power of p and q, the answer will be in the form of p/q.

PART 2 Exploring $\log_a x, \log_b x, \log_{ab} x$
Determine the numerical values of the following sequences. Explain how you got these values. Justify your answers using technology.

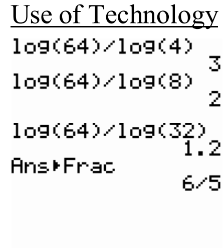
$\log_4 64, \log_8 64, \log_{32} 64$
$\log_7 49, \log_{49} 49, \log_{343} 49$
$\log_{\frac{1}{5}} 125, \log_{\frac{1}{125}} 125, \log_{\frac{1}{625}} 125$
$\log_8 512, \log_2 512, \log_{16} 512$

$\log_4 64 = x$
$4^x = 64$
$4^x = 4^3$
$2^{2x} = (2^2)^3$
$2x = 6$
$x = 3$

$\log_8 64 = x$
$8^x = 64$
$2^{3x} = 4^3$
$2^{3x} = (2^2)^3$
$3x = 6$
$x = 2$

$\log_{32} 64 = x$
$32^x = 64$
$2^{5x} = 4^3$
$2^{5x} = (2^2)^3$
$5x = 6$
$x = 6/5$

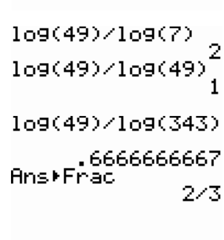
Use of Technology



$\log_7 49 = x$
$7^x = 49$
$7^x = 7^2$
$x = 2$

$\log_{49} 49 = x$
$49^x = 49$
$7^{2x} = 7^2$
$2x = 2$
$x = 1$

$\log_{343} 49 = x$
$343^x = 49$
$7^{3x} = 7^2$
$3x = 2$
$x = 2/3$



$\log_{1/5} 125 = x$

$1/5^{x} = 125$

$5^{-3x} = 5^{3}$

$-x = 3$

$x = -3$

$\log_{1/125} 125 = x$

$1/125^{x} = 125$

$5^{-x} = 5^{3}$

$-3x = 3$

$x = -1$

$\log_{1/625} 125 = x$

$1/625^{x} = 125$

$5^{-4x} = 5^{3}$

$-4x = 3$

$x = -3/4$

```
log(125)/log(1/5
)
                -3
log(125)/log(1/1
25)
                -1
log(125)/log(1/6
25)
              -.75
Ans▸Frac
              -3/4
```

$\log_8 512 = x$

$8^{x} = 512$

$2^{3x} = 8^{3}$

$2^{3x} = (2^{3})^{3}$

$3x = 9$

$x = 3$

$\log_2 512 = x$

$2^{x} = 512$

$2^{x} = 8^{3}$

$2^{x} = (2^{3})^{3}$

$x = 9$

$\log_{16} 512 = x$

$16^{x} = 512$

$2^{4x} = 8^{3}$

$2^{4x} = (2^{3})^{3}$

$4x = 9$

$x = 9/4$

```
log(512)/log(8)
                 3
log(512)/log(2)
                 9
log(512)/log(16)
              2.25
Ans▸Frac
               9/4
```

The third answer in each row can be obtained from the first two answers in that row. Explore several of these examples to conjecture a way to combine the first two answers to get the third. Confirm this conjecture in the remaining examples. Create two more examples to test your conjecture.

The formula that combines the first two answers to get the third is: (ab)/(a+b) where a is the first answer and b is the second answer of the sequence.

<u>Proof:</u>

<u>First Sequence</u>
First answer: 3
Second answer: 2
Third answer: $6/5$

$$\frac{3 \times 2}{3+2} = 6/5$$

<u>Second Sequence</u>
First answer: 2
Second answer: 1
Third answer: $2/3$

$$\frac{2 \times 1}{2+1} = 2/3$$

<u>Third Sequence</u>
First answer: -3
Second answer: -1
Third answer: $-3/4$

$$\frac{-3 \times -1}{-3+(-1)} = -3/4$$

<u>Fourth Sequence</u>
First answer: 3
Second answer: 9
Third answer: $9/4$

$$\frac{3 \times 9}{3+9} = 9/4$$

<u>Two more examples:</u>
$\log_3 81$, $\log_9 81$, $\log_{27} 81$

$\log_3 81 = x$

$3^{x} = 81$

$3^{x} = 3^{4}$

$x = 4$

$\log_9 81 = x$

$9^{x} = 81$

$3^{2x} = 3^{4}$

$2x = 4$

$x = 2$

$\log_{27} 81 = x$

$27^{x} = 81$

$3^{3x} = 3^{4}$

$3x = 4$

$x = 4/3$

First answer: 4
Second answer: 2

Third answer: $^4/_3$

$\frac{4 \times 2}{4+2} = {^8/_6} = {^4/_3}$

$\log_6 216$, $\log_{36} 216$, $\log_{216} 216$

$\log_6 216 = x$	$\log_{36} 216 = x$	$\log_{216} 216 = x$
$6^x = 216$	$36^x = 216$	$216^x = 216$
$6^x = 6^3$	$6^{2x} = 6^3$	$6^{3x} = 6^3$
$x = 3$	$2x = 3$	$3x = 3$
	$x = {^3/_2}$	$x = 1$

First answer: 3
Second answer: $^3/_2$
Third answer: 1

$\frac{3 \times {^3/_2}}{3 + {^3/_2}} = {^9/_2} \big/ {^9/_2} = 1$

Now consider the general case of $\log_a x, \log_b x, \log_{ab} x$.
Let $\log_a x = c, \log_b x = d$
Determine the general equation for $\log_{ab} x$ in terms of c and d.

The general equation for $\log_{ab} x$ in terms of c and d is: $x = {^{cd}/_{c+d}}$

<u>Proof:</u>

$\log_a x = c$, $a^c = x$, $a = x^{1/c}$
$\log_b x = d$, $b^d = x$, $b = x^{1/d}$
$\log_{ab} x = y$

$\log_{ab} x = y$
$(ab)^y = x$
$(x^{1/c} \times x^{1/d})^y = x$
$(x^{1/c + 1/d})^y = x$
$\left({^1/_c} + {^1/_d}\right)y = 1$
$y = \frac{1}{({^1/_c} + {^1/_d})}$
$y = \frac{1}{(c+d)/(cd)}$

$y = {^{cd}/_{(c+d)}}$

Test the validity of this equation using other values of a , b , and x .

Testing the equation: $y = {}^{cd}/_{(c+d)}$

$\log_{11} 1331$, $\log_{121} 1331$, $\log_{1331} 1331$

$\log_{11} 1331 = x$	$\log_{121} 1331 = x$	$\log_{1331} 1331 = x$
$11^x = 1331$	$121^x = 1331$	$1331^x = 1331$
$11^x = 11^3$	$11^{2x} = 11^3$	$11^{3x} = 11^3$
$x = 3$	$2x = 3$	$3x = 3$
	$x = {}^{3}/_{2}$	$x = 1$

First answer: 3

Second answer: 3/2

Third answer: 1

$$\frac{3 \times {}^{3}/_{2}}{(3 + {}^{3}/_{2})} = \frac{{}^{9}/_{2}}{{}^{9}/_{2}} = 1$$

Discuss the scope/limitations of a , b , and x .

Limitations:

$a, b, ab \text{ is} > 0 \text{ and} \neq 1$

a, b, ab:

- has to be always bigger than 0 but not equal to 1 because we cannot evaluate the logarithm of a negative base
- cannot equal to 1. Using the change of base formula, the logarithm of the bases is the denominator of the equation, $\log_y x = \frac{\log x}{\log y}$. When y equals to 1, the denominator is 0 as the logarithm of 1 is 0. Any number divided by 0 is undefined.

$x > 0$

x:

- is bigger than 0 because we cannot evaluate the logarithm of a negative number

Use the rules of logarithms to justify this equation.

Given: $\log_a x = c$, $\log_b x = d$, $\log_{ab} x = ?$

$$\log_a x = \frac{\log x}{\log a}$$

$$\therefore \quad \log a = \frac{\log x}{\log_a x} \quad \text{(equation 1)}$$

$$\log_b x = \frac{\log x}{\log b}$$

$$\therefore \ \log b = \frac{\log x}{\log_b x}$$ (equation 2)

$$\log_{ab} x = \frac{\log x}{\log ab}$$

$$= \frac{\log x}{\log a + \log b}$$

$$= \frac{\log x}{\frac{\log x}{\log_a x} + \frac{\log x}{\log_b x}}$$ (from equation 1 and 2)

$$= \frac{\log x}{\log x(\frac{1}{\log_a x} + \frac{1}{\log_b x})}$$

$$= \frac{1}{\frac{1}{\log_a x} + \frac{1}{\log_b x}}$$

$$= \frac{1}{\frac{1}{c} + \frac{1}{d}}$$

$$= \frac{1}{\frac{c+d}{cd}}$$

$$= \frac{cd}{c+d}$$