Logarithm Bases

Consider the following sequences. Write down the next two terms of each sequence.

$\text{Log}_2 8$, $\text{Log}_4 8$, $\text{Log}_8 8$, $\text{Log}_{16} 8$, $\text{Log}_{32} 8$, *$\text{Log}_{64} 8$, $\text{Log}_{128} 8$*

$\text{Log}_3 81$, $\text{Log}_9 81$, $\text{Log}_{27} 81$, $\text{Log}_{81} 81$, *$\text{Log}_{243} 81$, $\text{Log}_{729} 81$*

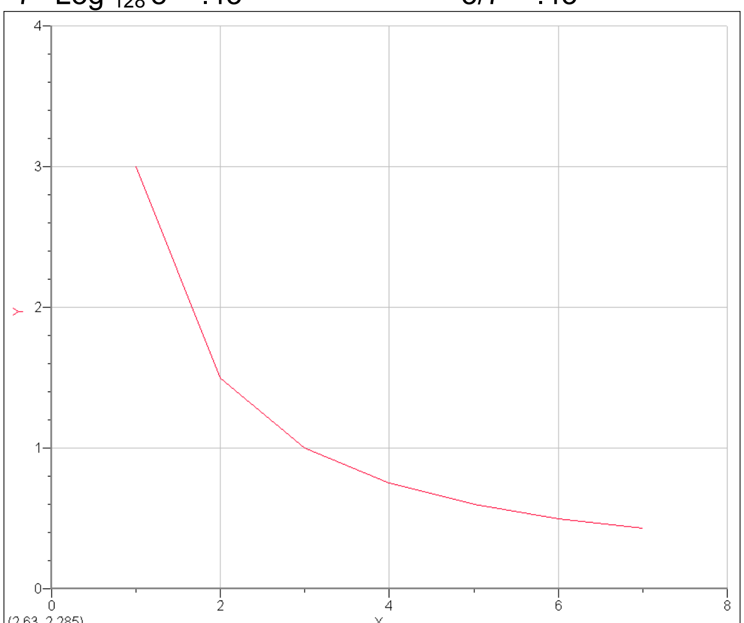
$\text{Log}_{5} 25$, $\text{Log}_{25} 25$, $\text{Log}_{125} 25$, $\text{Log}_{625} 25$, *$\text{Log}_{3125} 25$, $\text{Log}_{15625} 25$*

Find an expression for the nth term of each sequence. Write your expressions in the form P/Q.

$\text{Log}_{2} 8$, $\text{Log}_{4} 8$, $\text{Log}_{8} 8$, $\text{Log}_{16} 8$, $\text{Log}_{32} 8$, $\text{Log}_{64} 8$, $\text{Log}_{128} 8$

This sequence can be expressed 3/N. N being the nth term of the sequence.

1st $\text{Log}_{2} 8 = 3$	3/1 = 3
2nd $\text{Log}_{4} 8 = 1.5$	3/2 = 1.5
3rd $\text{Log}_{8} 8 = 1$	3/3 = 1
4th $\text{Log}_{16} 8 = .75$	3/4 = .75
5th $\text{Log}_{32} 8 = .6$	3/5 = .6
6th $\text{Log}_{64} 8 = .5$	3/6 = .5
7th $\text{Log}_{128} 8 = .43$	3/7 = .43



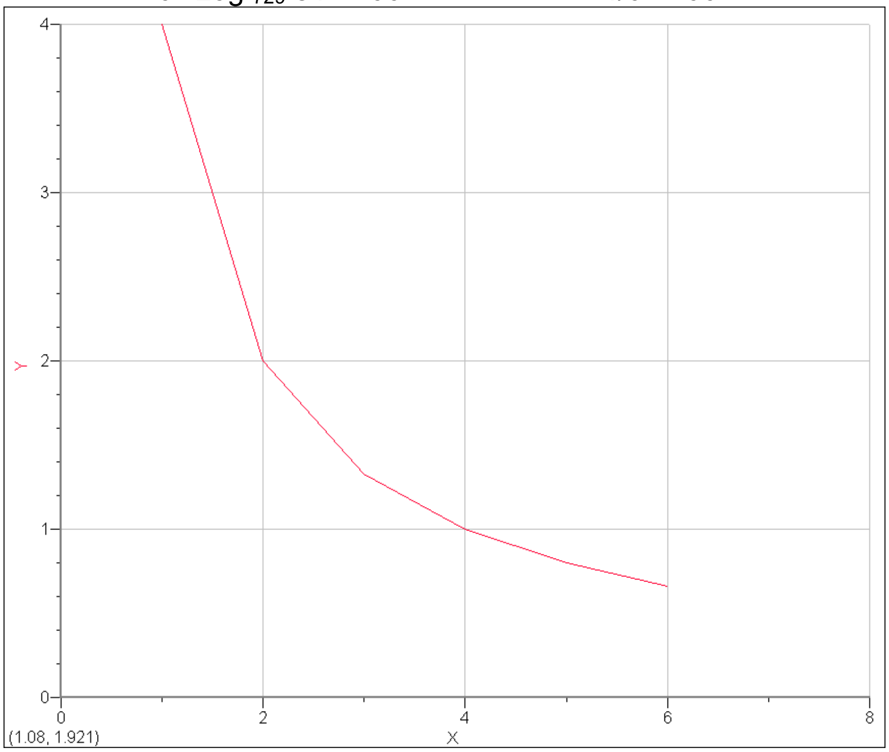
Find an expression for the nth term of each sequence. Write your expressions in the form P/Q.

$\text{Log}_{3} 81$, $\text{Log}_{9} 81$, $\text{Log}_{27} 81$, $\text{Log}_{81} 81$, $\text{Log}_{243} 81$, $\text{Log}_{729} 81$

This sequence can be expressed 4/N. N being the nth term of the sequence.

1st $\text{Log}_{3} 81 = 4$	4/1 = 1
2nd $\text{Log}_{9} 81 = 2$	4/2 = 2
3rd $\text{Log}_{27} 81 = 1.33$	4/3 = 1.33
4th $\text{Log}_{81} 81 = 1$	4/4 = 1

5th $\text{Log}_{243} 81 = .8$ 4/5 = .8
6th $\text{Log}_{729} 81 = .66$ 4/6 = .66

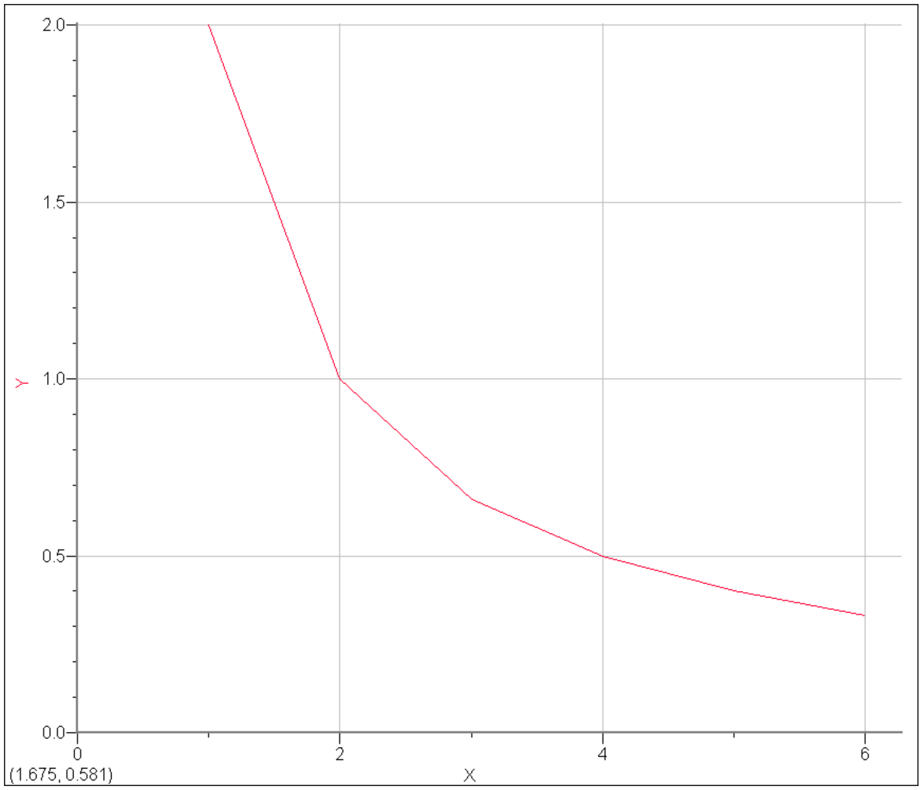


Find an expression for the nth term of each sequence. Write your expressions in the form P/Q.

$\text{Log}_5 25$, $\text{Log}_{25} 25$, $\text{Log}_{125} 25$, $\text{Log}_{625} 25$, $\text{Log}_{3125} 25$, $\text{Log}_{15625} 25$

This sequence can be expressed 2/N. N being the nth term of the sequence.

1st $\text{Log}_5 25 = 4$ 2/1 = 2
2nd $\text{Log}_{25} 25 = 2$ 2/2 = 1
3rd $\text{Log}_{125} 25 = 1.33$ 2/3 = .66
4th $\text{Log}_{625} 25 = 1$ 2/4 = .5
5th $\text{Log}_{3125} 25 = .8$ 2/5 = .4
6th $\text{Log}_{15625} 25 = .66$ 2/6 = .33



Now calculate the following. Giving your answers in the form of P/Q.

$\text{Log}_4 64 = 3$, $\text{Log}_8 64 = 2$, $\text{Log}_{32} 64 = 6/5$

$\text{Log}_7 49 = 2$, $\text{Log}_{49} 49 = 1$, $\text{Log}_{343} 49 = 2/3$

$\text{Log}_{1/5} 125 = -3$, $\text{Log}_{1/125} 125 = -1$, $\text{Log}_{1/625} 125 = -3/4$

$\text{Log}_8 512 = 3$, $\text{Log}_2 512 = 9$, $\text{Log}_{16} 512 = 9/4$

To obtain the third answer from each row using only the first two answers, you must first multiply the first and second answer together and then divide by the sum of the first and second answers.

Two more examples that fit the pattern are listed below:

$\text{Log}_2 4 = 2$, $\text{Log}_4 4 = 1$, $\text{Log}_8 4 = 2/3$ $(2)(1)/(2 + 1) = 2/3$

$\text{Log}_2 1024 = 10$, $\text{Log}_4 1024 = 5$, $\text{Log}_8 1024 = 3.33$ $(10)(5)/(10 + 5) = 3.33$

Let $\text{Log}_a x = c$ and $\text{Log}_b x = d$. Find the general statement that expresses $\text{Log}_{ab} x$, in terms of c and d.

If we do this than the general statement that expresses this is (c)(d)/(c + d).

For example let:

A = 2 C = 4
B = 8 D = 1.33
X = 16

$\text{Log}_2\ 16 = 4 = C$ $\text{Log}_8\ 16 = 1.33 = d$ $\text{Log}_{16}\ 16 = 1$

(4)(1.33)/(4 + 1.33) = 5.33/5.33 = 1

Discuss the scope and/ or limitations of a, b, and x.

Some basic restrictions for the general statement are that both a and b must be greater than 0. Logs have to have positive numbers as bases so a > 0, b > 0.

$\text{Log}_{-5}\ 25$ = Nonreal Answer

Another limitation of a and b is that (a)(b) cannot equal 1.

$\text{Log}_4\ 16 = 2$, $\text{Log}_{1/4}\ 16 = -2$, $\text{Log}_1\ 16 = ?$

1 raised to the power of any number will always be 1 so it can never equal 16 thus the equation is flawed when (a)(b) = 1

(2)(-2)/(2 + (-2)) = -4/0

Using the general statement only further proves how (a)(b) cannot = 1 because it would cause you to divide by 0 which we are not able to do.

The general statement that I applied throughout the assessment (c)(d)/(c + d) I arrived at through plugging numbers in. Through a system of trial and error I was able to figure out the pattern and discover how you can deduce the third answer from the first two in the sequence using logs. Of course the general statement only applies to sequences of logs that follow the pattern of $\text{Log}_a X = c$, $\text{Log}_b X = d$, $\text{Log}_{ab} X = (c)(d)/(c + d)$.