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Pd. 4th

Investigating Logarithm Bases

In this portfolio, I'll attempt to investigate logarithm bases numerically and graphically by TI-83 plus graphic calculator, and other graphing soft wares. In part one to two, I'll analyze the given logarithms and try to find an expression for the n^{th} term of each sequence. In part three to five, I'll use previous expression I found to find out how to get 3^{rd} answer from first two answers, and create more examples from previous exercise. (Part I to V… The rest of them will be finished on due date.)

Part I

Here, I will analyze and try to write down next two terms of each sequence correctly. I will use TI-83 calculator to double check my work.

- Sequence 1: $\text{Log}_2 8$, $\text{Log}_4 8$, $\text{Log}_8 8$, $\text{Log}_{16} 8$, $\text{Log}_{32} 8$…
- Sequence 2: $\text{Log}_3 81$, $\text{Log}_9 81$, $\text{Log}_{27} 81$, $\text{Log}_{81} 81$…
- Sequence 3: $\text{Log}_5 25$, $\text{Log}_{25} 25$, $\text{Log}_{125} 25$, $\text{Log}_{625} 25$…

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- Sequence 4: $\text{Log}_m m^k$, $\text{Log}_{m^2} m^k$, $\text{Log}_{m^3} m^k$, $\text{Log}_{m^4} p\ m^k$ …

As I looked at the each sequences, I found out that base if the logarithm increased by number of power by that many terms from original base. This rule I found was proved by the last example of sequence 4. So by following the rule, next two terms from each sequence will be as follows:

- Sequence 1: $\text{Log}_2 8$, $\text{Log}_4 8$, $\text{Log}_8 8$, $\text{Log}_{16} 8$, $\text{Log}_{32} 8$, $\mathbf{Log_{64} 8}$**,** $\mathbf{Log_{128} 8}$…
- Sequence 2: $\text{Log}_3 81$, $\text{Log}_9 81$, $\text{Log}_{27} 81$, $\text{Log}_{81} 81$, $\mathbf{Log_{243} 81}$**,** $\mathbf{Log_{729} 81}$…
- Sequence 3: $\text{Log}_5 25$, $\text{Log}_{25} 25$, $\text{Log}_{125} 25$, $\text{Log}_{625} 25$, $\mathbf{Log_{3125} 25}$**,** $\mathbf{Log_{15625} 25}$…

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- Sequence 4: $\text{Log}_m m^k$, $\text{Log}_{m^2} m^k$, $\text{Log}_{m^3} m^k$, $\text{Log}_{m^4} m^k$, $\mathbf{Log_{m^5} m^k}$**,** $\mathbf{Log_{m^6} m^k}$ …

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Part II

Now, I will write an expression for the n^{th} term for each sequence. I'm going to use calculator to figure out the exact number from each term.

- **Sequence 1**: $\log_2 8$, $\log_4 8$, $\log_8 8$, $\log_{16} 8$, $\log_{32} 8$, $\log_{64} 8$, $\log_{128} 8$…

n	1	2	3	4	5	6	7	…	n
p/q	3	3/2	1	3/4	3/5	1/2	3/7	…	?

- **Sequence 2**: $\log_3 81$, $\log_9 81$, $\log_{27} 81$, $\log_{81} 81$, $\log_{243} 81$, $\log_{729} 81$…

n	1	2	3	4	5	…	N
p/q	4	2	4/3	1	4/5	…	?

- **Sequence 3**: $\log_5 25$, $\log_{25} 25$, $\log_{125} 25$, $\log_{625} 25$, $\log_{3125} 25$, $\log_{15625} 25$…

n	1	2	3	4	5	…	N
p/q	2	1	2/3	1/2	2/5	…	?

I noticed that all have numerator as 3 in p/q in sequence 1. The n^{th} term is **$3/n^{th}$**. Similarly, sequence 2 have common numerator as 4. So the n^{th} term is **$4/n^{th}$** in sequence 2. Lastly, Sequence 3 had common numerator 2, which allowed me to guess that **$2/n^{th}$** is n^{th} term for sequence 3. If this pattern is true, I can state that in sequence 4, the n^{th} term in p/q expression is **$\log_m m^k / n^{th}$.**

Part III & IV

In part three, I will calculate the given logarithms in the form p/q, where p, q$\in\mathbb{Z}$. I will again use TI- 83 calculator to solve and double check the answer.

$\log_2 64$, $\log_8 64$, $\log_4 64$

Logarithm	**1st**	**2nd**	**3rd**
p/q	**6/1 (6)**	**6/3 (2)**	**6/2 (3)**

$\log_4 1024$, $\log_{32} 1024$, $\log_8 1024$

Logarithm	**1st**	**2nd**	**3rd**
p/q	**10/2 (5)**	**10/5 (2)**	**10/3**

$\log_7 343$, $\log_{1/7} 343$, $\log_{1/49} 343$

Logarithm	**1st**	**2nd**	**3rd**
p/q	**3/1 (3)**	**3/-1 (-3)**	**3/-2**

$\log_{1/5} 125$, $\log_{1/25} 125$, $\log_{1/125} 125$

Logarithm	**1st**	**2nd**	**3rd**
p/q	**-3/1 (-3)**	**-3/2**	**-3/3 (-1)**

$\log_2 512$, $\log_8 512$, $\log_4 512$

Logarithm	**1st**	**2nd**	**3rd**
p/q	**9/1 (9)**	**9/3 (3)**	**9/2**

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From the first table above, I noticed at first that you divide first two logarithms to get the third answer. If the first and second equation is divided, it will looks like:

$$\frac{\text{Log } 64 \text{ X Log } 8}{\text{Log } 2 \text{ X Log } 64} \rightarrow \frac{\cancel{\text{Log } 64} \text{ X Log } 8}{\text{Log } 2 \text{ X } \cancel{\text{Log } 64}}$$

The Log 64 cancel each other out and only **Log 8/ Log 2** is left. If you put **Log 8/ Log 2** into your calculator, the answer is 3, which is third logarithm's answer in p/q form. However, rest of 4 sequences did not worked like the first sequence. I was able to see the clear connections between the bases of logarithms, but failed to found the relationship between 1st and 2nd to come up with the 3rd logarithm. I found out that bases are closely related, and was able to establish the pattern just for the base: **2nd base divided by 1st base will give 3rd logarithm's base in all 5 sets**. Also it had the same value ***m*…**

So then I substituted the 1st and 2nd logarithm as ***a*** and ***b*** respectively. I tried to multiply, subtract, add, and combination of these method. Finally, when I tried multiplication, division and subtraction all together, the fitting pattern appeared. The pattern I found is like this:

a*= *1st* logarithm, *b*= 2nd logarithm

$$\frac{ab}{(a-b)} = 3^{rd} \text{ logarithm}$$

For example, I used 2nd set of logarithms, which is **$\log_4 1024$, $\log_{32} 1024$, $\log_8 1024$.**
If I substitute first and second logarithm with ***a*** and ***b*** respectively, I will get following numbers**:**

$$\frac{(5)(2)}{(5-2)} = \frac{10}{3}$$

Here, I can see that it gives me the 3rd answer in form of p/q form. So I ended up with two distinctive patterns. With this two patterns I found, I will now try to make same kind of sets see if it follows the rules when I place different numbers in the base.

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Part V

Here in part five, I'll try to create two more examples that fit the two patterns from **Part IV within of my knowledge of understanding it**. I'll use calculator to get the answer that has fraction, and it is rational.

Example 1: $\text{Log}_{1/9}729$, $\text{Log}_{1/729}729$, $\text{Log}_{1/81}729$

Logarithm	1st	2nd	3rd
p/q	-3	-1	-3/2

Example 2: $\text{Log}_{6}216$ $\text{Log}_{216}216$, $\text{Log}_{36}216$

Logarithm	1st	2nd	3rd
p/q	3	1	1.5

It seems that these two examples fit on all two patterns. Both sets pass the relationship of bases and the equation I discovered during the investigation.

Part VI

Now, I will try to find general statement that expresses $\textbf{Log}_{b/a}\textbf{X}$ in terms of ***c*** **($\text{Log}_a X$)** and ***d*** **($\text{Log}_b X$)**. There is really no way to do this in other ways other than algebraically, so I will write steps. First I changed all logarithms into exponents. Then I made expression $\textbf{Log}_{b/a}\textbf{X}$ into an equation: $\textbf{Log}_{b/a}\textbf{X} = \textbf{y}$

- $a^c = x$
- $b^d = x$
- $(b/a)^y = x$
- $a^c = b^d$
- $b = a^{(c/d)}$
- Find y……
- $(b/a)^y = x = a^c$
- $(b/a)^y = a^c$
- $(b^y)(1/a)^y = a^c$ ← stretching the equation
- $(a^{(c/d)y})(1/a)^y = a^c$ ← substituted ***b*** to $a^{(c/d)}$
- $a^{(cy/d)}(a)^{-y} = a^c$ ← change all expressions with base ***a***
- $(cy/d)-y = c$
- $cy-dy = cd$
- $y(c-d) = cd$
- $y = cd/(c-d)$

As it turns out, the equation I found here was equal to what I found in **part IV!** Also, the relationship between the bases that I found was also in it, here represented by ***c*** and ***d***. I think it is not a coincident that I found those two patterns in both parts.

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Part VII

In this section, I will put different types of integers in a b or x to see this equation is valid. I will use the TI- 83 plus calculator to find exact value or close to 3 significant figures.

- **Example 1:** Log $_{1/100}$ 1000, Log $_{1/1000}$ 1000, Log $_{1/10}$ 1000

Logarithm	1st	2nd	3rd
p/q	-1.5	-1	-3/2

- **Example 2:** Log $_2$ 216 Log $_8$ 216, Log $_4$ 216

Logarithm	1st	2nd	3rd
p/q	7.75	2.58	3.87

- **Example 3:** Log $_2$ 1 Log $_8$ 1, Log $_4$ 1

Logarithm	1st	2nd	3rd
p/q	0	0	0

Example 5: Log $_2$ -8 Log $_8$ -8, Log $_4$ -8

Logarithm	1st	2nd	3rd
p/q	Und.	Und.	Und.

- **Example 4:** Log $_2$ 0 Log $_8$ 0, Log $_4$ 0

Logarithm	1st	2nd	3rd
p/q	Und.	Und.	Und.

It fits the equation y = cd/(c-d). Also I see the expression **Log $_{b/a}$ X** as well**.** I double checked with the calculator and it is valid. I seems like if the **x** is same for all three expressions, and **have bases that 2nd /1st = 3rd** equation, the general statement **Log $_{b/a}$ X** = cd/(c-d) is true. However if any of these are incorrect, the expression no longer works. There are some limitations on x, a, and b.

Part VIII

Now, since I found the general expression of **Log $_{b/a}$ X,** I will attempt to look for the limitation of a, b, and x by studying the precious parts.

As I analyzed the precious parts, I found that ***a***, and ***b***'s range is (0, 1)U(1, ∞), range for the base of the logarithm. If ***a***, or ***b*** = 1 or ***a***, or ***b*** $\leq$ **0**, this expression will not work. Also, if **x** is not same in all three logarithms in the set, this statement will no longer work. Finally, if **x** is 1, 0, or negative integers, this expression no longer works as you see in example 3, 4 and 5.

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<u>Part IX</u> (how did I come this far?)

At first, I was given with the set of logarithms. I analyzed them and put them into form of p/q. At the part II, I found the sets' p/q form follows the expression **Log** $_m\, m^k$ **/ n**th**.** So In part 3 and 4, I tried to find a pattern that was in the given sets of logarithms. It took some time, but I found out that three logarithms had same x and 3rd Log's base was product from 2nd Log's base being divided by 1st Log's base.

After discovery, I attempted to find a pattern for the set by substituting first two Logs with ***a*** and ***b*** combinations of adding, subtracting, division and multiplication. It took me a long time, but I was able to find the expression **3rd Log = ab /(a-b).** Later in part six, I was able to confirm that the expression I found was the general expression I was looking for the whole time. After investigating further into the expression, I came to a conclusion that **Log** $_{b/a}$ **X** = cd/(c-d) is true. There is limitation to this expression, however. If a, b, or x $\neq$ (0, 1)U(1, ∞).

As I finished investigating the bases of common logarithms, I came to a conclusion that you can get third logarithm's answer from the two Logs if I use expression **Log** $_{b/a}$ **X =** cd/(c-d) if **a**, **b**, and **x = (0, 1)U(1, ∞),** and **x** has to be **constant in both Logs**.