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Mathematics Internal Assessment Type 2

Mathematical Modelling

Modelling Probabilities in games of tennis.

IB Math HL

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Introduction

In this task I will be investigating Probabilities and investigating models based on probabilities in a game of tennis. I will look to start with a relatively easy and simplistic models where Adam and Ben play each other in Club practice and have a set number of point that they will play. I will then look to find an expected value for the number of points that Adam wins. For this expected value I will calculate a standard deviation to see how much does a randomly selected point vary from the mean. I will then look at Non Extended play games where a maximum of 7 points can be played. I will show that there are 70 ways in which the game can be played. I will do this with the help of the binomial probability distribution formula.

I will also calculate the odds of Adam winning the game and then look to generalize my model so that it does not only apply to only Adam and Ben but to any player.

After making generalized model, I will look at extended games where in theory games could go on forever. Here I will look to use the sum of an infinite geometric series to come up with an appropriate model. I will then use that model to find the odds of Adam winning extended games and then I will look to generalize this model too.

I will also test the model for different values of point winning probabilities and find out the odds for each of them, I will then look for patterns in the values for odds.

Finally, I will evaluate the benefits and limitations models such as these.

Part 1

The ratio of the points won by Adam and Ben are 2:1 respectively. Therefore Adam wins twice as many points as Ben does. Therefore Adam wins $\frac{2}{3}$ of the points and subsequently Ben wins $\frac{1}{3}$ of the points.

The distribution of X, the number of points won by Adam would be derived by using the binomial probability function and substituting variables and constants to arrive at an appropriate model for the distribution of X, the number of points won by Adam.

The distribution chosen is the binomial probability distribution because in the case of the games of tennis,

- There is a repetition of a number of independent trials in which there are two possible results, success [the event occurs ex. Adam wins the point] or failure [the event does not occur ex. Adam does not win the point].
- The probability of success p, is a constant for all trials.
- The probability of failure q is a constant for all trials.
  $q = 1 - p \;\; since \; p + q = 1$

$$P(X = r) \; = C_r^n (p)^r (q)^{n-r}$$

Here p can substituted by the $\frac{2}{3}$ since that is probability of Adam winning each point and q can be substituted with $\frac{1}{3}$ since $q = 1 - p \;\; and \; therefore \; q = 1 - \frac{2}{3} = \frac{1}{3}$.

$$P(X = r) \; = C_r^n \left(\frac{2}{3}\right)^r \left(\frac{1}{3}\right)^{n-r}$$

$C_r^n = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

Therefore, C_r^n represents the binomial coefficient and it can also be written as $\frac{n!}{r!(n-r)!}$

Now, n is substituted with 10 because it's the number of points which are played between Adam and Ben and we need to find how many points each player wins so r is substituted with A as a variable where it represents the number of points which are won by Adam and therefore $(10 - A)$ would represent the number of points won by Ben.

$P(X = A) \; = C_A^{10} \left(\frac{2}{3}\right)^A \left(\frac{1}{3}\right)^{10-A}$

The possible limitations to this value might be that although, Ben and Adam know that Adam wins twice as many points as Ben does, still conditions in practice may vary and a change in these conditions such as weather change, injury, illness etc. might lead to change in the probability of the points won by each player. The mathematical model stated above, does not take into the above mentioned factors such as injury or illness [such as dehydration] and this is a limitation because it makes the model unrealistic and merely theoretical. Therefore although it can be used relatively accurately to predict the most probable outcome, still it is not set in stone.

Also unless, the total numbers of points played are multiples of three, the last 2 points or any 2 points will have to be distributed such that for example Adam wins 1.333 of the 2 points and Ben wins 0.667 of the 2 points. This is not practically possible and a limitation to the model.

I do not have concerns about its validity other than the fact that the point winning probability will not be constant for all trials.

Now the binomial distribution model which has been developed above must be used to find the distribution of the points.

$$P(X = A) = C_A^{10}\left(\frac{2}{3}\right)^A\left(\frac{1}{3}\right)^{10-A}$$

The probability of Adam winning 'A' number of points can be found by substituting 'A' with the number of points that Adam Wins and after solving, the answer will be Adams probability of winning the given number of points that were substituted into the equation.

Data Table 1

Number of Points Won by Adam [A]	Solution – the probability of Winning 'A' points
0	$P(0) = C_0^{10}\left(\frac{2}{3}\right)^0\left(\frac{1}{3}\right)^{10} = \frac{1}{59049}$
1	$P(1) = C_1^{10}\left(\frac{2}{3}\right)^1\left(\frac{1}{3}\right)^9 = \frac{20}{59049}$
2	$P(2) = C_2^{10}\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^8 = \frac{180}{59049}$
3	$P(3) = C_3^{10}\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^7 = \frac{960}{59049}$
4	$P(4) = C_4^{10}\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^6 = \frac{3360}{59049}$
5	$P(5) = C_5^{10}\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right)^5 = \frac{8064}{59049}$
6	$P(6) = C_6^{10}\left(\frac{2}{3}\right)^6\left(\frac{1}{3}\right)^4 = \frac{13440}{59049}$

7	$P(7) = C_7^{10}\left(\frac{2}{3}\right)^7\left(\frac{1}{3}\right)^3 = \frac{15360}{59049}$
8	$P(8) = C_8^{10}\left(\frac{2}{3}\right)^8\left(\frac{1}{3}\right)^2 = \frac{11520}{59049}$
9	$P(9) = C_9^{10}\left(\frac{2}{3}\right)^9\left(\frac{1}{3}\right)^1 = \frac{5120}{59049}$
10	$P(10) = C_{10}^{10}\left(\frac{2}{3}\right)^{10}\left(\frac{1}{3}\right)^0 = \frac{1024}{59049}$

Data Table 2

The number of points won by Adam	Possible distribution of X, the number of points won by Adam. [in decimal form]	Possible distribution of X, the number of points won by Adam. [in fractional form]
0	$1.693508781 \times 10^{-5}$	$\frac{1}{59049}$
1	$3.387017562\ 10^{-4}$	$\frac{20}{59049}$
2	0.00304833158	$\frac{180}{59049}$
3	0.0162576843	$\frac{960}{59049}$
4	0.056901895	$\frac{3360}{59049}$
5	0.1365645481	$\frac{8064}{59049}$

6	0.2276075801	$\frac{13440}{59049}$
7	0.2601229487	$\frac{15360}{59049}$
8	0.1950922116	$\frac{11520}{59049}$
9	0.0867076496	$\frac{5120}{59049}$
10	0.017341529	$\frac{1024}{59049}$

I can also check for reasonability of these values because I know the fact that they must add up to 1.

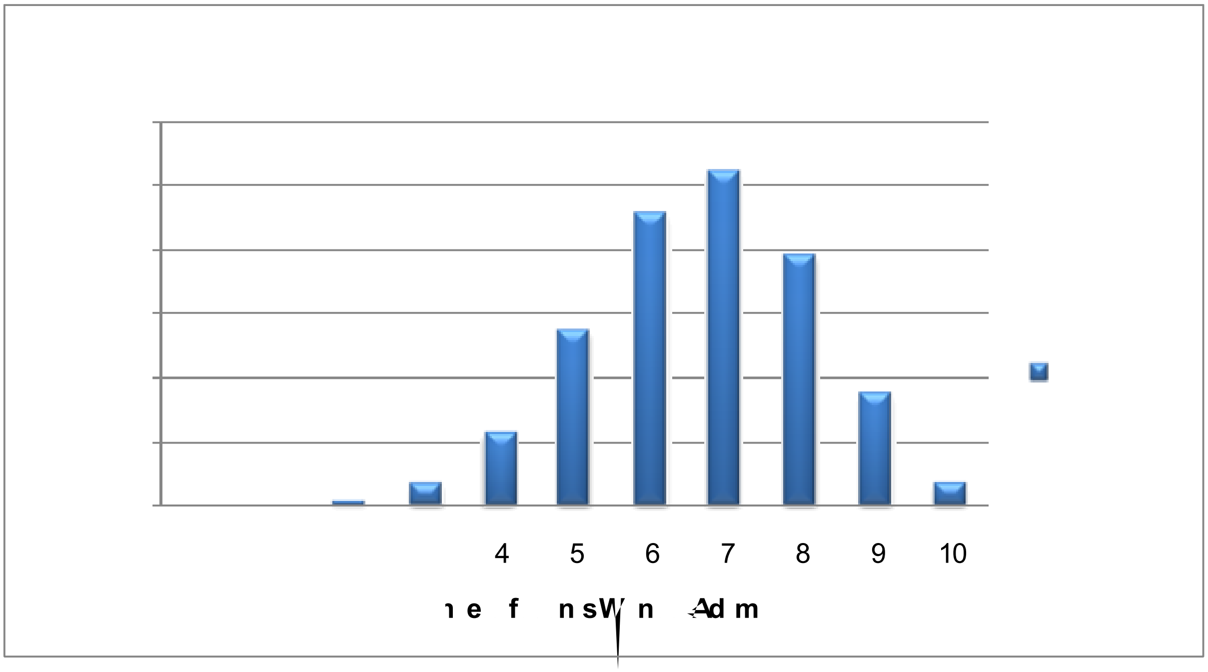
Therefore I will find the sum of the values of the distributions.

$$\frac{1}{59049}+\frac{20}{59049}+\frac{180}{59049}+\frac{960}{59049}+\frac{3360}{59049}+\frac{8064}{59049}+\frac{13440}{59049}+\frac{15360}{59049}+\frac{11520}{59049}+\frac{5120}{59049}+\frac{1024}{59049}$$

$$=\frac{59059}{59049}=1$$

Therefore the values for the distribution are reasonable as they add up to 1.

Although I have not yet calculated the mean or expected value for this binomial distribution, still by looking at the data table, I have a 'hunch' that it would be somewhere between 6 points and 7 points since, they are the events which are most likely to happen as they have probabilities of $\frac{13440}{59049}\ and\ \frac{15360}{59049}$ respectively. Also I notice that the probability of Adam winning 7 points is greater than him winning 6 points, therefore I think that the mean will be close to 7 than it is to 6.



Note: the histogram suggests that values for probability for 0 and 1 point won is 0 but this is not the case, it's just that the values are so small that they cannot be seen in the graph. These values can be observed in Data Table 2.

I had made a table in Microsoft Word and then transported it to Microsoft Excel with the help of which I created this histogram.

Now I will look to find the Expected value to see what the anticipated result is for the number of points that Adam wins.

The formula for the expected value is:

Expectation = np

Where n is the number of trials, therefore in this case it is the number of points played which will be 10]. p represents the probability of the event occurring and will be $\frac{2}{3}$ since we want to find the expected value for how many points Adam wins.

$$E = \mu = np$$

$$E = \mu = 10 * \frac{2}{3}$$

Therefore the expected value for the number of points that Adam wins will be $\frac{20}{3}\, points\ or\ 6.6\dot{6}\ which\ can\ also\ be\ represented\ as\ an\ approximated\ 6.67 points$

The value represents the mean for the binomial probability distribution

Therefore the expected value for the number of points that Adam wins is $\frac{20}{3}\, points$and now I will find the standard deviation to observe the typical amount by which a randomly selected point varies from the mean.

Consequently we can conclude that the expected value for Ben is $\frac{10}{3}$ because it would be $10 - \frac{20}{3}$ which would be $\frac{10}{3}\ or\ 0.333$

The standard deviation of this binomial random variable has the formula $\sigma = \sqrt{npq}$

Where n is the number of trials which would be 10 here, since there are 10 points played. p would be the probability of success and q the probability of failure and they would be $\frac{2}{3}\, and\, \frac{1}{3}\ respectively.$

Therefore:

$$\sigma = \sqrt{npq}$$

$$\sigma = \sqrt{10 * \frac{2}{3} * \frac{1}{3}}$$

$$\sigma = 1.491$$

Therefore a randomly selected point varies from the mean by 1.491points and the mean is 6.667. Thus normally Adam will win $[6.667 - 1.491]\ to\ [6.667 + 1.491]$ points which means that Adam makes between 5.176 to 8.158points and therefore we can conclude that Adam normally wins the practice games as he almost certainty gets more than half the points, which is more than 5 points.

Part 2

Now I will look at Non extended play games where to win a game, the player must win with at least 4 points and by at least 2 points, but to save court time, no game is allowed to go beyind 7 points. This means that if deuce is called and each player has 3 points then the next point determines the winner.

I want to show that the number of different ways in which the game can be played is 70 different ways and I will use primarily the binomial coefficient and lucid logic to show this.

The number of possibilities for the points played can be found by using the binomial coefficient formula which is $\binom{n}{k} = \frac{n!}{k!(n-k)!}$.

Therefore in order to find the number of possible games played, I will substitute n with the number of points up for contention [for the sake of calculation, this number is not always the total number of points played because the last point must always be won by the player whose probability of winning is being calculated therefore the n will always be the total points played subtracted by 1.]

However, k will be the number of points that need to be won by the player whose probability of winning is being calculated.

Data Table 3

Total Points played in the game. (Y)	The available points*	Binomial Distribution for Adam	The number of possibilities for the game being played when Adam loses all games.	Binomial distribution for Ben	The number of possibilities for the game being played when Ben loses all games.	The possibilities for losing for both the players	Total number of possibilities
4	_ _ _ -	$C_0^3 = \frac{3!}{3!0!}$	1	$C_0^3 = \frac{3!}{3!0!}$	1	2	2+8+20+40 = **<u>70</u>**
5	_ _ _ _ -	$C_1^4 = \frac{4!}{3!\,1!}$	4	$C_1^4 = \frac{4!}{3!\,1!}$	4	8	
6	_ _ _ _ _ -	$C_2^5 = \frac{5!}{3!\,2!}$	10	$C_2^5 = \frac{5!}{3!\,2!}$	10	20	
7	_ _ _ _ _ _ -	$C_3^6 = \frac{6!}{3!3!}$	20	$C_3^6 = \frac{6!}{3!3!}$	20	40	

*Each '_' here represents a possible point which either player could win but the last point is not up for contention in the distribution because for example if it was then in a combination where 5 points must be played there will be a combination such that one player wins 4 of the first 4 points and there then this would not be a game of 5 points. Therefore it is important to not include the last point in the calculation because it is the winning point and is always the last point and therefore it is imperative that its position be kept unchanged for the sake of calculation.

I have showed that there are 70 different ways that a game can be played by showing the different combinations possible when one player loses all the games. Therefore I found the combinations when Adam lost all the games and I found the combinations when Ben lost all the games and added the 2 values together because it represents all the different possibilities the game can be played because in each game one player loses and one player wins, therefore, there cannot be a draw since the total number of points cannot exceed 7 and by finding the different possibilities of each player losing, I have found the total number of ways in which the game can be played.

However, it is important also to note that, I could have reached the same results for the total number of games played by using a different approach. I could have looked at the ways in which both players can win all their games and I would have reached the same answer which would be 70 ways the game could be played.

Having found the number of ways the game can be played, now I will look to find the probability of Adam winning. I will look to find this by find the different ways in which Adam wins and finding the probabilities of the different ways in which Adam wins and then finding the sum of the probabilities.

The ways in which Adam can win is 4 – 0, 4 – 1, 4 – 2, 4 – 3 therefore I will find the sum of probability of these events occurring.

The model for the probability of the particular outcome of **Adam winning** occurring would be:

$$P(X=3) = C_3^n\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^{n-3} * C_1^1\left(\frac{2}{3}\right)^1\left(\frac{1}{3}\right)^0$$

Where n would be the points up for contention and r would be points that Adam must win in order to win the game which is 3 because the last point is not included as it is always last and must not be a possible combination. The highlighted part is the last point and this part of the model has no variables because it is the last point and always must be won by Adam.

$$P(X=r) = C_r^n\left(\frac{2}{3}\right)^r\left(\frac{1}{3}\right)^{n-r} * C_1^1\left(\frac{2}{3}\right)^1\left(\frac{1}{3}\right)^0$$

This model can also be written in this form, where n are the points for contention[or the total point – 1] and r is the points for contention that need to be won for adam, incidentally it always turns out to be 3.

The model for the probability of a particular outcome of **Ben winning** occurring would be:

$$P(X=3) = C_3^n\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^{n-3} * C_1^1\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^0$$

The highlighted part is the last point and this part of the model has no variables because it is the last point and must be always won by Ben

Now I will calculate each of ways Adam can win.

Probability of Adam winning 4- 0 $P(X=3)=C_3^3\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^0 * C_1^1\left(\frac{2}{3}\right)^1\left(\frac{1}{3}\right)^0$ $1*\left(\frac{2}{3}\right)^3 * \left(\frac{2}{3}\right)$ $=\left(\frac{2}{3}\right)^4=\frac{16}{81}$
Probability of Adam winning 4- 1 $P(X=3)=C_3^4\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^1 * C_1^1\left(\frac{2}{3}\right)^1\left(\frac{1}{3}\right)^0$ $=4*\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)*\left(\frac{2}{3}\right)$ $=4\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)=\frac{64}{243}$
Probability of Adam winning 4- 2 $P(X=3)=C_3^5\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^2 * C_1^1\left(\frac{2}{3}\right)^1\left(\frac{1}{3}\right)^0$ $=10*\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^2*\left(\frac{2}{3}\right)$ $=10\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^2=\frac{160}{729}$
Probability of Adam winning 4- 3 $P(X=3)=C_3^6\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^3 * C_1^1\left(\frac{2}{3}\right)^1\left(\frac{1}{3}\right)^0$ $=20*\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^3*\left(\frac{2}{3}\right)$ $=20\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^3=\frac{320}{2187}$
Therefore in order to find the probability of Adam winning the game, we must add the probabilities of him winning the different possible games.

$total\ probability\ of\ Adam\ winning\ the\ game = \frac{16}{81} + \frac{64}{243} + \frac{160}{729} + \frac{320}{2187}$
$= \frac{432}{2187} + \frac{576}{2187} + \frac{480}{2187} + \frac{320}{2187} = \frac{1808}{2187}$
Therefore the probability of Adam winning the game would be $\frac{1808}{2187}$.

The expression for the **Odds of Adam winning** is represented as
$\frac{probability\ of\ Adam\ winning}{probability\ of\ Adam\ losing}$

The probability of Adam winning has been stated above as $\frac{1808}{2187}$ and the probability of Adam losing will be 1 subtracted by the probability of Adam winning since there cannot be a draw of tie in this game.
Probability of Adam losing $= 1 - \frac{1808}{2187} = \frac{379}{2187}$
Odds = $\left[\frac{\left[\frac{1808}{2187}\right]}{\left[\frac{379}{2187}\right]}\right] = \frac{1808}{379} = 4.77$
Therefore the odds of Adam winning are 4.77:1

I will look to generalize the model which I have made so that it can be made applicable not only to the cases of Ben and Adam but other players and different probabilities.

$P(X = 3) = C_3^n \left(\frac{2}{3}\right)^3 \left(\frac{1}{3}\right)^{n-3} * C_1^1 \left(\frac{2}{3}\right)^1 \left(\frac{1}{3}\right)^0$

Now substituting $\frac{2}{3}$with c and $\frac{1}{3}$ with d.

$P(X = 3) = C_3^n (c)^3 (d)^{n-3} * C_1^1 (c)^1 (d)^0$

Now the probability of C winning would be the sum of the probabilities of different ways in which Player C would win the game. The ways in which player C could win the game would be 4-0, 4-1, 4-2 and 4-3. Therefore the total number of points would be 4,5,6 and 7. The points in contention therefore be 3, 4, 5 and 6 since 1 point, the winning point is fixed in each game.

Therefore the probability of winning would be:

For 4-0 result.
$P(X = 3) = C_3^3 (c)^3 (d)^0 * C_1^1 (c)^1 (d)^0$

$= c^4$
For 4-1 result $P(X=3) = C_3^4(c)^3(d)^1 * C_1^1(c)^1(d)^0$ $= \mathbf{4c^4d}$
For 4-2 result $P(X=3) = C_3^5(c)^3(d)^2 * C_1^1(c)^1(d)^0$ $= \mathbf{10c^4d^2}$
For 4-3 result $P(X=3) = C_3^6(c)^3(d)^3 * C_1^1(c)^1(d)^0$ $= \mathbf{20c^4d^3}$

Therefore, the probability of Player C winning the game would be:

$$\mathbf{c^4 + 4c^4d + 10c^4d^2 + 20c^4d^3}$$

Part 3

Having looked at Non-extended games of play, I will now investigate the probability of each player winning when the games are extended or can in theory go on forever because the players need to win 2 points in a row if and when deuce is achieved. Theoretically, the game could go on forever as the player could keep winning one point each and keep getting back to deuce.

I will divide the question into two parts, non-deuce games and deuce games as suggested by the question.

Non Deuce	Deuce
The possible results without deuce are 4 – 0 , 4 – 1 and 4 – 2 This would be: $c^4 + 4c^4d + 10c^4d^2$ Now I will substitute c and d with $\frac{2}{3}$ *and* $\frac{1}{3}$ *respectively* as they represent the chances of Adam and Ben winning each point.	The deuce will occur when the score is 3-3 Therefore the probability of this occurring would be: $C_3^6\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^3 = \frac{160}{729}$ This is the probability of deuce occurring and out of this a certain fraction Adam wins and the rest Ben wins. Now I will calculation what that fraction is, so that I can find the probability of Adam winning the deuce and add that value to the probability of Adam winning the non-deuce games to subsequently find the

	odds of him winning which is my eventual goal.
Therefore the expression for the probability of Adam winning the non-deuce games is: $\left(\frac{2}{3}\right)^4 + 4\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right) + 10\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^2$ $\frac{16}{81}+\frac{64}{243}+\frac{160}{729}=\frac{496}{729}$	Therefore after deuce has been achieved, Adam would need to win 2 points in a row, therefore the probability of that would be $C_2^2\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^0 = c^2$ Therefore with c^2 Adam would have won the game, but there is the possibility that Adam and Ben won 1 point each after the initial deuce and are back to deuce. This would be represented by $C_1^2\left(\frac{2}{3}\right)^1\left(\frac{1}{3}\right)^1 = 2cd$ Therefore the probability of Adam winning the deuce would be: $\frac{160}{729}(c^2 + 2cd(c^2 + 2cd(c^2 + 2cd(c^2 \ldots.$ The infinite geometric series goes on because Adam would have to win 2 points in a row and the deuce, in theory could go on forever, therefore the sum of the infinite geometric series must be found. The formula for the sum of an infinite geometric series is $S = \frac{u_1}{1-r}$ Where u_1 is the first term and r is the ratio. $S = \frac{c^2}{1-2cd}$ Now I will substitute the values for c and d with the probabilities of Adam and Ben winning each point. $S = \frac{\left(\frac{2}{3}\right)^2}{1-2\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)}$

	$S=\frac{\left(\frac{4}{9}\right)}{\left(\frac{5}{9}\right)}$ $S=\frac{4}{5}$ Therefore Adam wins $\frac{4}{5}$ of the deuce games. Therefore Adam will win $\frac{4}{5}*\frac{160}{729}=\frac{128}{729}$
Now I will add the value of the probability of Adam winning deuce and non-deuce games to find his total probability of winning and then I will find his odds of winning.	
Probability of Adam winning non deuce games = $\frac{496}{729}$	Probability of Adam winning deuce games = $\frac{128}{729}$
Overall Probability of Adam winning = $\frac{496}{729}+\frac{128}{729}=\frac{624}{724}$ Overall Probability of Adam losing = $1-\frac{624}{729}=\frac{105}{729}$	

Odds of Adam winning = $\frac{Probability\ of\ Adam\ Winning}{probability\ of\ Adam\ losing}$

Odds of Adam winning = $\frac{\frac{624}{729}}{\frac{105}{729}}$

Odds of Adam winning = $\frac{624}{105}$ = 5.94

Therefore Odds of Adam winning are 5.94:1 where are close to 6:1

I have found the odds of Adam winning the game and this is a very specific case, in order to generalize the model, the number must be replaced with apt variables.

Now I will generalize this model to any player C's probability of winning a point c and player D's probability of winning a point d.

The probability of player C winning non deuce games would be :

For 4-0 result. $P(X=3)=C_3^3(c)^3(d)^0*C_1^1(c)^1(d)^0$ $=\boldsymbol{c^4}$

For 4-1 result
$P(X=3) = C_3^4(c)^3(d)^1 * C_1^1(c)^1(d)^0$
$= \mathbf{4c^4d}$

For 4-2 result
$P(X=3) = C_3^5(c)^3(d)^2 * C_1^1(c)^1(d)^0$
$= \mathbf{10c^4d^2}$

Therefore, $\mathbf{c^4 + 4c^4d + 10c^4d^2}$ is the probability of a **non – deuce win.** The **probability of a deuce** is $C_3^6(c)^3(d)^3$ and therefore, $\mathbf{20c^3d^3}$ $when\ simplified.$

The probability of Player C winning given that the deuce is called is represented by:

$20c^3d^3 * \frac{c^2}{1-2cd}$ where, $\frac{c^2}{1-2cd}$ is the sum of the infinite geometric series where theoretically the game of tennis could go on indefinitely because of the possibility that each player keeps winning one point each after the initial deuce. When simplified the **probability of deuce win for the player C** becomes $\mathbf{\frac{20c^5d^3}{1-2cd}}$.

Now I will use the generalized formulas above and technology in the form of Microsoft excel to find the probabilities of winning for values of c as 0.5, 0.55, 0.6, 0.7 and 0.9.

What I will do is insert the formulas I have developed above into the excel columns and then substitute different values of c to find the probability of player C winning and the odds of player C winning,

Table 4

Column1	Column2	Column3	Column4	Column5
c	d	Probability of Player D winning	Probability of Player C winning	Odds of Player C winning
0.1	0.9	0.976773	0.001447805	0.001482232
0.2	0.8	0.978221	0.021778824	0.022263701
0.3	0.7	0.889566	0.110433947	0.124143616
0.4	0.6	0.735729	0.264270769	0.359195691
0.5	0.5	0.5	0.5	1
0.55	0.45	0.376851	0.623148502	1.653565149
0.6	0.4	0.264271	0.735729231	2.783997765
0.7	0.3	0.099211	0.900788966	9.079523968
0.8	0.2	0.021779	0.978221176	44.91616249

Therefore what I did was selected the various possibilities for c that I wanted to test and I set-up column 2 to function as 1-c because the game is such that there are 2 point probabilities of the 2 players. Then I set up column 4 to function as the winning probability function for player C where I defined it as the expression $c^4 + 4c^4d + 10c^4d^2 + \frac{20c^5d^3}{1-2cd}$.

Since the probabilities of Player C winning and the probability of Player D winning must add up to 1, I set up column 3 as $1 - \left(c^4 + 4c^4d + 10c^4d^2 + \frac{20c^5d^3}{1-2cd}\right)$.

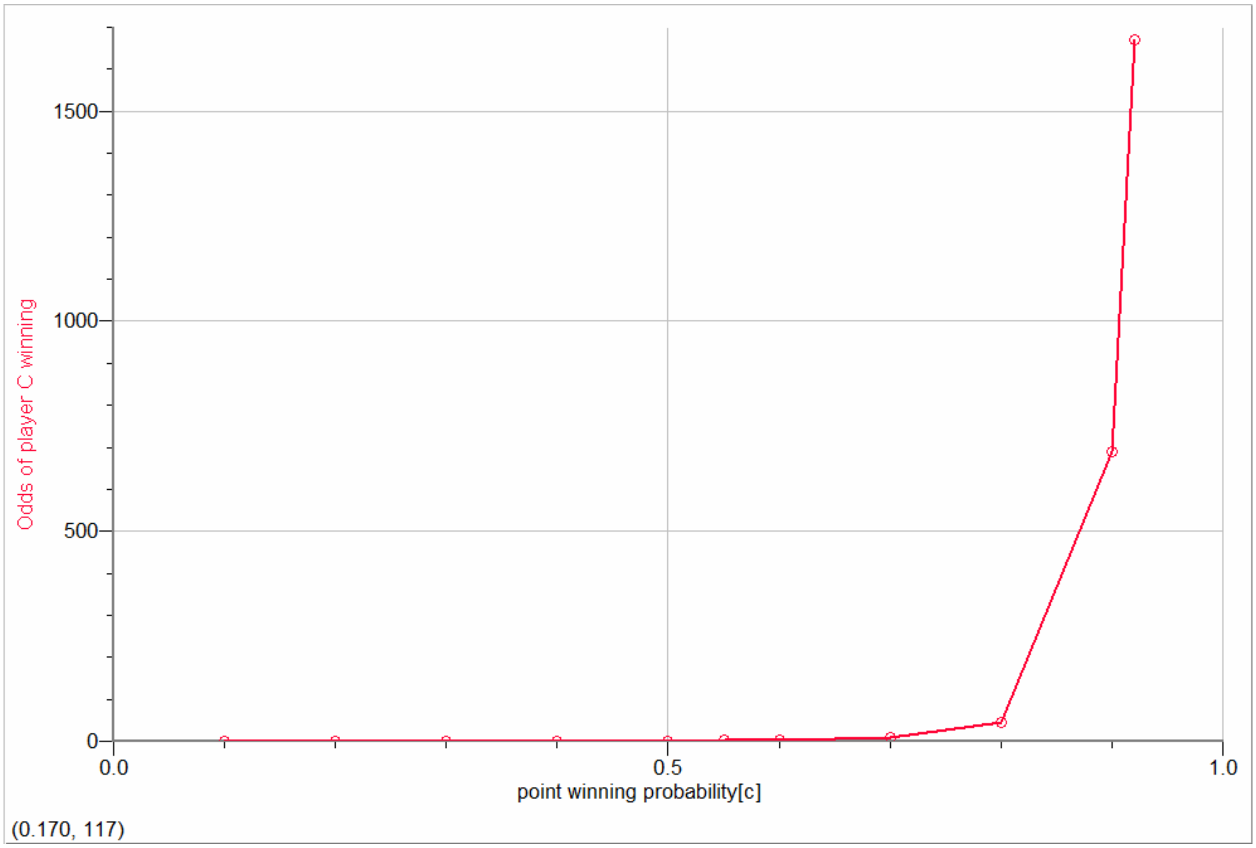
Then I set up column 5 as the odds function, which is defined as $\frac{\text{the probability of player C winning}}{\text{the probability of player C losing}}$

Since the Probability of Player C losing and the probability of Player D winning are equal I will now substitute the values into the equation to find the expression for Odds.

Odds of Player C winning = $\frac{c^4 + 4c^4d + 10c^4d^2 + \frac{20c^5d^3}{1-2cd}}{1 - \left(c^4 + 4c^4d + 10c^4d^2 + \frac{20c^5d^3}{1-2cd}\right)}$.

Then I put this equation for the function for column 5 and got my results for the various values of c.

From the values in table 4, I observe that as the values of c increase, the winning probabilities get close together and the odds increase very rapidly, to a point where when c is 0.95, the odds of Player C winning are 10830.027:1, which is huge number, considering that odds of winning for relatively high values of c such as 0.7 are 9.079523968:1. Therefore as values of winning probability for c get close, the values for odds in favor of Player C winning grow very rapidly, seemingly exponentially. Also as values of c get small, the values of odds of winning are getting very close to zero and are asymptotic to the x-axis.



Having made all the required models, I will now analyze them.

Some of the benefits of such types of model might include that it could provide mathematical evidence and guidance for gamblers and they could make educated bets on the player that is most likely to win. Although this is a benefit to the gambler, gambling can often be illegal, and the presence of such models might encourage illegal gambling, as certain individuals might want to take advantage of the fact that they know the most probably outcome or the expected value. These types of models might also help statisticians as they would not have to keep all the data and just record the average point probability of that player or players because other data such as winning odds and probabilities can be calculated from there.

Also, players and fans can have a realistic idea of the abilities and expectations. For example

There are however some limitations to the model, the main reason why they are not entirely realistic is that the point probabilities are not always constant in reality because of factors such as changing weather conditions, possible injuries and illnesses.

Also, it has been stated that the '2 players, Adam and Ben have played each other often enough to know that Adam wins about twice as many points as Ben does'. Even if we assume that they have practiced against each other and played non-extended games against each other enough to know that their point winning ratio is 2:1, still for extended games such with the possibility of games going on indefinitely, we cannot assume that the their point winning ratio will hold as time goes on, simply because they Adam and Ben could not have played indefinitely, in fact they may not have play each other beyond a couple of deuces. Therefore the fact that the point winning ratio is constant in the model but realistically is changing is a limitation to such probability models.

Another limitation or rather downside to such a model would be that the element of surprise of anticipation of the fans would not exist as the expected outcome would tell us the most likely results and watching the game would no longer be enjoyable, the players would be like robots merely doing their job and playing as math dictates!