Logarithm Bases

$\log_4 64 = 3, \log_8 64 = 2, \log_{32} 64 = \frac{6}{5}$

$\log_7 49 = 2, \log_{49} 49 = 1, \log_{343} \frac{2}{3}$

$\log_{\frac{1}{5}} 125 = -3, \log_{\frac{1}{125}} 125 = -1, \log_{\frac{1}{625}} 125 = \frac{-3}{4}$

$\log_8 512 = 3, \log_2 512 = 9, \log_{16} 512 = \frac{9}{4}$

Describe how to obtain the third answer in each row from the first two answers. Create two more examples that fit the pattern above.

In order to obtain the answer of the third logarithm, the product of the first and second logarithms must be divided by the sum of the answers of the first and second logarithms.

Two other examples that fit the pattern above:

$\log_2 16 = 4$, $\log_4 16 = 2$, $\log_8 16 = \frac{4}{3}$

$\log_9 81 = 2$, $\log_{27} 81 = \frac{4}{3}$, $\log_{243} 81 = \frac{4}{5}$

Let $\log_a x = c$ and $\log_b x = d$. Find the general statement that expresses $\log_{ab} x$, in terms of *c* and *d*.

The general statement that expresses $\log_{ab} x$ is: $\log_{ab} x = \frac{cd}{c+d}$

To test this statement substitute the values of a, x, and c for the values *a*= 2, *b*=4, and *x*= 256:

Thus, $\log_2 256 = 8 \; and \; \log_4 256 = 4 \; so \; \log_{2\bullet 4} 256 = \frac{8 \bullet 4}{8+4} = \log_8 256 = \frac{32}{12} = \frac{8}{3}$

Scopes and/or limitations of *a*, *b*, and *x*

1. $a > 0$, $b > 0$

The base of a logarithmic equation has to be greater than zero thus, a has to be greater than 0 and b has to be greater than 0.

2. $ab \neq 1$
The base of a logarithmic equation cannot be one because any argument to the base 1 is undefined. If you take an equation such as $\log_1 x$ and by using the change of base formula we get $\frac{\ln(x)}{\ln(1)}$. The natural log of 1 is 0, thus this equation is undefined because the denominator is 0.
For example,
If we let a=4 and let b= $\frac{1}{4}$ and let x=64 in the follow logarithmic equations: $\log_a x = c$ and $\log_b x = d$, then by substituting all the values of *a*, *b*, and *x* we get: $\log_4 64 = 3$, $\log_{\frac{1}{4}} 64 = -3$.

Thus, by using the equation $\log_{ab} x = \frac{cd}{c+d}$, and substituting the values we get:

$\log_1 64 = \frac{-9}{3-3} =$ undefined. This proves that any logarithmic equation with a base of 1 will be undefined.

Explain how you arrived at your general statement

I obtained the general statement, $\log_{ab} x = \frac{cd}{c+d}$, by finding the answers of the first two logarithms using the change of base formula. By taking the natural log of the argument and dividing it by the natural log of the base we obtain the answers of the first and second answers. Then by looking at the answer of the third answer, I assumed that the answer was obtained by taking the product of the first two logarithms and dividing it by the sum of the first two logarithms.

$$\log_2 8 = \log_{2^1} 2^3 = \frac{3}{1}; \log_4 8 = \log_{2^2} 2^3 = \frac{3}{2}; \log_8 8 = \log_{2^3} 2^3 = \frac{3}{3}; \log_{16} 8 = \log_{2^4} 2^3 = \frac{3}{4}; \ldots\ldots\ldots$$

$$\log_{2^n} 8 = \log_{2^n} 2^3 = \frac{3}{n}.$$

$$\log_3 81 = \log_{3^1} 3^4 = \frac{4}{1}; \log_9 81 = \log_{3^2} 3^4 = \frac{4}{2}; \log_{27} 81 = \log_{3^3} 3^4 = \frac{4}{3}; \log_{81} 81 = \log_{3^4} 3^4 = \frac{4}{4}; \ldots\ldots\ldots$$

$$\log_{3^n} 81 = \log_{3^n} 3^4 = \frac{4}{n}.$$

$$\log_5 25 = \log_{5^1} 5^2 = \frac{2}{1}; \log_{25} 25 = \log_{5^2} 5^2 = \frac{2}{2}; \log_{125} 25 = \log_{5^3} 5^2 = \frac{2}{3}; \log_{625} 25 = \log_{5^4} 5^2 = \frac{2}{4}; \ldots\ldots\ldots$$

$$\log_{5^n} 25 = \log_{5^n} 5^2 = \frac{2}{n}.$$

$$\log_4 256 = \log_{4^1} 4^4 = \frac{4}{1}; \log_{16} 256 = \log_{4^2} 4^4 = \frac{4}{2}; \log_{64} 256 = \log_{4^3} 4^4 = \frac{4}{3}; \log_{256} 256 = \log_{4^4} 4^4 = \frac{4}{4}; \ldots\ldots\ldots$$

$$\log_{4^n} 256 = \log_{4^n} 4^4 = \frac{4}{n}.$$

$$\log_{m^1} m^k = \frac{k}{1}; \log_{m^2} m^k = \frac{k}{2}; \log_{m^3} m^k = \frac{k}{3}; \log_{m^4} m^k = \frac{k}{4}; \ldots\ldots\ldots \log_{m^n} m^k = \frac{k}{n}.$$

Find and expression for the nth term of each sequence Write your expressions in the form $\frac{p}{q}$; where p,q $\in \mathbb{Z}$. Justify your using technology.

Using a TI-84 calculator we verified the answers we got by taking each logarithmic expression and by using the change of base formula. By doing so we took the natural log of the argument and divided it by the natural log of the base. For example:

$$\log_2 8 = \frac{\ln(8)}{\ln(2)} = 3$$

$$\log_4 256 = \frac{\ln(256)}{\ln(4)} = 4$$

$$\log_{125} 25 = \frac{\ln(25)}{\ln(125)} = \frac{2}{3}$$