SL type 1 portfolio

Matrix Binomial

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Introduction

Matrices are rectangular tables of numbers or any algebraic quantities that can be added or multiplied in a specific arrangement. A matrix is a block of numbers that consists of columns and rows used to represent raw data, store information and to perform certain mathematical operations.

In this portfolio we are asked to generate different expressions from a series of matrices to the power of *n* and from this generate general statements that are appropriate to each question. Using a graphical display calculator all the basic mathematical calculation was made easier.

The first part of the question asks us to find $\boldsymbol{X^2}$,$\boldsymbol{X^3}$, $\boldsymbol{X^4}$, $\boldsymbol{Y^2}$ $\boldsymbol{Y^3}$, $\boldsymbol{Y^4}$given that:

$$X = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$X^2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

$$X^3 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix}$$

$$X^2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

$$X^4 = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix}$$

and

$$Y = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$Y^2 = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$$

$$Y^3 = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix}$$

$$Y^4 = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}$$

From the above an obvious trend emerges whereby when we increase the power of the matrix there is an increase in the resulting matrix. For the *X* matrices I observed that when X^2, the number in the resulting matrix are all 2's; when X^3, the numbers in the resulting matrix are all 4's; when X^4, the numbers in the resulting matrix are all 8's. A similar pattern has emerged in the *Y* matrix, except there are two negative signs present per matrix, in the same position, even as the matrix power increases.

From this information we asked to find expressions for $X^n, Y^n \; and \; (X+Y)^n$, by considering integer powers of *X* and *Y*. To do this I will substitute 7, 8 and 9 as powers of both *X* and *Y*.

$$X^7 = \begin{pmatrix} 64 & 64 \\ 64 & 64 \end{pmatrix} \qquad Y^7 = \begin{pmatrix} 64 & -64 \\ -64 & 64 \end{pmatrix}$$

$$X^8 = \begin{pmatrix} 128 & 128 \\ 128 & 128 \end{pmatrix} \qquad Y^8 = \begin{pmatrix} 128 & -128 \\ -128 & 128 \end{pmatrix}$$

$$X^9 = \begin{pmatrix} 256 & 256 \\ 256 & 256 \end{pmatrix} \qquad Y^9 = \begin{pmatrix} 256 & -256 \\ -256 & 256 \end{pmatrix}$$

From these values I deduced that the expressions would be:

$X^n = \begin{pmatrix} 2^{n-1} & 2^{n-1} \\ 2^{n-1} & 2^{n-1} \end{pmatrix}$ $\quad Y^n = \begin{pmatrix} 2^{n-1} & -(2^{n-1}) \\ -(2^{n-1}) & 2^{n-1} \end{pmatrix}$

However, we must note that this formula only gives us the progression for the scalar values which will be multiplied to the matrix, X. In order to find the final expression for X^n, we must multiply the general scalar value 2^{n-1} by matrix X and Y respectively:

$\mathrm{X}^n = 2^{n-1}\mathrm{X}$ and $\mathrm{Y}^n = 2^{n-1}\mathrm{Y}$

To generate the general statement for $(X+Y)^n$ I will substitute 7, 8 and 9 as powers of X and Y and add the two together.

$$(X+Y)^7 = \begin{bmatrix} 64 & 64 \\ 64 & 64 \end{bmatrix} + \begin{bmatrix} 64 & -64 \\ -64 & 64 \end{bmatrix} = \begin{bmatrix} 128 & 0 \\ 0 & 128 \end{bmatrix}$$

$$(X+Y)^8 = \begin{bmatrix} 128 & 128 \\ 128 & 128 \end{bmatrix} + \begin{bmatrix} 128 & -128 \\ -128 & 128 \end{bmatrix} = \begin{bmatrix} 256 & 0 \\ 0 & 256 \end{bmatrix}$$

$$(X+Y)^9 = \begin{bmatrix} 256 & 256 \\ 256 & 256 \end{bmatrix} + \begin{bmatrix} 256 & -256 \\ -256 & 256 \end{bmatrix} = \begin{bmatrix} 512 & 0 \\ 0 & 512 \end{bmatrix}$$

From these values I deduced that the expression for $(X+Y)^n = \begin{bmatrix} 2^n & 0 \\ 0 & 2^n \end{bmatrix}$, however we must again consider that this formula only gives us the progression for the scalar values which will be multiplied to the matrix, (X+Y) in order to yield the product $(X+Y)^n$. In order to find the final expression for $(X+Y)^n$, we must multiply the general scalar value 2^{n-1} by matrix (X+Y):

$(X+Y)^n = (2^{n-1} \cdot X)(2^{n-1} \cdot Y) = 2^{n-1}(X+Y)$

The next part of the question asks that we consider *A*=*aX* and *B*=*bX*, whereby *a* and *b* are constant, and $X = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ *and* $Y = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$. It then asks we calculate $\boldsymbol{A^2}$,$\boldsymbol{A^3}$,$\boldsymbol{A^4}$;$\boldsymbol{B^2}$,$\boldsymbol{B^3}$,$\boldsymbol{B^4}$using different values of *a* and *b*.

i. Let a=1

$$\therefore \quad A = 1\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$A^2 = 1\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

$$A^3 = 1\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix}$$

$$A^2 = 1\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

$$A^4 = 1\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix}$$

ii. Let a=2

$$\therefore \quad A \ = a\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$A^2 = 2\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix}$$

$$A^3 = 2\begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix} = \begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix}$$

$$A^4 = 2\begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix} = \begin{bmatrix} 16 & 16 \\ 16 & 16 \end{bmatrix}$$

iii. Let b=1

∴ B= $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$$B^2 = 1\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$$

$$B^3 = 1\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix}$$

$$B^4 = 1\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}$$

iv. Let b=2

∴ $B \;= b\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$$B^2 = 2\begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix} = \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}$$

$$B^3 = 2\begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix} = \begin{bmatrix} 32 & -32 \\ -32 & 32 \end{bmatrix}$$

$$B^4 = 2\begin{bmatrix} 32 & -32 \\ -32 & 32 \end{bmatrix} = \begin{bmatrix} 128 & -128 \\ -128 & 128 \end{bmatrix}$$

We are then asked to find expressions for A^n, B^n and $(A+B)^n$, by considering integer powers of *A* and *B*. To do this I will use 2, 3 and 4 as integer powers. Remember essentially α*X*=*A* therefore A^2 is the same as $(\mathrm{aX})^2$.

When α= 1 then $A^2 = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

$$\frac{1}{1^2} \times A^2 = \begin{pmatrix} 2/_1 & 2/_1 \\ 2/_1 & 2/_1 \end{pmatrix} = X^2 = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}$$

When α= 2 then $A^2 = \begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix}$

$$\frac{1}{2^2} \times A^2 = \begin{pmatrix} 4/_2 & 4/_2 \\ 4/_2 & 4/_2 \end{pmatrix} = X^2 = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}$$

Because $\boldsymbol{A^2}$can be expressed as $\boldsymbol{a^2X^2}$ then $\boldsymbol{A^n}$ can be expressed like $\boldsymbol{a^nX^n}$ which can be expressed in a matrix form, like it is shown below.

$$\boldsymbol{A^n} = a^nX^n = a^n \begin{bmatrix} (2^{n-1}) & (2^{n-1}) \\ (2^{n-1}) & (2^{n-1}) \end{bmatrix} = \begin{bmatrix} a^n(2^{n-1}) & a^n(2^{n-1}) \\ a^n(2^{n-1}) & a^n(2^{n-1}) \end{bmatrix}$$

However, we must note that this formula only gives us the progression for the scalar values which will be multiplied to the matrix, A. In order to find the final expression for A^n, we must multiply the general scalar value $(2a)^{n-1}$ $by\ A\ or\ aX$ the final general expression being: $A^n = 2^{n-1} \cdot a^n \cdot A$

A similar technique can be used to determine the expression for B^n, which is equal to bY therefore $A^2 = (bY)^2$:

When b= 1 then $B^2 = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$

$$\frac{1}{1^2} \times A^2 = \begin{pmatrix} {}^2/_1 & -{}^2/_1 \\ -{}^2/_1 & {}^2/_1 \end{pmatrix} = X^2 = \begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}$$

When b= 2 then $B^2 = \begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix}$

$$\frac{1}{2^2} \times A^2 = \begin{pmatrix} {}^4/_2 & -{}^4/_2 \\ -{}^4/_2 & {}^4/_2 \end{pmatrix} = X^2 = \begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}$$

Because B^2can be expressed as $\boldsymbol{b^2X^2}$ then $\boldsymbol{B^n}$ can be expressed like $\boldsymbol{bX^n}$ which can be expressed in a matrix form, like it is shown below.

$$\boldsymbol{B^n} = b^n X^n = b^n \begin{bmatrix} (2^{n-1}) & (2^{n-1}) \\ (2^{n-1}) & (2^{n-1}) \end{bmatrix} = \begin{bmatrix} b^n(2^{n-1}) & b^n(2^{n-1}) \\ b^n(2^{n-1}) & b^n(2^{n-1}) \end{bmatrix}$$

However, we must note that this formula only gives us the progression for the scalar values which will be multiplied to the matrix, B. In order to find the final expression for A^n, we must multiply the general scalar value $(2b)^{n-1}$ $by\ B\ or\ bX$, the final general expression being: $B^n = 2^{n-1} \cdot b^n \cdot B$

To find the expression for $(A + B)^n$ a similar technique as before may be used; we know that *A=aX* and *B=bX* ∴ $(A + B)^n = (aX + bX)^n$. From this we can develop an expression for $(A + B)^n$:

$$\boldsymbol{(A + B)^n = (aX + bY)^n}$$

$$(aX + bY)^n = \left[a\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix} + b\begin{pmatrix}1 & -1\\-1 & 1\end{pmatrix}\right]^n$$

$$\left[a\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix} + b\begin{pmatrix}1 & -1\\-1 & 1\end{pmatrix}\right]^n = \left[\begin{pmatrix}a & a\\a & a\end{pmatrix} + \begin{pmatrix}b & -b\\-b & b\end{pmatrix}\right]^n$$

$$\left[\begin{pmatrix}a & a\\a & a\end{pmatrix} + \begin{pmatrix}b & -b\\-b & b\end{pmatrix}\right]^n = \begin{bmatrix}(a+b) & (a-b)\\(a-b) & (a+b)\end{bmatrix}^n$$

$$\boldsymbol{(A + B)^n} = \begin{bmatrix}(a+b) & (a-b)\\(a-b) & (a+b)\end{bmatrix}^n$$

The expression for $\boldsymbol{(A + B)^n}$ is then $\begin{bmatrix}(a+b) & (a-b)\\(a-b) & (a+b)\end{bmatrix}^n$

$$(A + B)^n = (2^{n-1} \cdot a^n \cdot X) \cdot (2^{n-1} \cdot b^n \cdot Y)$$

$$= 2^n \cdot a^n \cdot (X + Y)$$

Example:

When a=-1

$A^2 = 2^{2-1} \cdot -1^2 \cdot \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = 2 \cdot \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}$

When b=2

$B^3 = 2^{3-1} \cdot 2^2 \cdot \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = 4 \cdot 8 \cdot \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{pmatrix} 32 & -32 \\ -32 & 32 \end{pmatrix}$

When a=-2 and b= -2:

$(A+B)^4 = 2^{4-1} \cdot -2^4 \cdot \left(\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \right)$

$=128 \cdot \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 256 & 0 \\ 0 & 256 \end{bmatrix}$

Because $A^4 = \begin{bmatrix} 128 & 128 \\ 128 & 128 \end{bmatrix}$ and $B^4 = \begin{bmatrix} 128 & -128 \\ -128 & 128 \end{bmatrix}$

$\begin{bmatrix} 128 & 128 \\ 128 & 128 \end{bmatrix} + \begin{bmatrix} 128 & -128 \\ -128 & 128 \end{bmatrix} = \begin{bmatrix} 256 & 0 \\ 0 & 256 \end{bmatrix}$

The expression $(A+B)^n = 2^{n-1} \cdot a^n (X+Y)$is proved to be true.

For the next part of this Matrix Binomial question I am first asked to consider the following matrix: $M = \begin{bmatrix} a+b & a-b \\ a-b & a+b \end{bmatrix}$

From this matrix I must show that $M = A + B$, remembering A is aX and $B$is bYso

$A + B = (aX + bY)$:

M = A+B

$$\begin{pmatrix} a+b & a-b \\ a-b & a+b \end{pmatrix} = aX + bY$$

$$\begin{pmatrix} a+b & a-b \\ a-b & a+b \end{pmatrix} = \begin{pmatrix} a & a \\ a & a \end{pmatrix} + \begin{pmatrix} b & -b \\ -b & b \end{pmatrix}$$

$$\begin{pmatrix} a+b & a-b \\ a-b & a+b \end{pmatrix} = \begin{pmatrix} a+b & a-b \\ a-b & a+b \end{pmatrix}$$

$\therefore M = A + B$

To further illustrate that M= A+B I have substituted integers as values of a and b.

$Example_1$ Let a= -1 and b= -1

$$M = \begin{bmatrix} -1+(-1) & -1-(-1) \\ -1-(-1) & -1+(-1) \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$$

As before $A = \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$

$$\therefore \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} + \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$$

Proving that $M = A + B$

$Example_2$ Let a= -2 and b= -2

$$M = \begin{bmatrix} -2+(-2) & -2-(-2) \\ -2-(-2) & -2+(-2) \end{bmatrix} = \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix}$$

As before $A = \begin{bmatrix} -2 & -2 \\ -2 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix}$

$$\therefore \begin{bmatrix} -2 & -2 \\ -2 & -2 \end{bmatrix} + \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} = \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix}$$

Proving that once again that $M = A + B$

$Example_3$ Let a= -3 and b= -3

$$M = \begin{bmatrix} -3 + (-3) & -3 - (-3) \\ -3 - (-3) & -3 + (-3) \end{bmatrix} = \begin{bmatrix} -6 & 0 \\ 0 & -6 \end{bmatrix}$$

As before $A = \begin{bmatrix} -3 & -3 \\ -3 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 3 \\ 3 & -3 \end{bmatrix}$

$$\therefore \begin{bmatrix} -3 & -3 \\ -3 & -3 \end{bmatrix} + \begin{bmatrix} -3 & 3 \\ 3 & -3 \end{bmatrix} = \begin{bmatrix} -6 & 0 \\ 0 & -6 \end{bmatrix}$$

Once again proving that $M = A + B$

To show that $M^2 = A^2 + B^2$ I have used the same substitution method as shown above.

$M^2=A^2+B^2$

$$\begin{pmatrix} a+b & a-b \\ a-b & a+b \end{pmatrix} \begin{pmatrix} a+b & a-b \\ a-b & a+b \end{pmatrix} = (aX)^2 + (bY)^2$$

$$\begin{pmatrix} 2a^2+2b^2 & 2a^2-2b^2 \\ 2a^2-2b^2 & 2a^2+2b^2 \end{pmatrix} = \begin{pmatrix} 2a^2 & 2a^2 \\ 2a^2 & 2a^2 \end{pmatrix} + \begin{pmatrix} 2b^2 & -2b^2 \\ -2b^2 & 2b^2 \end{pmatrix}$$

$$\begin{pmatrix} 2a^2+2b^2 & 2a^2-2b^2 \\ 2a^2-2b^2 & 2a^2+2b^2 \end{pmatrix} = \begin{pmatrix} 2a^2+2b^2 & 2a^2-2b^2 \\ 2a^2-2b^2 & 2a^2+2b^2 \end{pmatrix}$$

$\therefore M^2 = A^2 + B^2$

To further illustrate that $M^2 = A^2 + B^2$ I have substituted integer values of a and b.

$Example_1$ Let a= -1 and b= -1

$$M^2 = \begin{bmatrix} -1+(-1) & -1-(-1) \\ -1-(-1) & -1+(-1) \end{bmatrix}^2 = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$$A^2 + B^2 = \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}^2 + \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}^2 = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \therefore \boldsymbol{M^2 = A^2 + B^2}$$

Finally we are asked to come to a general statement for M^n in terms of aX and bX.

From all the calculation thus far we can deduce that $\boldsymbol{M^n = (A^n + B^n)}$, therefore we can find the general statement by adding the expressions developed for $A^n \ and \ B^n$:

$$M^n = (a \cdot X) + (b \cdot X)$$

$$(A+B)^n = (a \cdot X) + (b \cdot X) \ = (2a)^{n-1} \cdot aX + (2b)^{n-1} \cdot bY$$

$$M^n = (2a)^{n-1} \cdot aX + (2b)^{n-1} \cdot bY = A^n + B^n$$

To validate my general statement I must now substitute values for *n*, a and *b:*

$Example_1$: $n = 1$ a=4 and b= 6

$$A = 4\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix} \qquad B = 6\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 6 & -6 \\ -6 & 6 \end{bmatrix}$$

$(A+B)^1 = \begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix} + \begin{bmatrix} 6 & -6 \\ -6 & 6 \end{bmatrix} = \begin{bmatrix} 10 & -2 \\ -2 & 10 \end{bmatrix}$

$\boldsymbol{A+B} = \begin{bmatrix} \mathbf{10} & \mathbf{-2} \\ \mathbf{-2} & \mathbf{10} \end{bmatrix}$

Using my general statement I should be able to come up with the same value for A+B:

A^n = aX $\therefore A^n = 4\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ and B^n = bX $\therefore B^n = 6\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$A^n + B^n = 4\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}^1 + 6\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}^1 = \begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix} + \begin{bmatrix} 6 & -6 \\ -6 & 6 \end{bmatrix} = \begin{bmatrix} 10 & -2 \\ -2 & 10 \end{bmatrix}$

$\boldsymbol{A^n + B^n} = \begin{bmatrix} \mathbf{10} & \mathbf{-2} \\ \mathbf{-2} & \mathbf{10} \end{bmatrix}$, Proving that my general statement is indeed correct.

$\text{Example}_{2:}$ n=3 a = 10 and b = 3

$A = 10\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 10 & 10 \\ 10 & 10 \end{bmatrix}$ $B = 3\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix}$

$A + B = \begin{bmatrix} 10 & 10 \\ 10 & 10 \end{bmatrix}^3 + \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix}^3 = \begin{bmatrix} 52 & 28 \\ 28 & 52 \end{bmatrix}$

$\boldsymbol{A+B} = \begin{bmatrix} \mathbf{52} & \mathbf{28} \\ \mathbf{28} & \mathbf{52} \end{bmatrix}$

Again using my general statement I should be able to get the same value for A+B:

A^n = aX $\therefore A^n = 10\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ and B^n = bX $\therefore B^n = 3\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$A^n + B^n = \begin{bmatrix} 10 & 10 \\ 10 & 10 \end{bmatrix}^3 + \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix}^3 = \begin{bmatrix} 52 & 28 \\ 28 & 52 \end{bmatrix}$

$A + B = \begin{bmatrix} 52 & 28 \\ 28 & 52 \end{bmatrix}$, once again proving that my general statement is indeed correct.

Limitations/Scope

- As is one of the characteristics of matrices, fractions cannot be put as a power only integers can be powers. Thus using a non integer on the formula would not generate an appropriate answer.
- The general formula also doesn't work for negative number as matrices can't have negative powers.
- Another limitation is that A B and n must not equal each other.