Math HL
Portfolio
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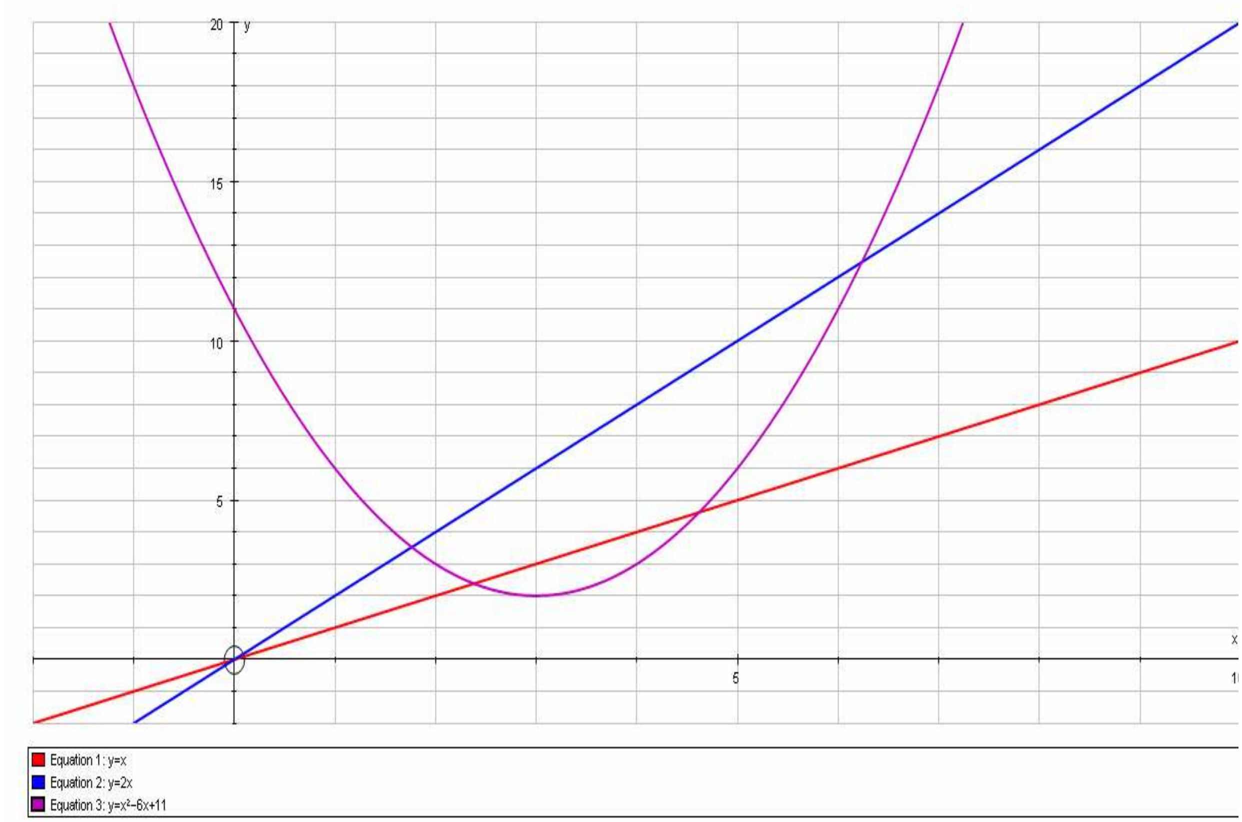
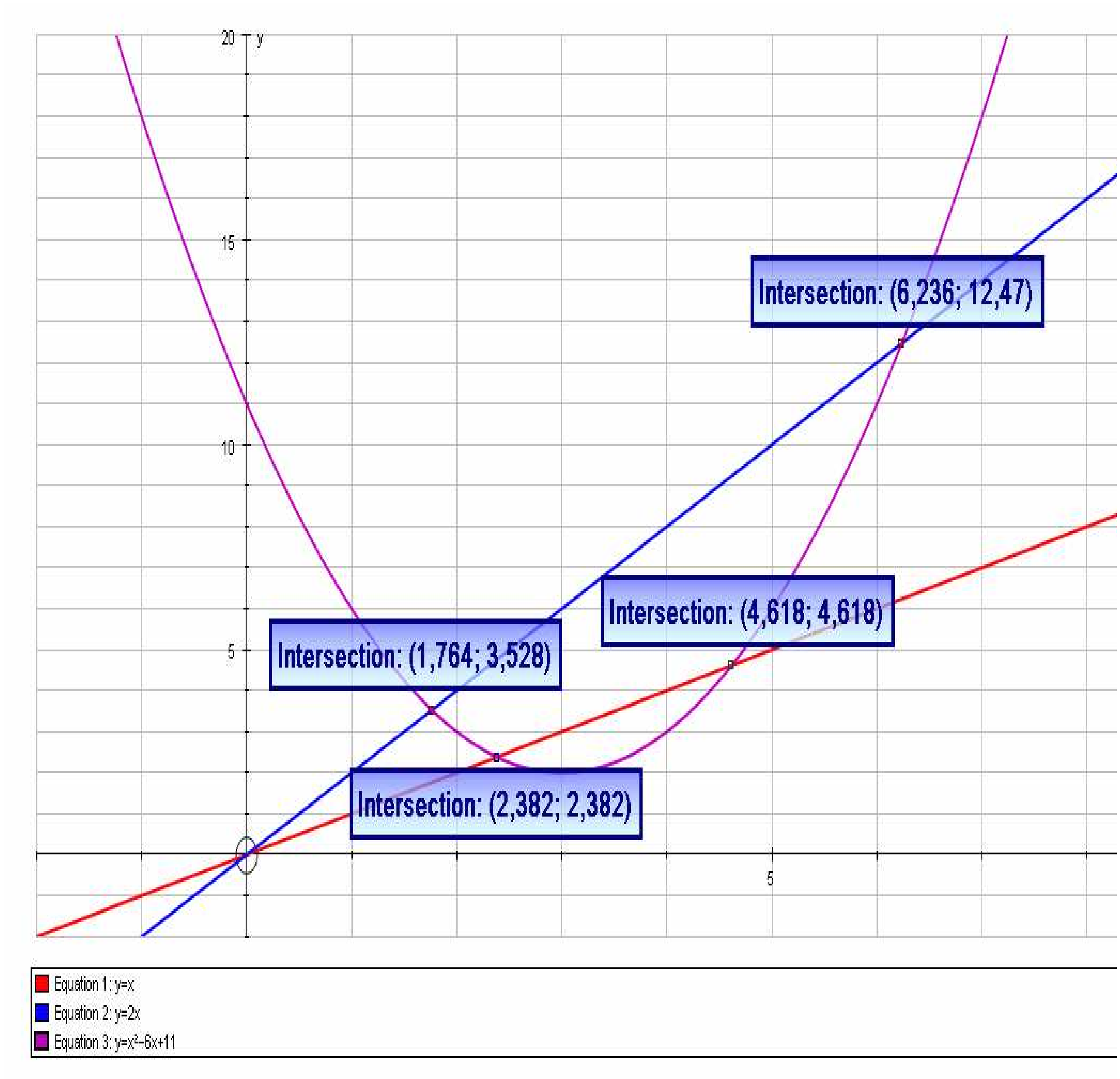
Description:
In this task you will investigate the patterns in the intersection of parabolas and the lines y=x and y=2x. Then you will be asked to prove your conjectures and to broaden the scope of the investigation to include other lines and other types of polynomials.
The main aim of my investigation is to conclude an answer for the relations between the graph intersections of the graphs y=x and y=2x
Method
1. Consider the parabola $y=(x-3)^2+2=x^2-6x+11$ and the lines y=x and y=2x.

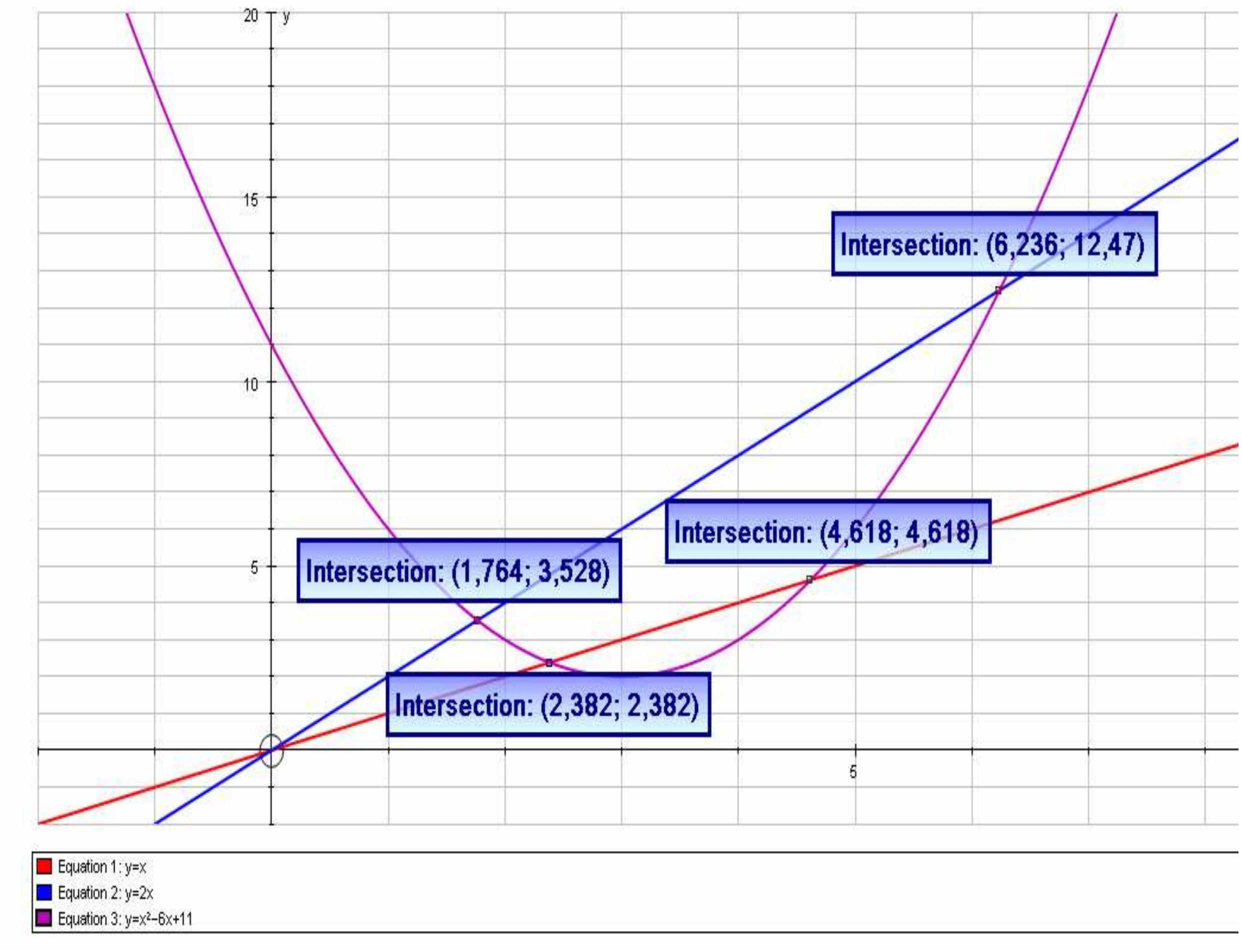


Using Technology find the four intersections illustrated on the right.

Using the Microsoft based program autograph® the four intersections between the three given graphs were found.


20
15
10
5
y
5
Intersection: (6,236; 12,47)
Intersection: (4,618; 4,618)
Intersection: (1,764; 3,528)
Intersection: (2,382; 2,382)
Equation 1: y=x
Equation 2: y=2x
Equation 3: y=x²−6x+11

Label all the x-values of these intersections as they appear from left to right on the x-axis as x_1, x_2, x_3 and x_4. (Label on the actual graph)



Find the values of $x_2 - x_1$ *and* $x_4 - x_3$ *and name them respectievly* S_L *and* S_R.

$x_1 = \{1,764\}$ $\quad x_2 - x_1 = 2,382 - 1,764 = 0,618 = S_L$

$x_2 = \{2,382\}$ $\quad x_4 - x_3 = 6,236 – 4,618 = 1,618 = S_R$

$x_3 = \{4,618\}$

$x_4 = \{6,236\}$

Finally, calculate D = $| S_L - S_R |$

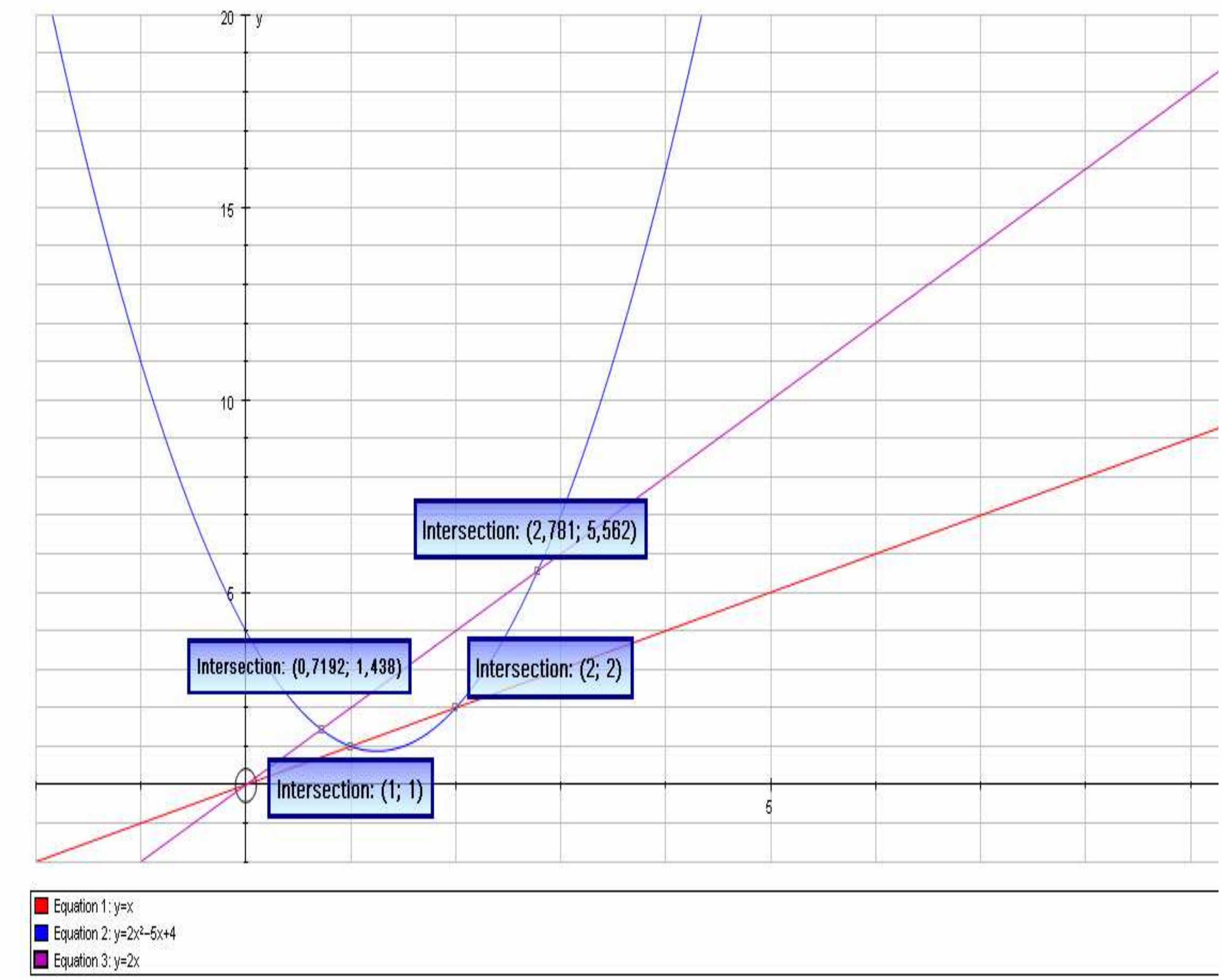
D= $| S_L - S_R | = |0,618 – 1,618| = |– 1| = 1$ D = 1

2. Find Values for D for other parabolas of the form $y=ax^2+bx+c$, $a > 0$, with vertices in quadrant 1, intersected by the lines $y=x$ and $y=2x$. Consider various values of a, beginning with a=1. Make a conjecture about the value of D for these parab olas.

I used several parabolas to derive to a relation or formula.

The first parabola I used in the form of $y=ax^2+bx+c$ is

$y=2x^2-5x+4$



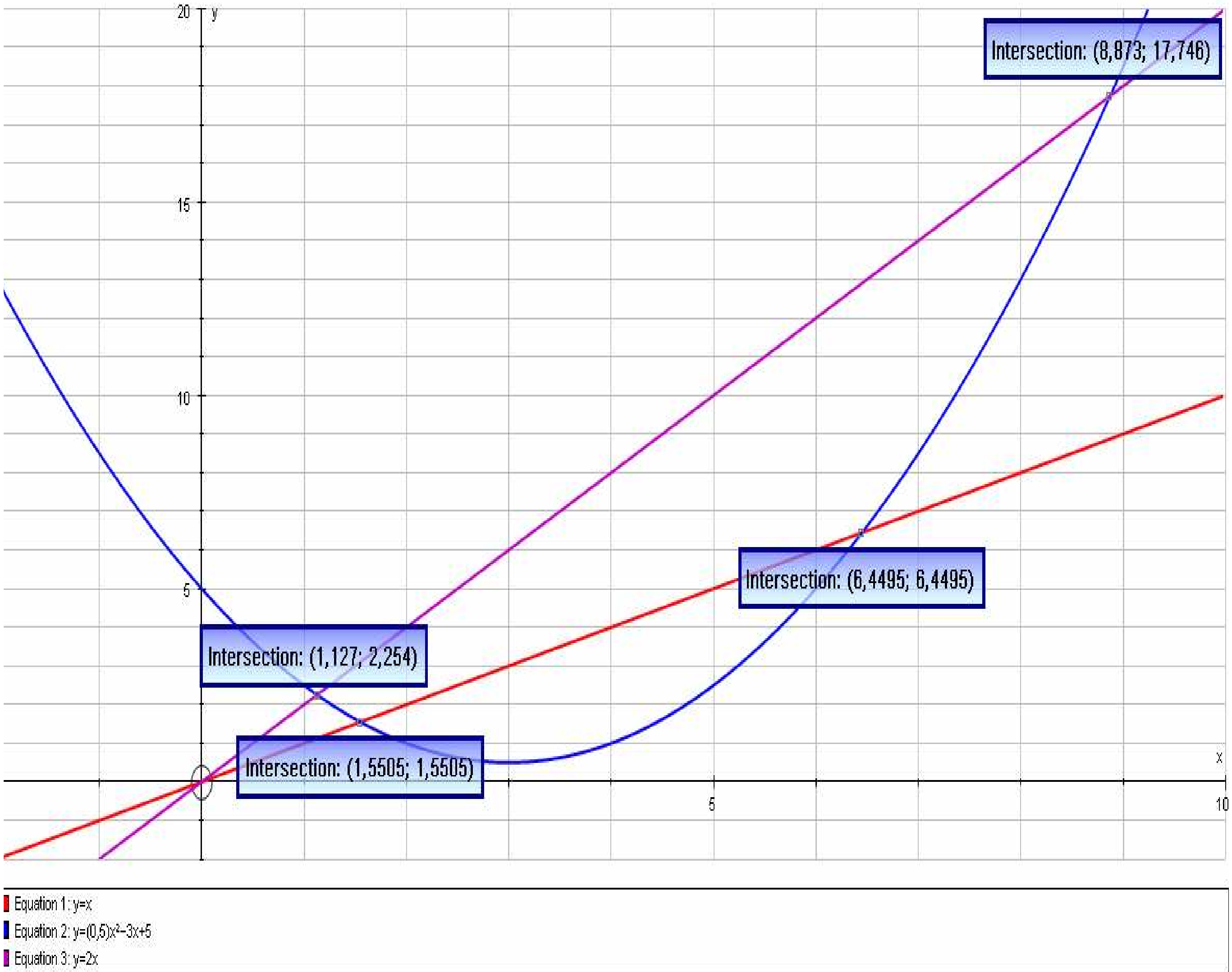
$x_1 = \{0,719\}$ $\quad x_2 - x_1 = 1 - 0.719 = 0.281 = S_L$

$x_2 = \{1\}$ $\quad x_4 - x_3 = 2,781 - 2 = 0.781 = S_R$

$x_3 = \{2\}$

$x_4 = \{2,781\}$ $\quad$ D= $|S_L - S_R| = |(x_2 - x_1) - (x_4 - x_3)| = |0.2808 - 0.781| = 0.5$

The third parabola I used in the form of $y=ax^2+bx+c$ is
Y= 0.5x²-3x+5



$x_1 = \{1{,}127\}$

$x_2 = \{1{,}5505\}$

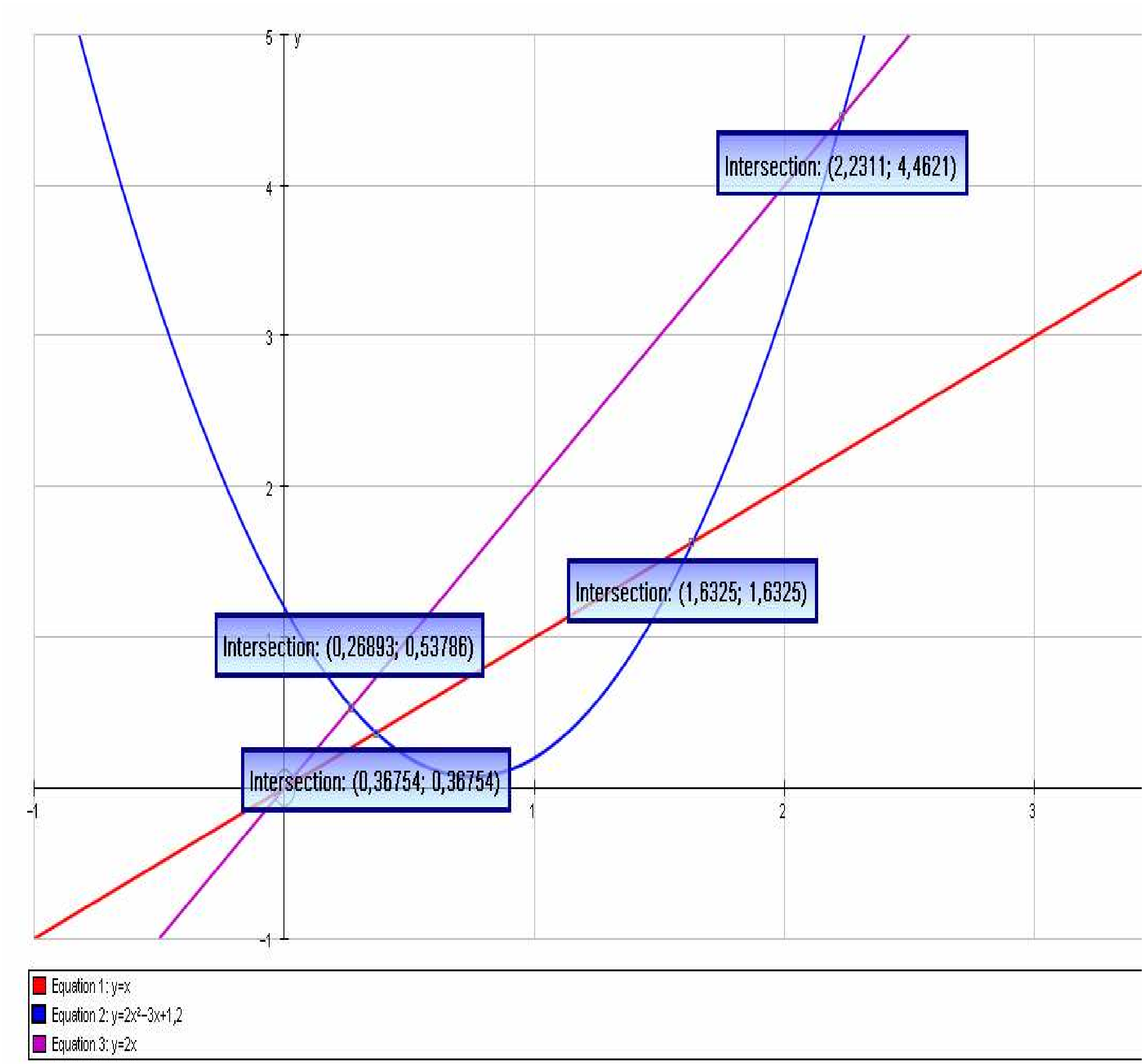
$x_3 = \{6{,}4495\}$

$x_4 = \{8{,}873\}$

D= $| S_L - S_R | = |(x_2 - x_1) - (x_4 - x_3)| = |0.4235 - 2.4235| = 2$

The fourth parabola I used in the form of $y=ax^2+bx+c$ is

$Y=2x^2-3x+1.2$



$x_1 = \{0,26893\}$

$x_2 = \{0,36754\}$

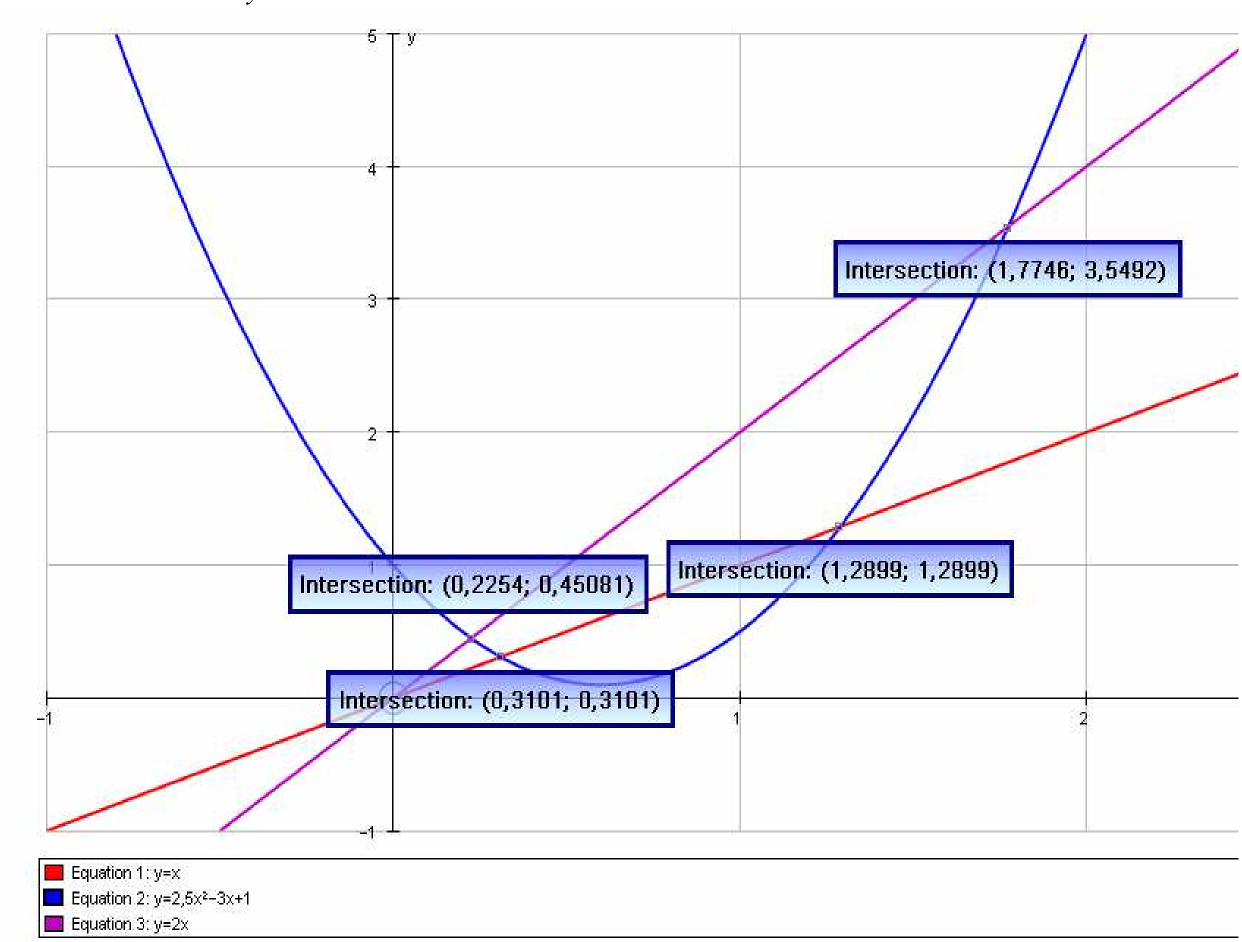
$x_3 = \{1,6325\}$

$x_4 = \{2,2311\}$

D= $| S_L - S_R | = |(x_2 - x_1) - (x_4 - x_3)| = |0.09861 - 0.59861| = 0.5$

The fifth parabola I used in the form of $y=ax^2+bx+c$ is

$y=2.5x^2-3x+1$



$x_1 = \{0{,}2254\}$

$x_2 = \{0{,}3101\}$

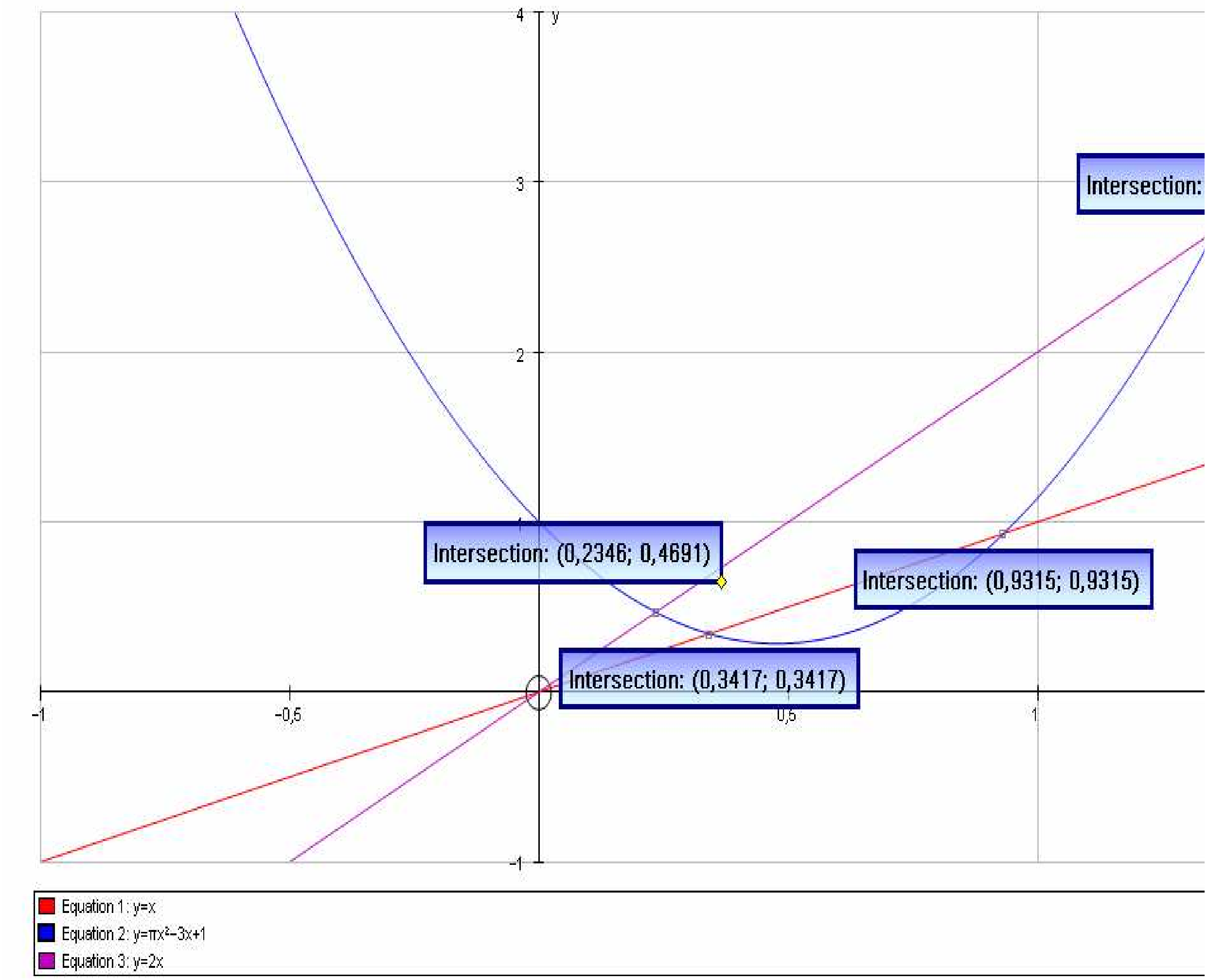
$x_3 = \{1{,}2899\}$

$x_4 = \{1{,}7746\}$

D= $| S_L - S_R | = |(x_2 - x_1) - (x_4 - x_3)| = |0.0847 - 0.4847| = 0.4$

The sixth parabola I used in the form of $y=ax^2+bx+c$ is

$y=\pi x^2-3x+1$



$x_1 = \{0,23457\}$

$x_2 = \{0,3417\}$

$x_3 = \{0,93154\}$

$x_4 = \{1,357\}$

$D = |S_L - S_R| = |(x_2 - x_1) - (x_4 - x_3)| = |0.10713 - 0.42546| = 0.31833$

The conjecture which can be derived by analysing the current occurring patterns is simple but is bounded to limitations.

$D = \frac{1}{a}$

How is that relation derived ?

The first example states that *x²-6x+11*

$D = |S_L - S_R| = |0{,}618 - 1{,}618| = |-1| = 1$ $\qquad$ $D = 1$

So applying my assumption for this case $D = \frac{1}{a}$

In the parabola $x^2-6x+11=y$

a = 1 , b = -6 , c = 11

implying that the regular quadric consists of $ax^2+bx+c=y$

We know that D = 1 by using the formula given before

Using the new conjecture $1 = \frac{1}{a}$

$1 = \frac{1}{1}$

$1 = 1$

Now lets use the example of a = 2

Like in $Y=2x^2-3x+1.2$ (example of page 7) where a = 2, b = -3 , c = 1.2

$D = |S_L - S_R| = |(x_2 - x_1) - (x_4 - x_3)| = |0.09861 - 0.59861| = 0.5$

$D = \frac{1}{2}$ that means $\frac{1}{a} = \frac{1}{2}$, a = 2

Therefore $\frac{1}{2} = \frac{1}{2}$

Now lets use the example of a = π

Like in Y= y=πx²−3x+1 (example of page 5) where a = π, b = -3 , c = 1

Π ∈ Sign for rational numbers

D= $| S_L - S_R | = |(x_2 - x_1) - (x_4 - x_3)| = |0.10713 - 0.42546| = 0.31833$

D = 0.31833

$0.31833 = \frac{1}{a}$

$0.31833 = \frac{1}{\pi}$

$0.31833 = 0.31833$

After now having changed "a" for 7 different parabolas, I can show the relation in visual diagram (table)

In order to make the results more obvious, I created the following table where I listed all my results:

Value for "a"	Value for "D"
0.5	2
1	1
2	0.5
2.5	0.4
Π	0.31833

This conjecture is only possible if the vertex is in the 1st quadrant and if the values of b and c are bounded to limitations.

B has to be smaller than zero in mathematical terms b<0

For example see the same quadratic equation just the variable b is changed

Y= x²-4x+5 (a=1, b= -4, c=5)

And Y= x²+4x+5 (a=1, b= 4, c=5)

The Vertex of the first quadratic lies in the First Quadrant

You can complete the square to convert $ax^2 + bx + c$ to vertex form, but it's simpler to just use a formula (derived from the completing-the-square process) to find the vertex.

For a given quadratic $y = ax^2 + bx + c$, the vertex (h, k) is found by computing $h = {}^{-b}/_{2a}$, and then evaluating y at h to find k where $k = (4ac - b^2) / 4a$.

So the vertex for the first one is is $h = \dfrac{-b}{2a} = \dfrac{-(-4)}{2(1)} = \dfrac{4}{2} = 2$

$$K = \frac{4ac - b^2}{4a} == \frac{4(1)(5) - (-4)^2}{4(1)} = \frac{20 - 16}{4} = \frac{4}{4} = 1$$

So the Vertex is (2,1)

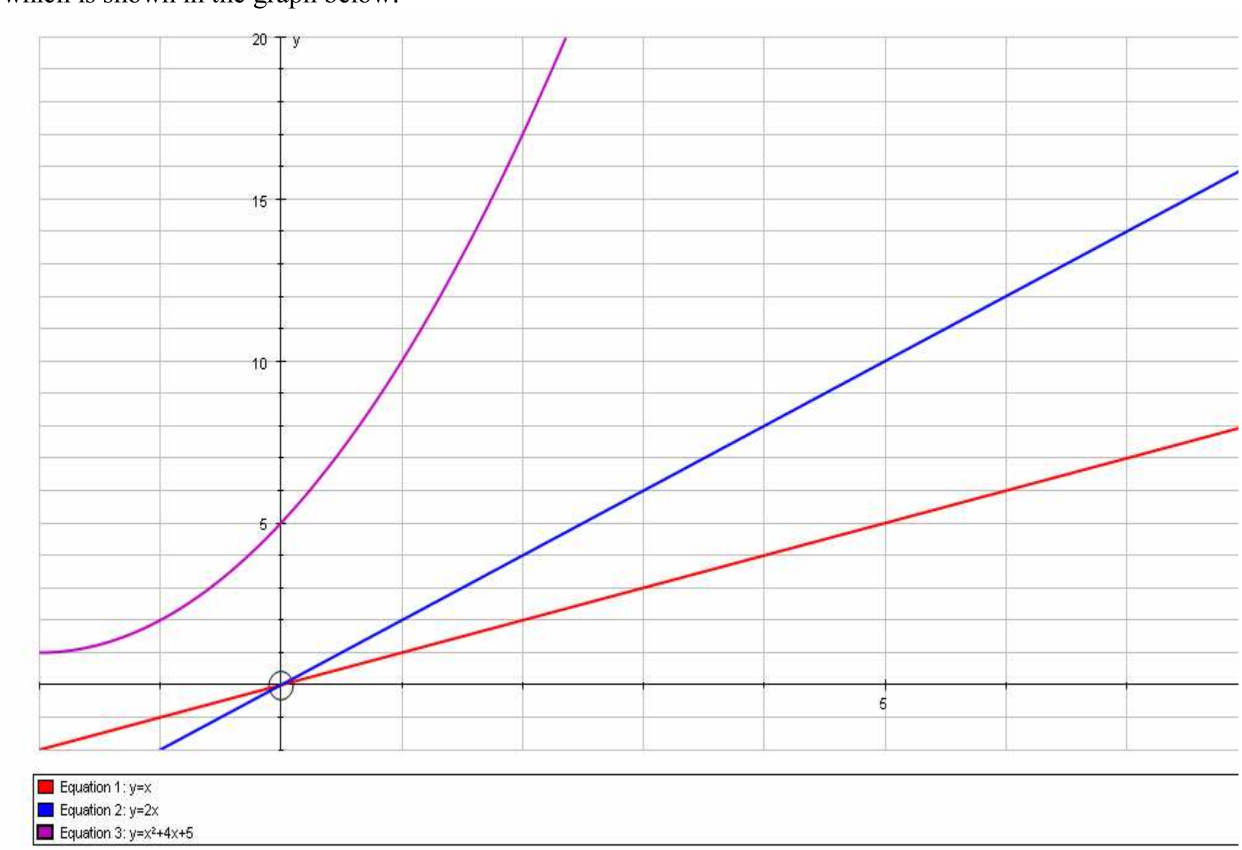
The Vertex of the second quadratic does not lie in the first quadrant

Y= x^2+4x+5 (a=1, b= 4, c=5)

$$h = \frac{-b}{2a} = \frac{-(4)}{2(1)} = \frac{-4}{2} = -2$$

$$K = \frac{4ac - b^2}{4a} == \frac{4(1)(5) - (4)^2}{4(1)} = \frac{20 + 16}{4} = \frac{36}{4} = 4$$

That the vertex is not in the first quadrant can also be proved by using technology (autograph) which is shown in the graph below:

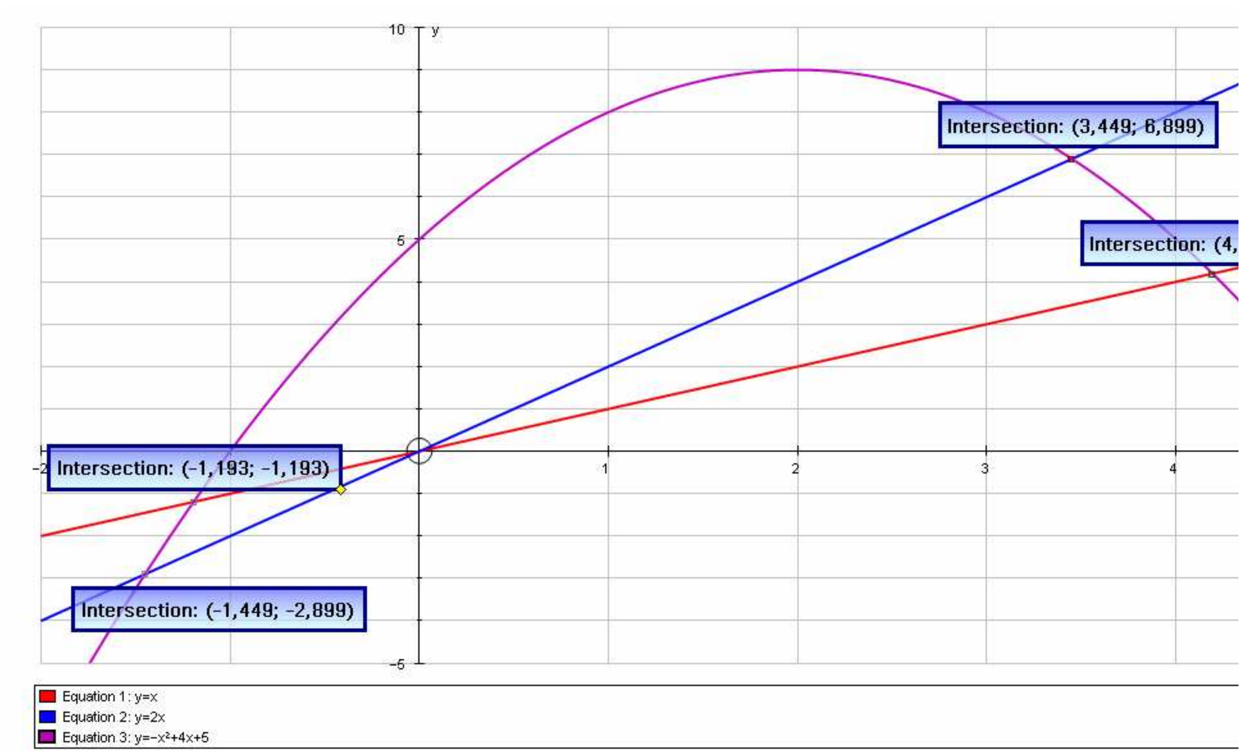


3. Investigate your conjecture for any real value of a and any placement of the vertex. Refine your conjecture as necessary and prove it. Maintain the labelling convention used in parts 1 and 2 by having the intersections of the first line to be x_2 *and* x_3 *, and the intersections with the second line to be* x_1 *and* x_4.

This Question implies that I have to investigate and modify my conjecture for any real value for "a" and any placement of the vertex. This means that I am going to change not only "a", but also the variables "b" and "c" in the parabola. The lines of y = x and y = 2x are still going to be the same. Similarly, I will note the intersections of those lines with the parabola and also calculate D again.

I used several parabolas to derive to a modified conjecture to the case of a vertex in different quadrants.

Y=−x²+4x+5



x_1 = {-1,449}

x_2 = {-1.193}

x_3 = {4,193}

x_4 = {3.449}

D= $|\ S_L - S_R\ | = |(x_2 - x_1) - (x_4 - x_3)| = |0.256 - (-0.744)| = 1$

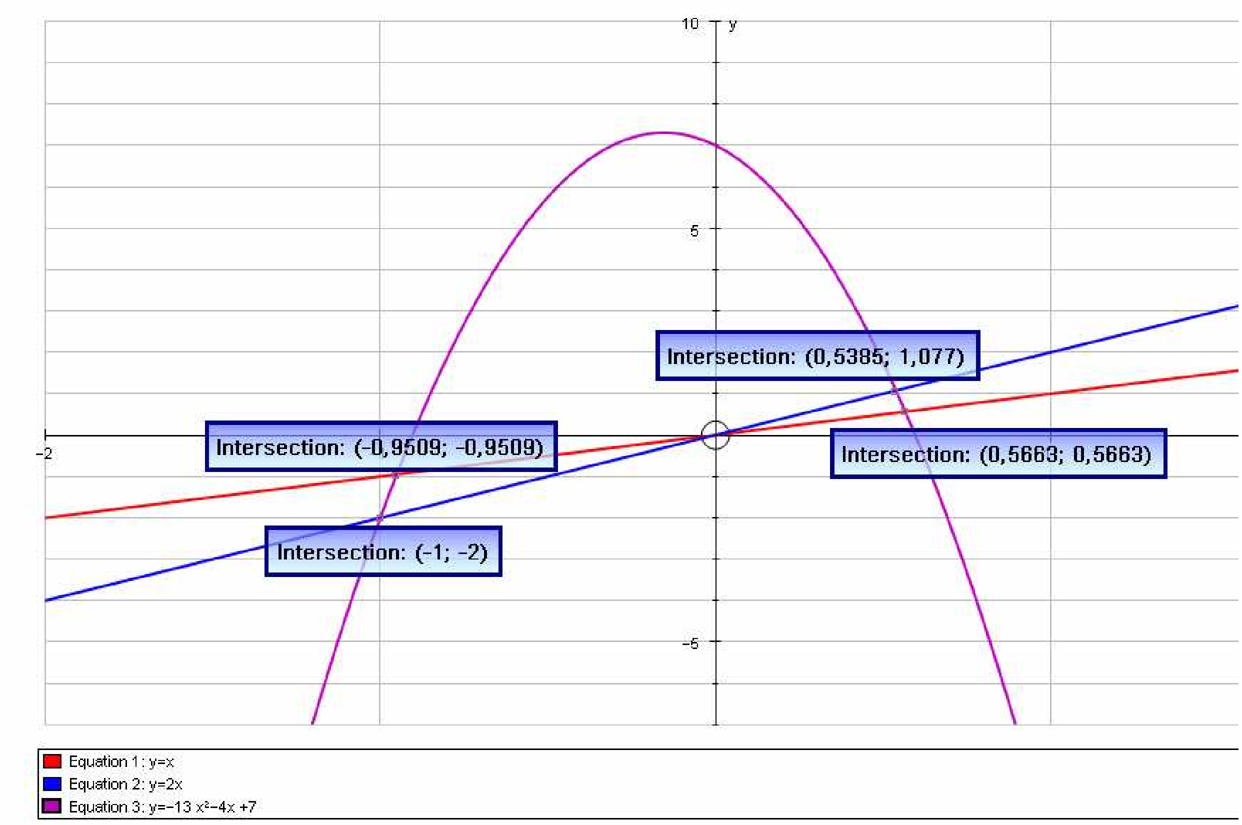
$\frac{1}{a} = -1 \quad D \neq -1$

My previously conjecture has to be modified from $\frac{1}{a} = D$

To $|\ \frac{1}{a}\ | = D$

In this case it would be $|\ \frac{1}{a}\ | = |\ -1| = 1 = D$

The second parabola I used in the form of $y=ax^2+bx+c$ is
$y=-13x^2-4x+7$



$x_1 = \{-1\}$

$x_2 = \{-0,9509\}$

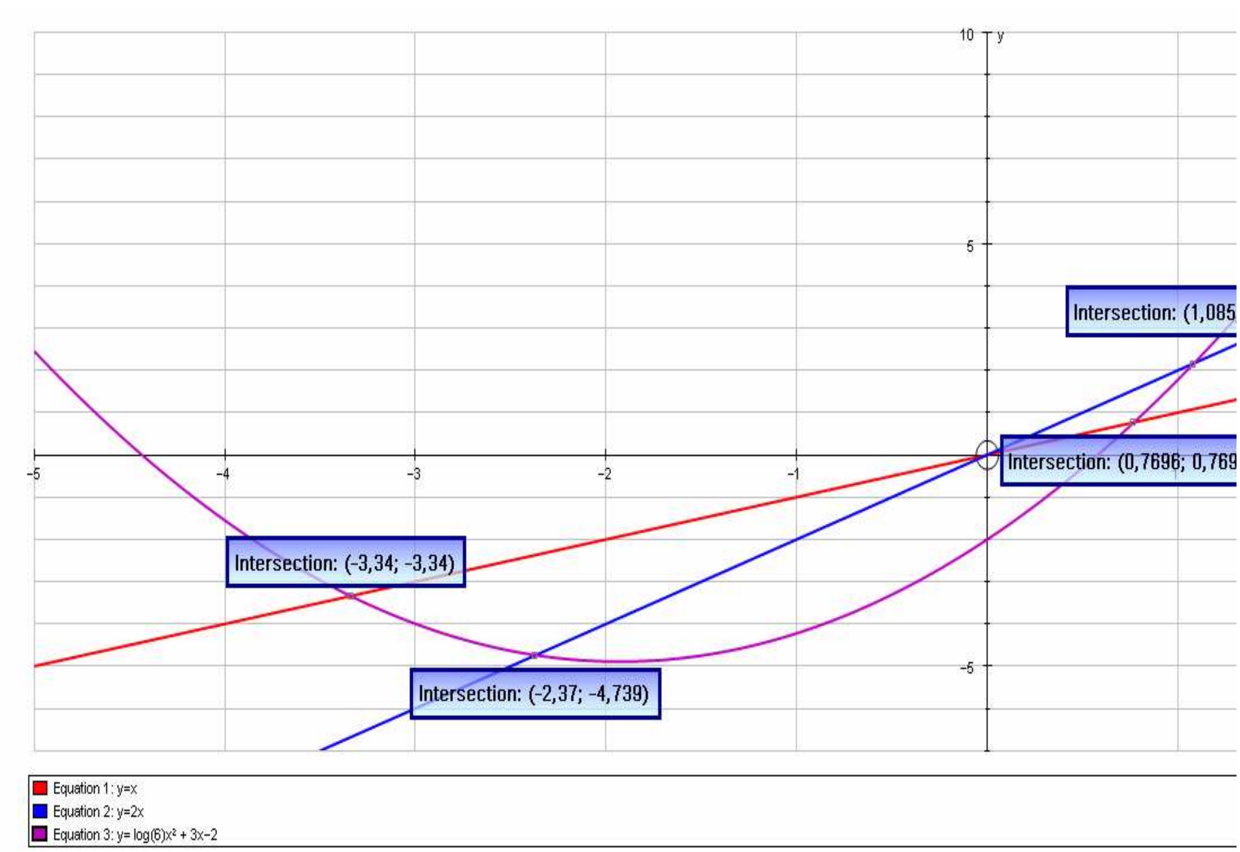
$x_3 = \{0,5663\}$

$x_4 = \{0{,}5385\}$

D= $| S_L - S_R | = |(x_2 - x_1) - (x_4 - x_3)| = |0.0491 - (-0.0278)| = 0.0769$

$\left| \frac{1}{a} \right| = \left| \frac{1}{-13} \right| = 0.0769 = D$

Y = log(6)x² + 3x – 2



$x_1 = -2.3697$

$x_2 = -3.3398$

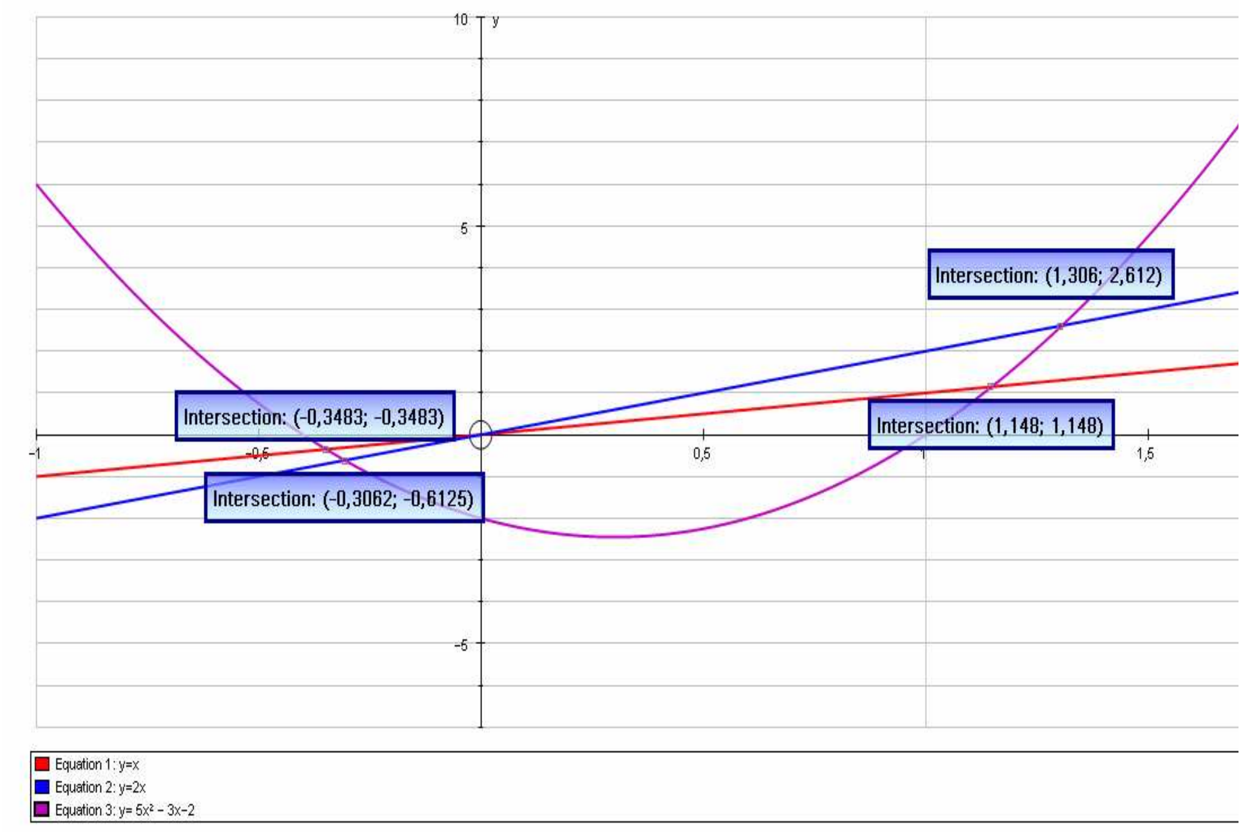
$x_3 = 0.7696$

$x_4 = 1.0846$

D = $| S_L - S_R | = |(x_2 - x_1) - (x_4 - x_3)| = |(-0.9701) - (0.3150)| = |(-1.2851)| = 1.2851$

D= $\left| \frac{1}{a} \right| = \left| \frac{1}{\log(6)} \right| = 1.2851$

Y = 5x² - 3x – 2

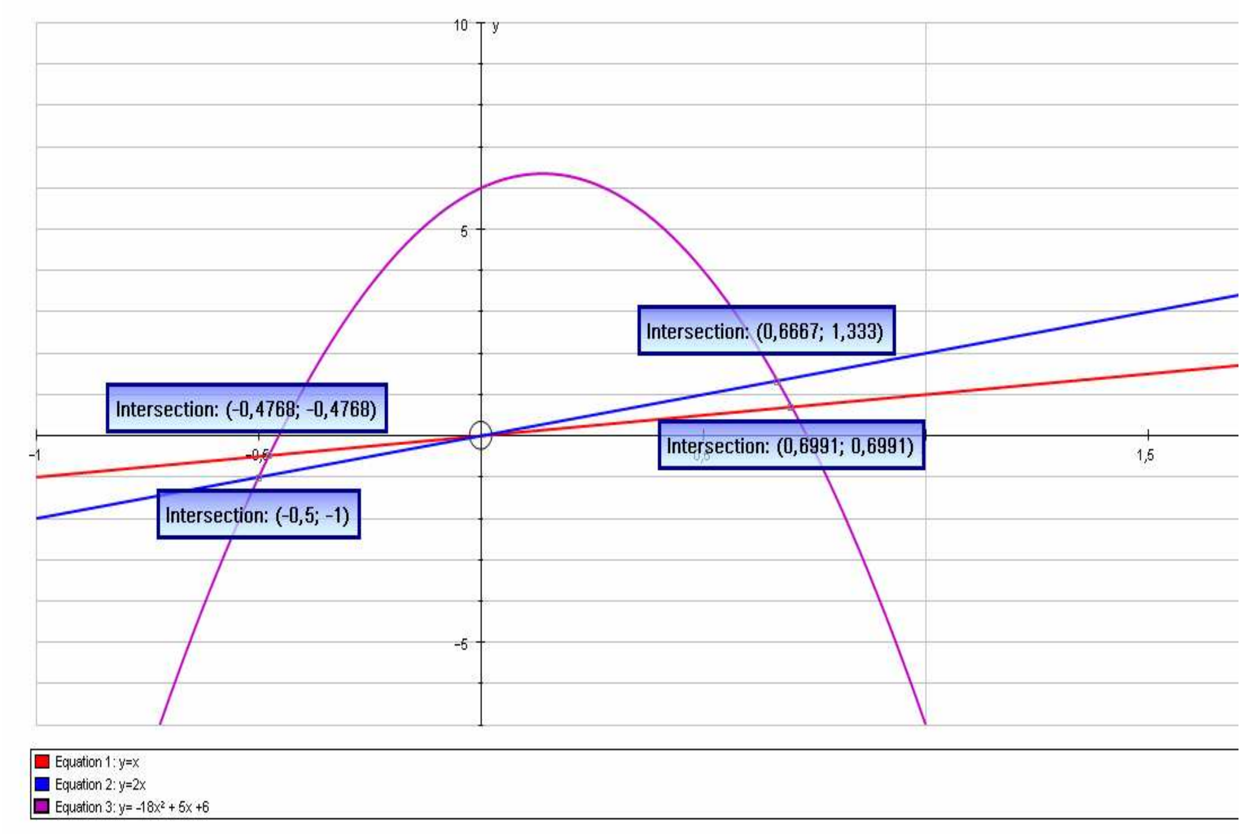


x_1 = -0.3062
x_2 = -0.3483
x_3 = 1.1483
x_4 = 1.3062

D = $| S_L - S_R | = |(x_2 - x_1) - (x_4 - x_3)|$ = | (-0.0421) – (0.1579) | = | -0.2000 | = 0.2

D= $\left| \frac{1}{a} \right| = \left| \frac{1}{5} \right|$ = 0.2

$Y = -18x^2 + 5x + 6$



$x_1 = -0.5$
$x_2 = -0.4768$
$x_3 = 0.6991$
$x_4 = 0.6667$

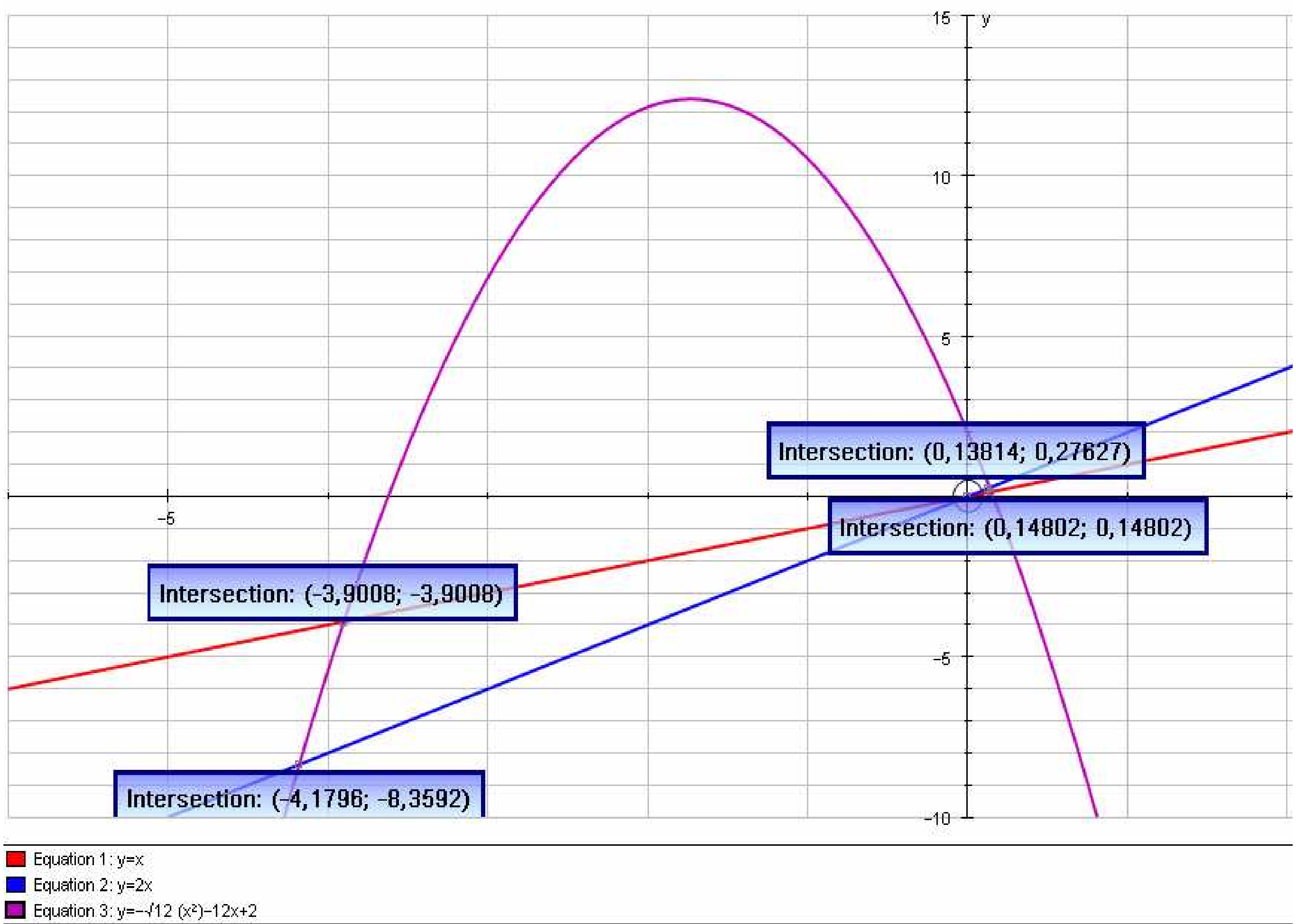
D $= | S_L - S_R | = | (x_2 - x_1) - (x_4 - x_3) | = | 0.0232 - (-0.0324) | = | -0.0556 | = 0.0556$

D$= \left| \frac{1}{a} \right| = \left| \frac{1}{18} \right| = 0.0556$ regarding only 4 significant figures.

My modified conjecture works so far which is shown in table of results below:

Value for "a"	Value for "D"
-18	0.0556
-13	0.0769
-1	1
Log(6)	1.2851
5	0.2

To prove my conjecture I picked a random parabola:
$y=-\sqrt{12}(x^2)-12x+2$



x_1 = -4.1796
x_2 =-3.9008
x_3 = 0.14802
x_4 = 0.13814

D =$\mid S_L - S_R \mid = \mid (x_2 - x_1) - (x_4 - x_3) \mid$ = | 0.2788 – (-0.0988) | = | 0.28868 | = 0.28868

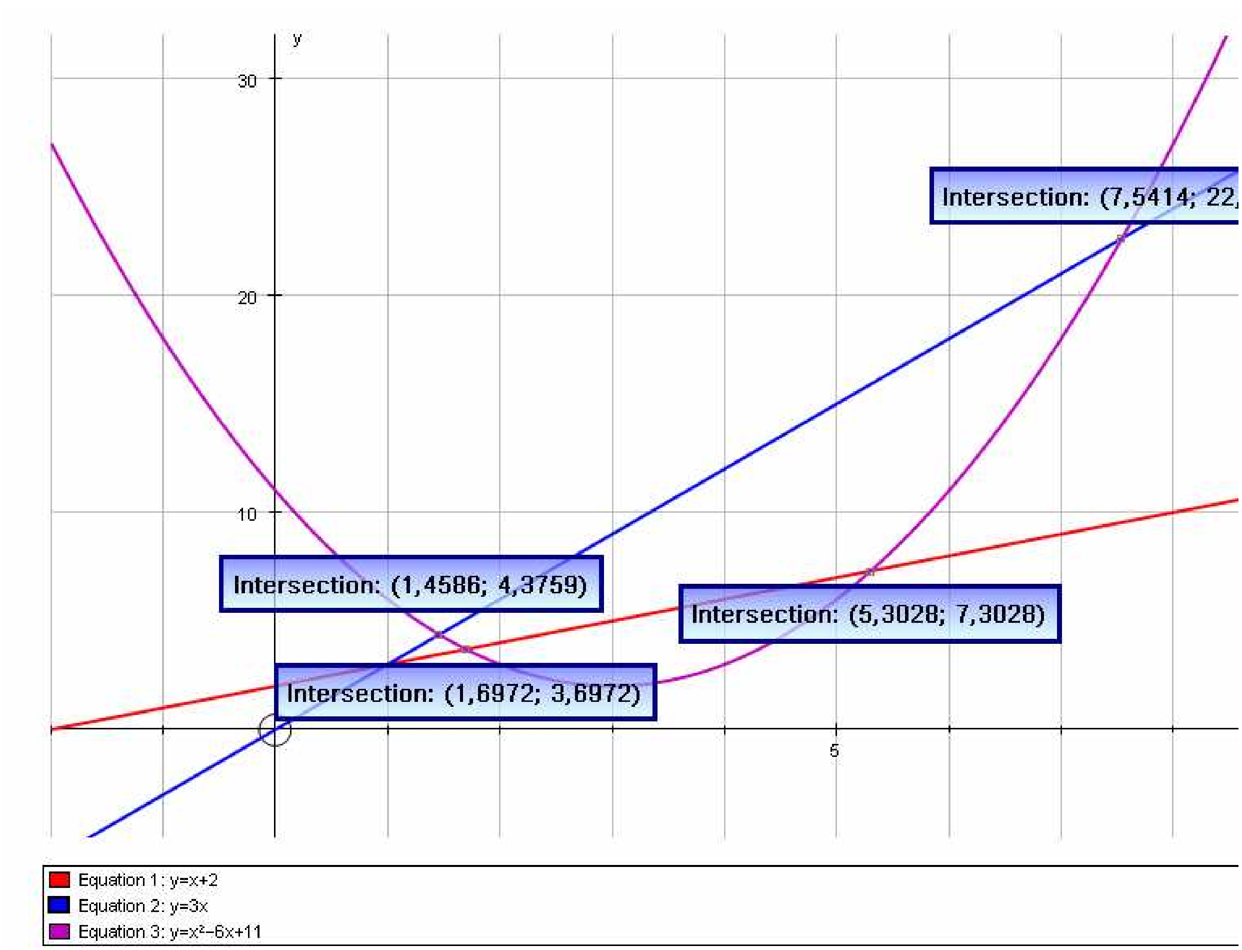
D=$\mid \frac{1}{a} \mid = \mid \frac{1}{-\sqrt{12}} \mid$ = 0.28868

Hereby my conjecture is proved.

4. Does your conjecture hold if the lines are changed? Modify your conjecture if necessary and prove it.
The questions asks if there is a change in the conjecture if the two lines y=x and y=2x are changed. Therefore, I am going to change the lines and see if any modification to the conjecture has to be made. In the beginning I will keep the parabola the same and only change the lines. Afterwards, I will change both the lines and the conjecture. If necessary, I will modify my conjecture and at the end prove it.

The Parabola for the first two example is going to stay the same:
$Y= x^2-6x+11$

I changed the lines to y=x +2 and y= 3x



Now I am going to use my conjecture from question 3 and check for validity:

x_1=1.4586
x_2=1.6972
x_3=5.3028
x_4=7.5414

D =| S_L - S_R | =|(x_2 - x_1) - (x_4 - x_3)| = | 0.2386 – (2.2386) | = | -2 | = 2

D=| $\frac{1}{a}$ | = | $\frac{1}{2}$ | = 0.5

My conjecture is not working if the lines are changed !!!

Now I am trying to modify my conjecture by analysing patterns which are occurring when the Parabola and/or the lines are changed.

Second Example

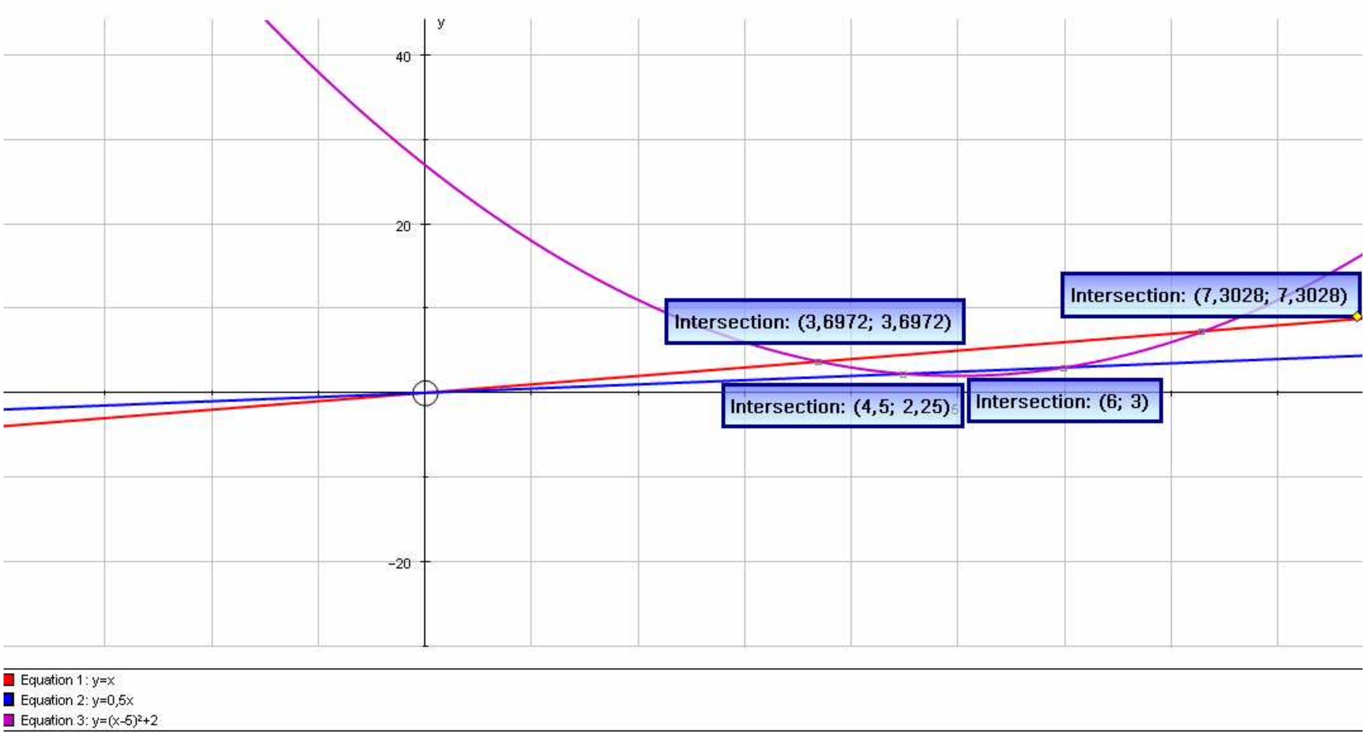
The Parabola is: $y = (x-5)^2+2$

The lines that I chose are:

y=x

y=0.5x

The x-coordinates of these intersections as they appear from left to right on the x-axis are: x_1, x_2, x_3, and x_4, respectively.



The x-values as they appear on the graph are as follows:

$x_1 = 3.6972$

$x_2 = 4.5$

$x_3 = 6$

$x_4 = 7.3028$

The values of $x_2 - x_1$ is named S_L and the values of $x_4 - x_3$ is named S_R.

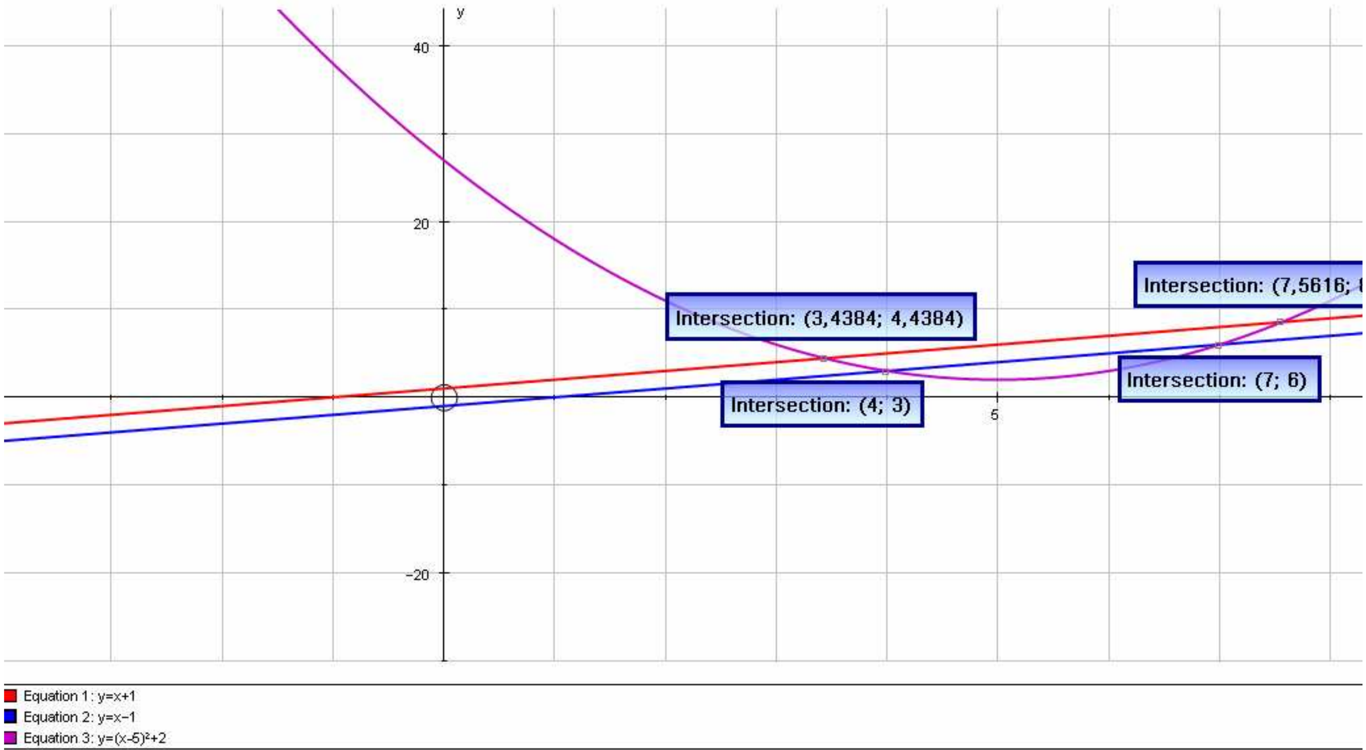
$S_L = x_2 - x_1 = 4.5 - 3.6972 = 0.8028$

$S_R = x_4 - x_3 = 7.3028 - 6 = 1.3028$

$D = | S_L - S_R| = |0.8028 - 1.3028| = 0.5$

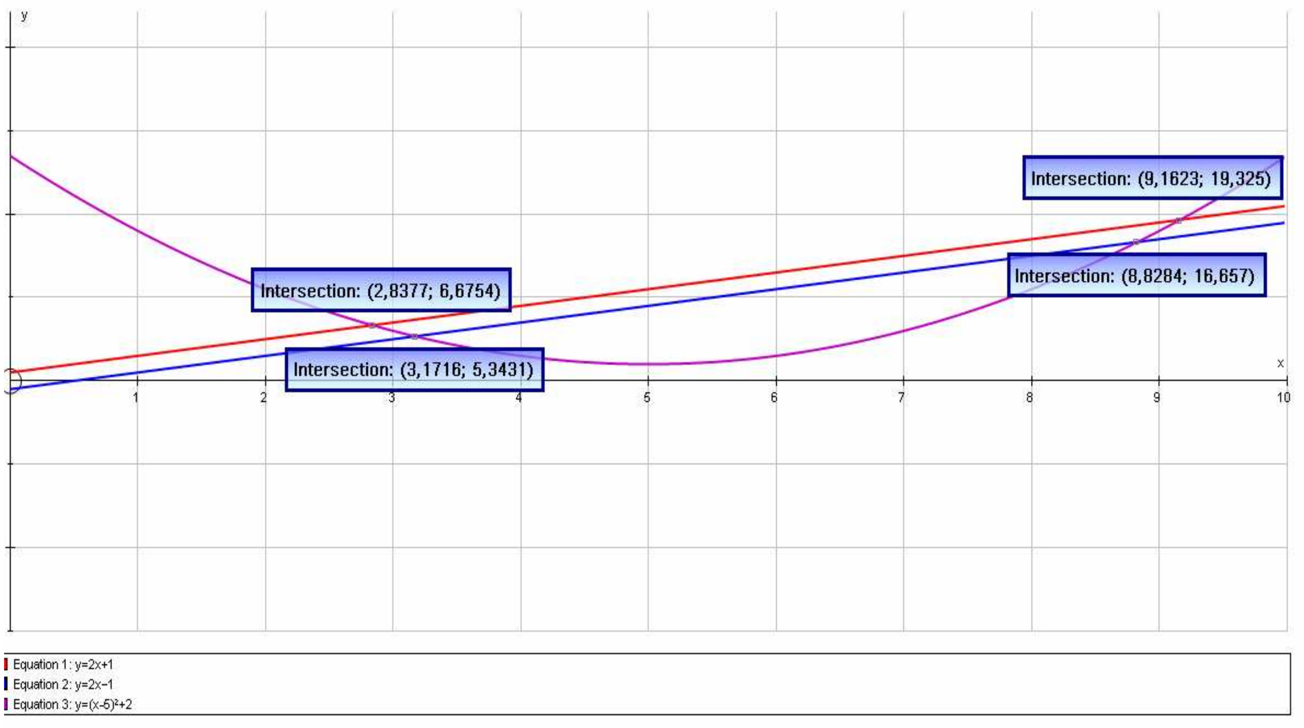
Third Example
The Parabola is: $y = (x-5)^2+2$
The lines that I chose are:
$y=x+1$
$y=x-1$

The x-coordinates of these intersections as they appear from left to right on the x-axis are: x_1, x_2, x_3, and x_4, respectively.



The x-values as they appear on the graph are as follows:
$x_1 = 3.4384$
$x_2 = 4$
$x_3 = 7$
$x_4 = 7.5616$
The values of $x_2 - x_1$ is named S_L and the values of $x_4 – x_3$ is named S_R.
$S_L = x_2 - x_1 = 4 - 3.4384 = 0.5616$
$S_R = x_4 – x_3 = 7.5616 – 7 = 0.5616$
$D = | S_L - S_R| = |0.5616 - 0.5616| = 0$

Fourth Example
The Parabola is: $y = (x-5)^2+2$
The lines that I chose are:
$y=2x+1$
$y=2x-1$
The x-coordinates of these intersections as they appear from left to right on the x-axis are: x_1, x_2, x_3, and x_4, respectively.



The x-values as they appear on the graph are as follows:
$x_1 = 2.8377$
$x_2 = 3.1716$
$x_3 = 8.8284$
$x_4 = 9.1623$
The values of x_2 - x_1 is named S_L and the values of $x_4 – x_3$ is named S_R.
$S_L = x_2 - x_1 = 3.1716 - 2.8377 = 0.3339$
$S_R = x_4 – x_3 = 9.1623 - 8.8284 = 0.3339$
$D = | S_L - S_R| = |0.3339 - 0.3339| = 0$

After now having investigated four different parabolas; I found a relation that might not seem obvious in the first place but if you realize it than you might get the hint.

Equations of the lines	Value of "a" of the parabola	Value for "D"
	$y = (x-5)^2+2$	
Y=x and Y=0.5x	1	0.5
Y=x+1 and Y=x-1	1	0
Y=2x+1 and Y=2x-1	1	0
Y=2x+3 and Y=2x+5	1	0
Y=3x+2 and Y=0.5x+5	1	2.5

I analyzed the patterns and I asked myself what actually a line is and what its components are. So I figured y=x + d is actually not everything there is this one part that I did not take in consideration in the conclusion before. This can best be explained by using a general parabola: $y=a(x+b)^2+c$ and general lines: $y=d_1x+e$ and $y=d_2x+f$. None of the values of "b", "c", "e" or "f" have any impact on the value of "D".

The conjecture which I made from the previous Question was: D=1/|a|

However, this conjecture is not valid anymore as the lines are changed and therefore has to be modified: The critical values which have to be paid attention on are: "d_1", "d_2" and "a" because they determine the gradient of the lines and the steepness of the parabola.

Through calculating and experimenting with these values I found out that the new conjecture will have to be:

$$D = |d_1 - d_2|/|a|$$

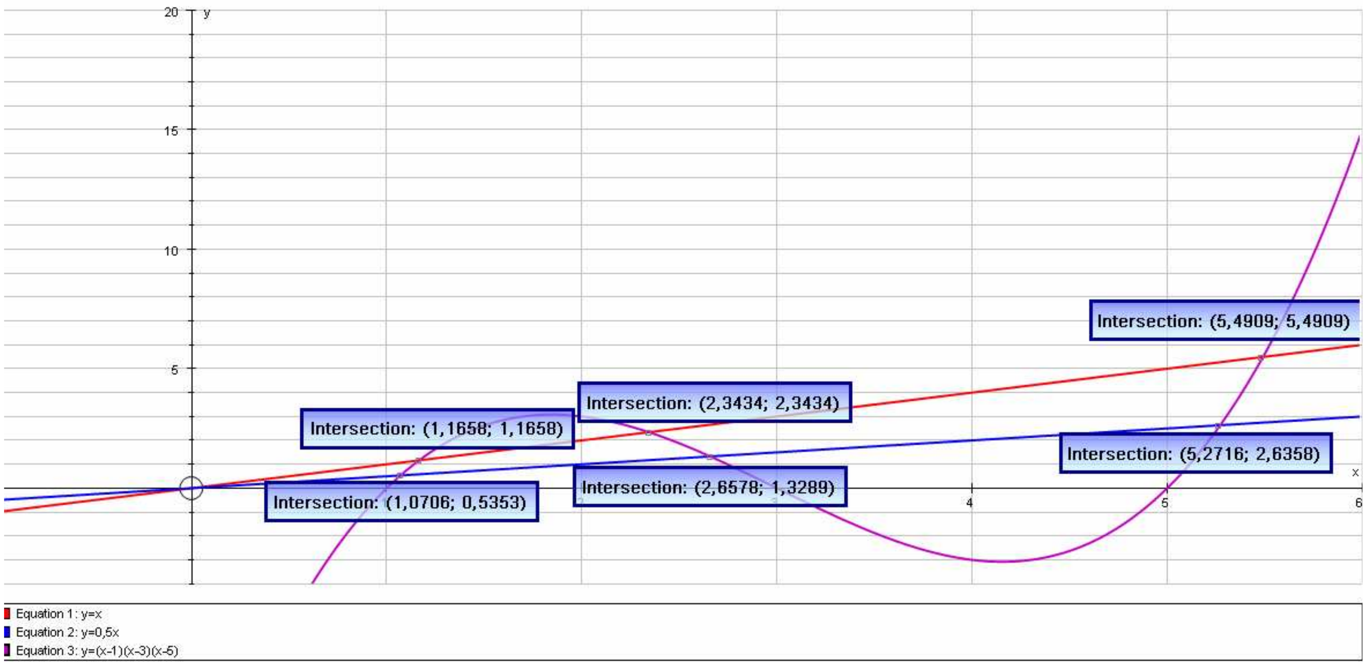
In order to prove this I will use the formula to see if my value for "D" equals the value I got from graphing the parabolas

uations of the lines	Value of "a" of the parabola $y = (x-5)^2+2$	Value for "D"	$D = \lvert d_1 - d_2 \rvert / \lvert a \rvert$
x and 0.5x	1	0.5	0. 5
x+1 and x-1	1	0	0
2x+1 and 2x-1	1	0	0
2x+3 and 2x+5	1	0	0
3x+2 and 0.5x+5	1	2.5	2.5

5. Determine whether a similar conjecture can be made for cubicpolynomials.

This question wants me to create a similar conjecture for cubic polynomials. Therefore I am going to keep the lines constant for the first few trials and only change the cubic polynomial and then I will change both the lines and the cubic polynomial. The difficulty hereby is that the cubic polynomial cuts the two lines more than four times. Therefore list all the intersections and try to find some kind of a pattern by considering both the consecutive terms and the ones following it.
In order to be able to find a conjecture, I am going to keep the cubic polynomial as simple as possible.

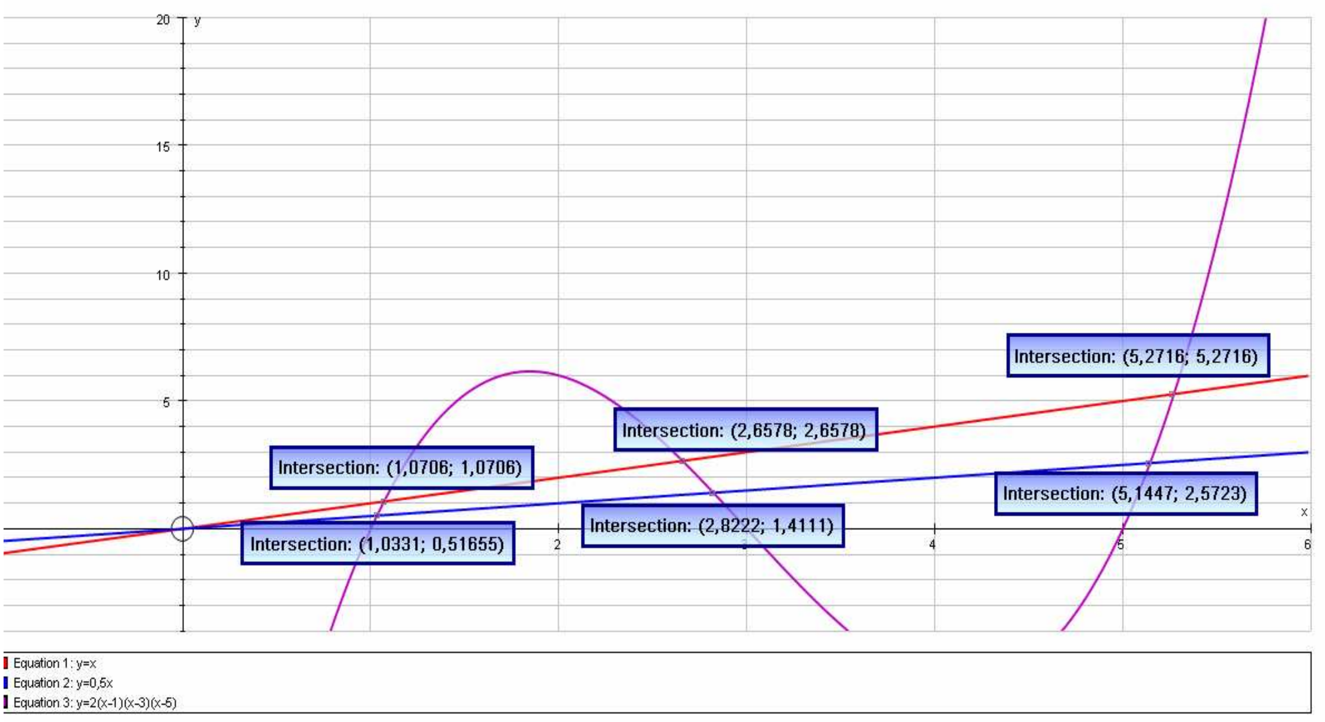
First Example
Y=(x-1)(x-3)(x-5)
Y=x and y = 0.5 x



The x-coordinates of these intersections as they appear from left to right on the x-axis are: x_1, x_2, x_3, x_4, x_5 and x_6 respectively.

The x-values as they appear on the graph are as follows:
$x_1 = 1.0706$
$x_2 = 1.1658$
$x_3 = 2.3434$
$x_4 = 2.6578$
$x_5 = 5.2716$
$x_6 = 5.4909$
The values of x_2 - x_1 is named S_L and the values of $x_4 – x_3$ is named S_R.
$S_L = x_2 - x_1 = 1.1658 - 1.0706 = 0.0952$
$S_R = x_4 – x_3 = 2.6578 - 2.3434 = 0.3144$
$D = | S_L - S_R| = |0.0952 - 0.3144| = 0.2192$
Basically I said that "D" is the first 4 Terms of the polynomial !
However, I am also interested in seeing how the last two points of intersections are related to "D", therefore I subtracted them from each other and found out that the last point of intersections subtracted from the point of intersections just before is also equal to D.
$x_6 - x_5 = 5.4909 - 5.2716 = 0.2193$
$D = x_6 - x_5 = 5.4909 - 5.2716 = 0.2193$
Therefore I can generalize my findings to:
$D = | x_6 - x_5 | = | x_4 – x_3| - | x_2 - x_1|$

Second Example
$Y=2(x-1)(x-3)(x-5)$
$Y=x$ and $Y=0.5x$
The x-coordinates of these intersections as they appear from left to right on the x-axis are: x_1, x_2, x_3, x_4, x_5 and x_6 respectively.

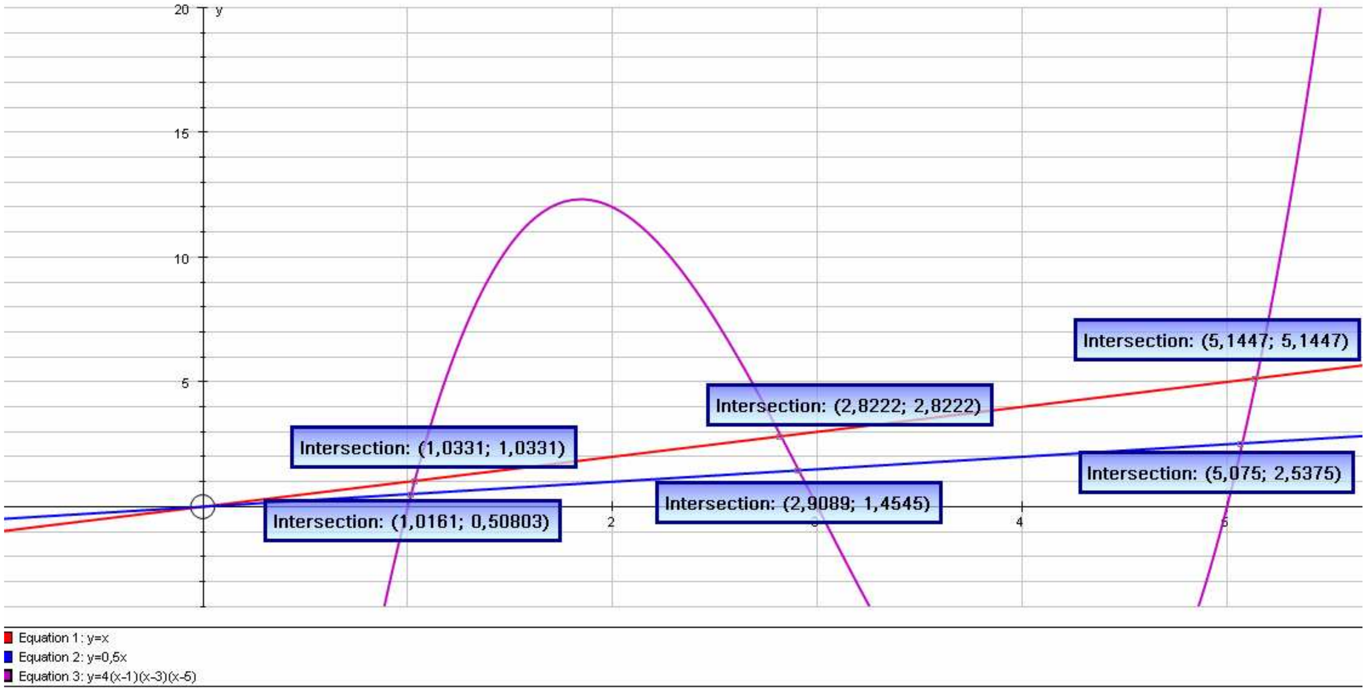


The x-values as they appear on the graph are as follows:
$x_1 = 1.0331$
$x_2 = 1.0706$
$x_3 = 2.6578$
$x_4 = 2.8222$
$x_5 = 5.1447$
$x_6 = 5.2716$
The values of $x_2 - x_1$ is named S_L and the values of $x_4 - x_3$ is named S_R.
$S_L = x_2 - x_1 = 1.0706 - 1.0331 = 0.0375$
$S_R = x_4 - x_3 = 2.8222 - 2.6578 = 0.1644$
$D = | S_L - S_R| = |0.0375 - 0.1644| = 0.1269$

Again, I want to see whether $x_6 - x_5$ gives me the same value as D, therefore I calculated:
$x_6 - x_5 = 5.2716 - 5.1447 = 0.1269$
As it can be seen, my findings have been repeated to be true:
$D = | x_6 - x_5 | = | x_4 - x_3| - | x_2 - x_1|$

Third Example
Y=4(x-1)(x-3)(x-5)
Y=x and Y=0.5x

The x-coordinates of these intersections as they appear from left to right on the x-axis are: x_1, x_2, x_3, x_4, x_5 and x_6 respectively.



The x-values as they appear on the graph are as follows:
$x_1 = 1.0161$
$x_2 = 1.0331$
$x_3 = 2.8222$
$x_4 = 2.9089$
$x_5 = 5.075$
$x_6 = 5.1447$
The values of $x_2 - x_1$ is named S_L and the values of $x_4 - x_3$ is named S_R.
$S_L = x_2 - x_1 = 1.0331 - 1.0161 = 0.017$
$S_R = x_4 - x_3 = 2.9089 - 2.8222 = 0.0867$
$D = | S_L - S_R| = |0.017 - 0.0867| = 0.0697$

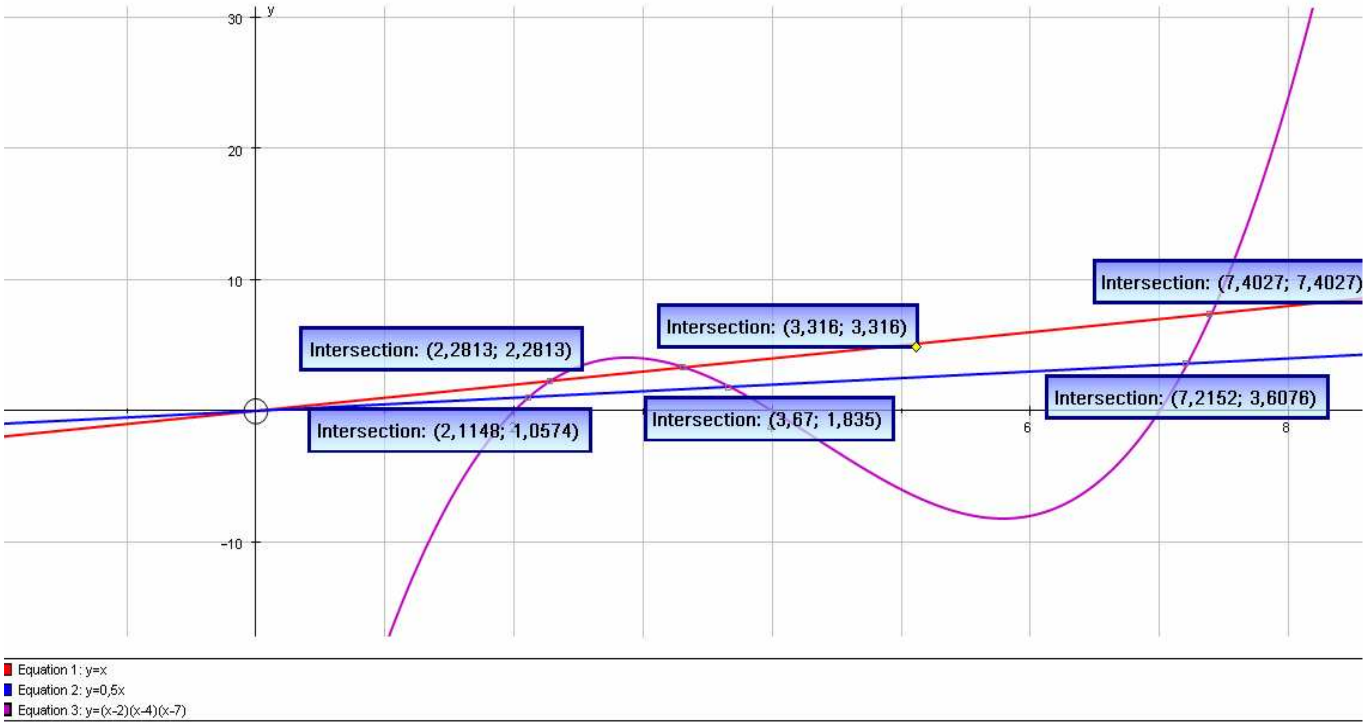
Again, I want to see whether $x_6 - x_5$ gives me the same value as D, therefore I calculated:
$x_6 - x_5 = 5.1447 - 5.075 = 0.0697$
As it can be seen, my findings have been repeated to be true:
$D = | x_6 - x_5 | = | x_4 - x_3| - | x_2 - x_1|$

Fourth Example

Y=(x-2)(x-4)(x-7)

Y=x and Y=0.5x

The x-coordinates of these intersections as they appear from left to right on the x-axis are: x_1, x_2, x_3, x_4, x_5 and x_6 respectively.



The x-values as they appear on the graph are as follows:

$x_1 = 2.1148$

$x_2 = 2.2813$

$x_3 = 3.316$

$x_4 = 3.67$

$x_5 = 7.2152$

$x_6 = 7.4027$

The values of $x_2 - x_1$ is named S_L and the values of $x_4 – x_3$ is named S_R.

$S_L = x_2 - x_1 = 2.2813 - 2.1148 = 0.1665$

$S_R = x_4 – x_3 = 3.67 - 3.316 = 0.354$

$D = | S_L - S_R| = |0.1665 - 0.354| = 0.1875$

Again, I want to see whether $x_6 - x_5$ gives me the same value as D, therefore I calculated:

$x_6 - x_5 = 7.4027 - 7.2152 = 0.1875$

As it can be see, my findings has been repeated to be true:

$D = | x_6 - x_5 | = | x_4 – x_3| - | x_2 - x_1|$

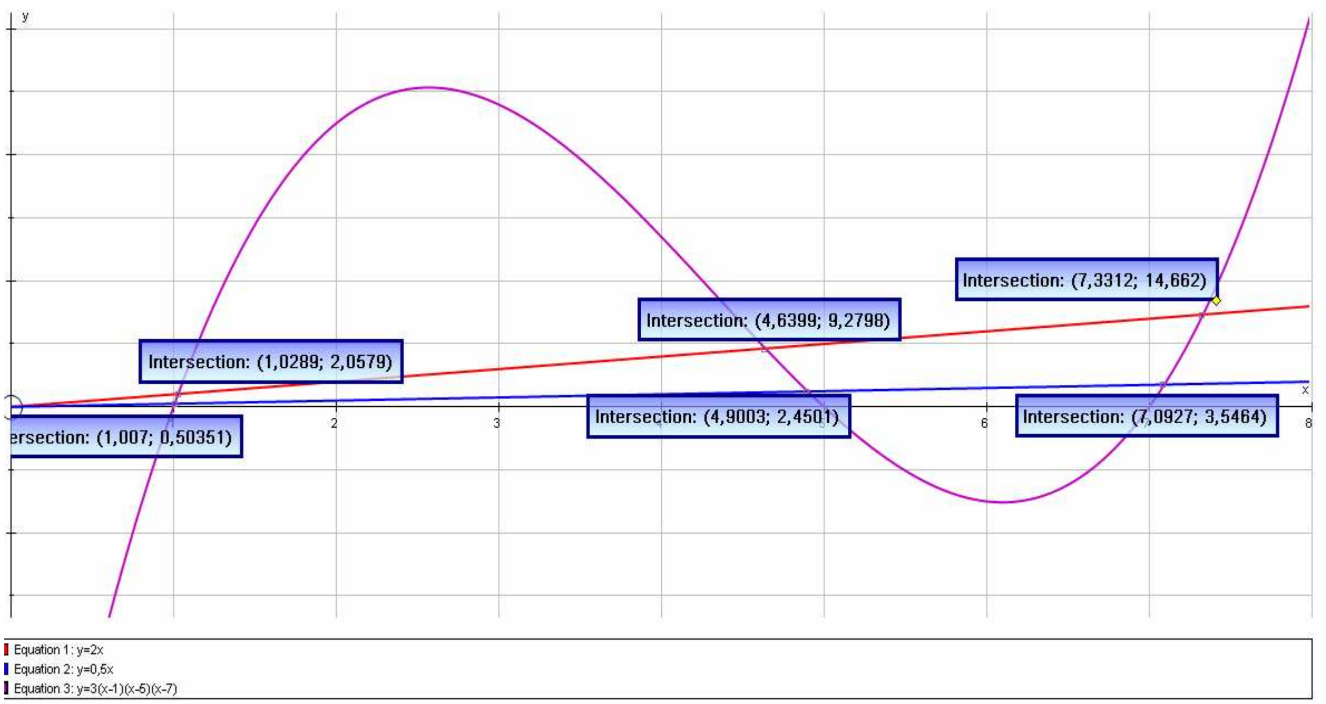
Now I am going to change both the lines and the cubic polynomial

Fifth Example

The polynomial is: Y=3(x-1)(x-5)(x-7)

The lines are: Y=2x and Y=0.5x

The x-coordinates of these intersections as they appear from left to right on the x-axis are: x_1, x_2, x_3, x_4, x_5 and x_6 respectively.



The x-values as they appear on the graph are as follows:

$x_1 = 1.007$

$x_2 = 1.0289$

$x_3 = 4.6399$

$x_4 = 4.9003$

$x_5 = 7.0927$

$x_6 = 7.3312$

The values of $x_2 - x_1$ is named S_L and the values of $x_4 – x_3$ is named S_R.

$S_L = x_2 - x_1 = 1.0289 - 1.007 = 0.0219$

$S_R = x_4 – x_3 = 4.9003 - 4.6399 = 0.2604$

$D = | S_{L\ -} S_R| = |0.0219 - 0.2604| = 0.2385$

Again, I want to see whether $x_6 - x_5$ gives me the same value as D, therefore I calculated:

$x_6 - x_5 = 7.3312 - 7.0927 = 0.2385$

As it can be see, my findings has been repeated to be true:

$D = | x_6 - x_5 | = | x_4 – x_3| - | x_2 - x_1|$

Conclusion

To find a conjecture I realized that D = | x_6 - x_5 | but how is this related to the principle of trigonometry so

First of all, I wanted to find the gradient of the curve:

Gradient of the curve (a) = rise/run = $y_6 – y_5 / x_5 – x_6 = y_6 – y_5 / D$

y_6 and y_5 are the y-coordinates of the points of intersection x_6 and x_5 respectively-

In order to now find “D” I am going to rearrange the equation to:

$D = y_6 – y_5 / a$

Now I want to have a look at the gradients of the lines intersecting the polynomial:

$m1 = y_5 / x_5$ So: $y_5 = m1 * x_5$

$m2 = y_6 / x_6$ So: $y_6 = m2 * x_6$

Furthermore it can be said that x_5 and x_6 have about the same value and so I want to set them equal, in order to make a simpler conjecture. Therefore $x_6 = x_5 = w$

Now I am going back to my original equation and insert my new findings:

D = ((d2 * w)– (d1 * w)) / a

D = (w*(d2-d1)) / a

D = (d2 – d1) / (a/w)

I found my conjecture to be the following:

D = |d2-d1| / |2a|

In order to see if my conjecture is valid, I am going to create a table to check my results:

Polynomial	Lines	D	Calculated value for D by using conjecture
Y = (x-1)(x-3)(x-5)	Y = x and y = 0.5x	0.2192	0.25
Y = 2(x-1)(x-3)(x-5)	Y = x and y = 0.5x	0.1269	0.125
Y = 4(x-1)(x-3)(x-5)	Y = x and y = 0.5x	0.0697	0.0625
Y = (x-2)(x-4)(x-7)	Y = x and y = 0.5x	0.1875	0.25
Y = 3(x-1)(x-5)(x-7)	Y = 2x and y= 0.5x	0.2385	0.25

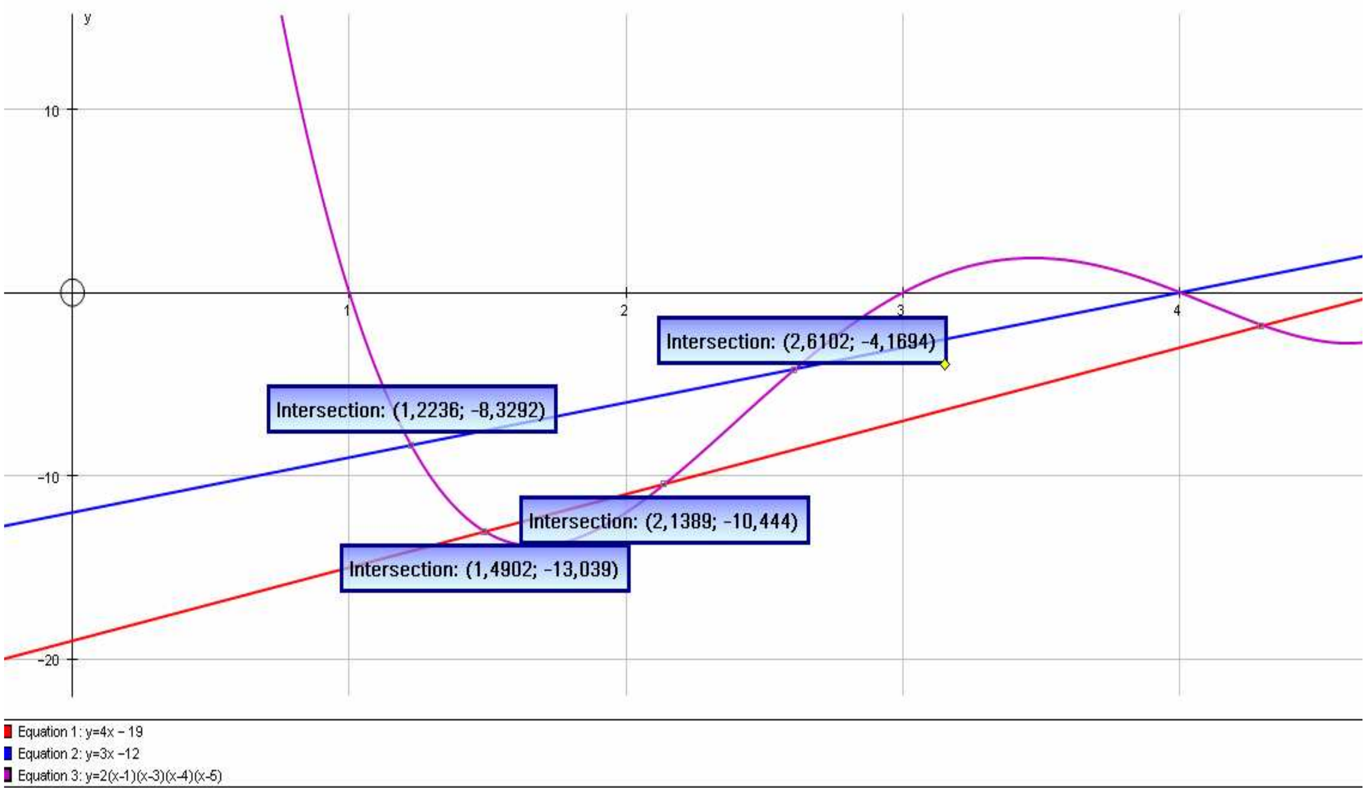
My value of “D” unfourtanetly is only close to the real value of my derived conjecture but still this can give you an approximate idea of the concept of cubicpoynomials.

6. Consider whether the conjecture might be modified to include higher order polynomials.
I am trying to find a conjecture for 4th degree polynomials first using the GDC.

First example
Polynomial: $y = (x-1)(x-3)(x-4)(x-5)$
The Lines: $y = x-6$ and $y = 0.5x-6$
The x-coordinates of these intersections as they appear from left to right on the x-axis are: x_1, x_2, x_3, and x_4, respectively.

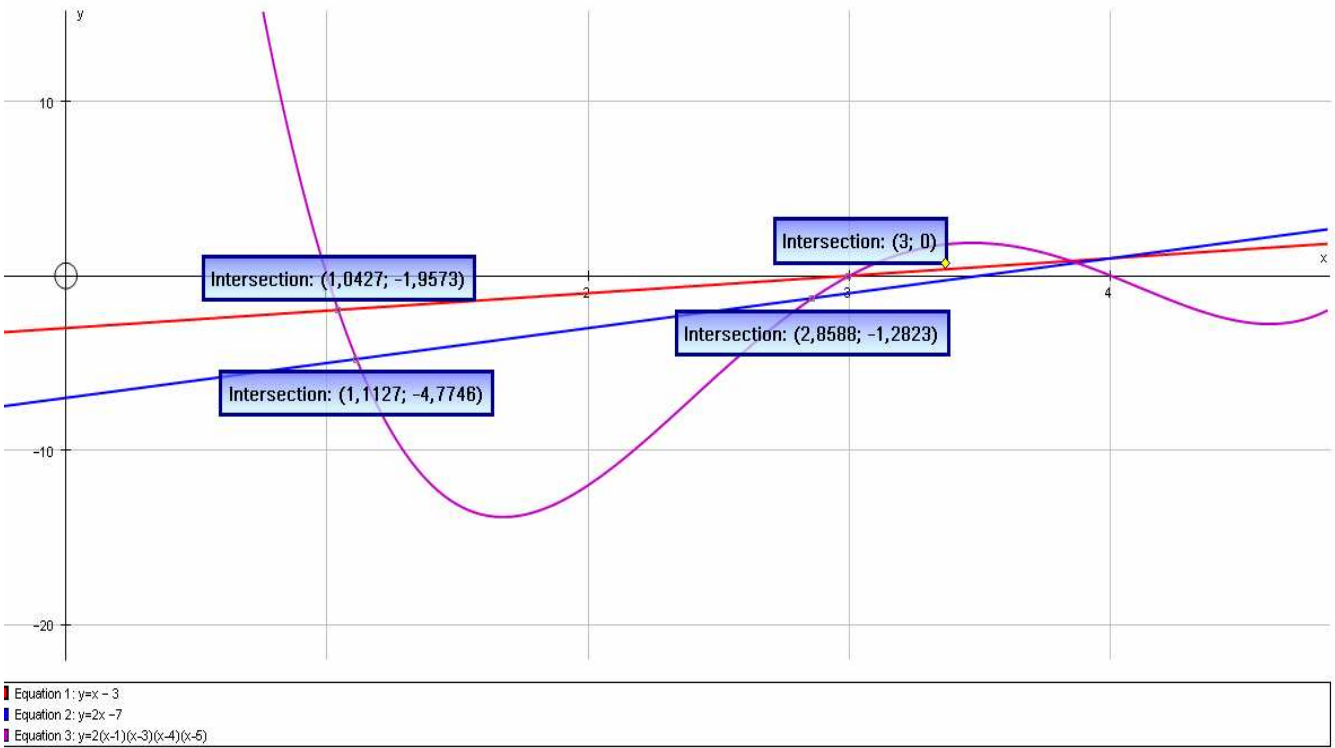
The x-values as they appear on the graph are as follows:
$x_1 = 1.2679$
$x_2 = 1.3236$
$x_3 = 2.1896$
$x_4 = 2.382$
The values of $x_2 - x_1$ is named S_L and the values of $x_4 – x_3$ is named S_R.
$S_L = x_2 - x_1 = 1.3236 - 1.2679 = 0.0557$
$S_R = x_4 – x_3 = 2.382 - 2.1896 = 0.1924$
$D = | S_L - S_R| = |0.0557 - 0.1924| = 0.1367$

Second Example
Polynomial: y = 2(x-1)(x-3)(x-4)(x-5)
Lines: y= 4x – 19 and 3x - 12
The x-coordinates of these intersections as they appear from left to right on the x-axis are: x_1, x_2, x_3, and x_4, respectively.



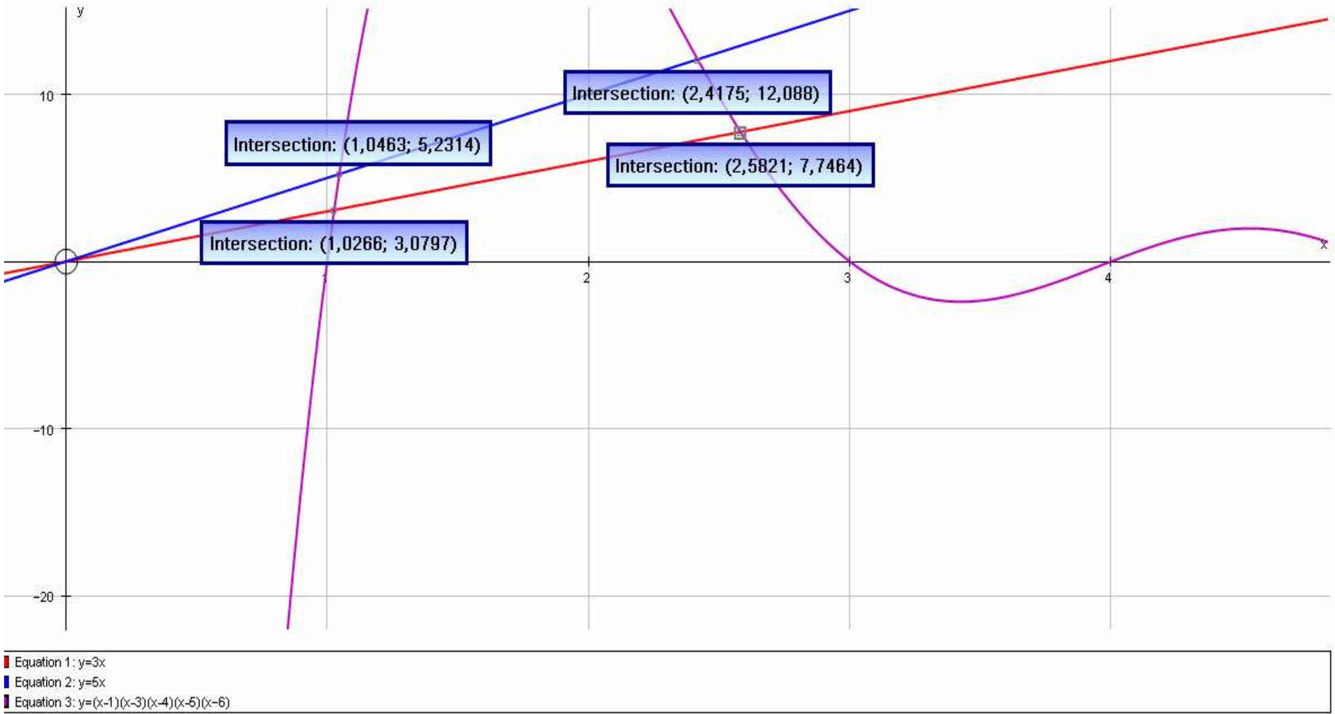
The x-values as they appear on the graph are as follows:
$x_1 = 1.2236$
$x_2 = 1.4902$
$x_3 = 2.1389$
$x_4 = 2.6102$
The values of $x_2 - x_1$ is named S_L and the values of $x_4 - x_3$ is named S_R.
$S_L = x_2 - x_1 = 1.4902 - 1.2236 = 0.2666$
$S_R = x_4 - x_3 = 2.6102 - 2.1389 = 0.4713$
$D = | S_L - S_R| = |0.2666 - 0.4713| = 0.2047$

Third Example
Polynomial: y = (x-1)(x-3)(x-4)(x-5)
Lines: y= 4x – 19 and 3x - 12
The x-coordinates of these intersections as they appear from left to right on the x-axis are: x_1, x_2, x_3, and x_4, respectively.



The x-values as they appear on the graph are as follows:
$x_1 = 1.1285$
$x_2 = 1.4009$
$x_3 = 2.2886$
$x_4 = 3$
The values of $x_2 - x_1$ is named S_L and the values of $x_4 – x_3$ is named S_R.
$S_L = x_2 - x_1 = 1.4009 - 1.1285 = 0.2724$
$S_R = x_4 – x_3 = 3 - 2.2886 = 0.7114$
$D = | S_L - S_R| = |0.2724 - 0.7114| = 0.439$

Fourth Example
Polynomial: y = (x-1)(x-3)(x-4)(x-5)(x-6)
Lines: y = 3x and y= 5x
The x-coordinates of these intersections as they appear from left to right on the x-axis are: x_1, x_2, x_3, and x_4, respectively.



The x-values as they appear on the graph are as follows:
$x_1 = 1.0266$
$x_2 = 1.0463$
$x_3 = 2.4175$
$x_4 = 2.5821$
The values of $x_2 - x_1$ is named S_L and the values of $x_4 – x_3$ is named S_R.
$S_L = x_2 - x_1 = 1.0463 - 1.0266 = 0.0197$
$S_R = x_4 – x_3 = 2.5821 - 2.4175 = 0.1646$
$D = | S_L - S_R| = |0.0197 - 0.1646| = 0.1449$

Conclusion

Each time the order of the polynomial has changed, the conjecture has changed as well. Therefore I assume that the power plays an important role in the conjecture.

The conjecture that I found in question 5 was: D = |d2-d1| / |2a|

At that case, the polynomial was to the power of 3. Therefore is the coefficient of a is (power-1).

Therefore my new general conjecture for any polynomial is: D = |d2-d1| / |(power-1)*a|, Where d1 and d2 are the gradients of the lines intersecting the polynomial and a is the gradient of the polynomial and the power is the order of the polynomial.

Now I want to summarize my findings form question 6 and check whether my conjecture is true.

Polynomial	Lines	D	Calculated value for D
Y=(x-1)(x-3)(x-4)(x-5)	Y=x-6 and y=0.5x-6	0.1367	0.166
Y=2(x-1)(x-3)(x-4)(x-5)	Y=4x-19 and y=3x-12	0.2047	0.166
Y=0.7(x-1)(x-3)(x-4)(x-5)	Y=2x-7 and y=x-3	0.439	0.952
Y=(x-1)(x-3)(x-4)(x-5)(x-6)	Y=3x and y=5x	0.1449	0.5

Again the calculated value for D did not come out to be exactly the same as the value for D that I got from drawing the polynomials and intersecting it with the lines. This is due to the fact that the gradient of the curve does not stay constant and so is only approximated. Therefore, my conjecture can only come out to be close, but not exactly the same.

Conclusion

After now having investigated in the patterns of the intersections of parabolas with two lines, I want to conclude that the general conjecture for any polynomial is:

D = (d2-d1) / ((power-1)*a), where d1 and d2 are the gradients of the lines intersecting the polynomial and a is the gradient of the polynomial and the power is the order of the polynomial.

The difficulties in this Project were that it was impossible to find an exact conjecture for D. It was always only an approximate. This is due to the fact that the gradient of the line, a, is approximated and changed throughout the whole curve. Therefore the calculated values for D will always vary slightly from the actual value for D.

However, the conjecture that I found comes out to be very close.

Parabola Investigation

Patrick Vollmer

Math Higher Level