Logan's Logo

Abstract

This internal assessment focuses on functions and areas under curves. The task at hand was to develop models of functions to best fit the characteristics/behaviours of the curve of Logan's logo. Using an appropriate set of axes data points of the curve were measured using a ruler and then identified and recorded. 11 data points were recorded for each curve. The main objective of this IA was to determine which model, cubic or sin, would be the best fit. It was observed that the cubic function was superior. Another task was to refine the model function to fit on a t-shirt and a business card. The area between the two curves had to be calculated. In doing this assignment many pieces of technology were incorporated when doing this assignment, ranging from a pencil and a ruler to calculators and even spreadsheets and other programs.

Introduction:

A diagram of a 10cm by 10cm square is divided into three regions by two curves. The logo is the shaded region between the two curves. This investigation is aimed at answering several questions but mainly to develop mathematical functions to model the two curves represented by the two curves. Some key terms that should be understood are: sin function, cubic function, MAE (Mean Absolute Error). These terms will be elaborated on later on this assignment.

A few data points were taken from the curves, with the base of the square representing the x-axis and each unit being one centimetre, and the y-axis represented as the left side of the square with each unit being a centimetre as well. The x-axis begins at 0 and goes on till 10 centimetres; the same is applicable for the y-axis. The lower curve is denoted at $f(x)$ and the upper curve denoted as $g(x)$. The following points were observed using a ruler

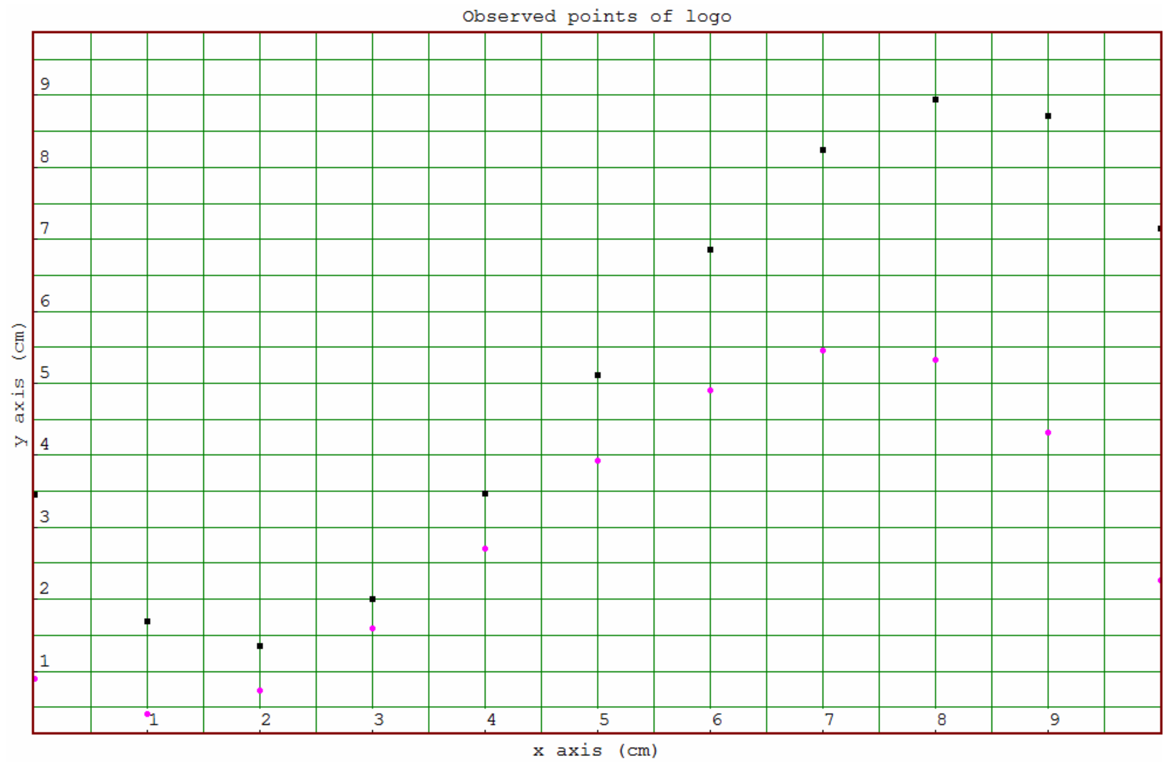
X	0	1	2	3	4	5	6	7	8	9	10
Y	0.9	0.41	0.73	1.6	2.7	3.93	4.91	5.45	5.32	4.31	2.27

Table 1: observed data points for the curve $f(x)$

X	0	1	2	3	4	5	6	7	8	9	10
Y	3.45	1.69	1.35	2.00	3.47	5.12	6.85	8.25	8.95	8.71	7.15

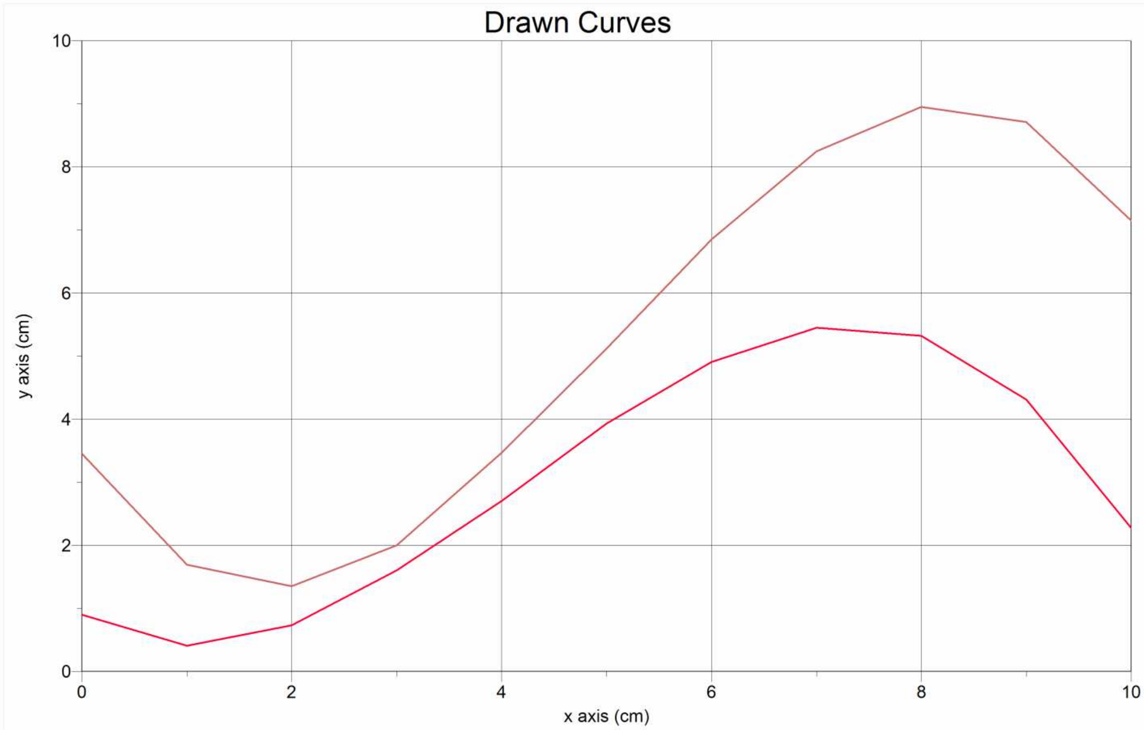
Table 2: observed data points for the curve $g(x)$

The points of these curves are graphed on the following graph:



Graph 1: Observed points of logo

In the graph above the points were plotted in a coordinate system with a domain of $0 \leq x \leq 10$ and a range of $0 \leq y \leq 10$, this was done according to the size of the diagram of the logo given. The points were plotted on a graph using the program Graphamatica. These points were then connected using a best fit curve with the program Logger Pro. By graphing the best fit curve line we can see what type of function this curve might be. This graph can be seen below



Graph 2: Points of **Graph 1** connected

It should be noted that the curve is not very smooth but rather rigid this is because these are not the exact points of the logo these are the points observed using a ruler, making the values slightly inaccurate resulting in a very rigid curve rather than a smooth one. Nonetheless an idea of the characteristics of the curve can be formulated. The curves best fit the family of periodical functions such as sine and cosine, as well as a polynomial function with an order of 3 or greater such $x^3, x^4, x^5, etc.$

For the purpose of simplicity only a sine and cubic function model will be used to see which is appropriate to represent the behaviour of the two curves of the logo.

Cubic Model:

A cubic function has the generic function of: $y = ax^3 + bx^2 + cx + d$. Where x and y are the variables representing the distance horizontally and vertically from the origin, respectively. The parameters a, b, c and d transform the graphs in to best fit the observed points. It should be noted that the cubic model has the constraints $0 \leq x, y \leq 10$ because there is no information regarding the curves exceeding the set boundaries. In order to find a function that showcases the behaviours of the curves, the values of the parameters a, b, c, d have to be calculated. This can be calculated by solving a simultaneous equation for four equations, since there are four unknowns. It is best if the points chosen are further apart from each other good idea to select points which are further away from each other. This comparison is made below:

Lower Curve

$0.41 = a(1)^3 + b(1)^2 + c(1) + d$

$0.73 = a(2)^3 + b(2)^2 + c(2) + d$

$2.7 = a(4)^3 + b(4)^2 + c(4) + d$

$3.93 = a(5)^3 + b(5)^2 + c(5) + d$

From here we can use matrices to solve for a, b, c, and d

$$[A] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 8 & 4 & 2 & 1 \\ 64 & 14 & 4 & 1 \\ 125 & 25 & 5 & 1 \end{bmatrix}$$

$$[B] = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$$[C] = \begin{bmatrix} 0.41 \\ 0.73 \\ 2.7 \\ 3.93 \end{bmatrix}$$

$[A] \times [B] = [C]$ $[A]^{-1} \times [C] = [B]$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 8 & 4 & 2 & 1 \\ 64 & 14 & 4 & 1 \\ 125 & 25 & 5 & 1 \end{bmatrix}^{-1} \times \begin{bmatrix} 0.41 \\ 0.73 \\ 2.7 \\ 3.93 \end{bmatrix} = \begin{bmatrix} 0.0267 \\ 0.0520 \\ -0.0232 \\ 0.3544 \end{bmatrix}$$

This gives the function: $f(x) = 0.0267x^3 + 0.0520x^2 - 0.0232x + 0.3544$

However if the points chosen are more wide-spread different values for the parameters are obtained:

$1.6 = a(3)^3 + b(3)^2 + c(3) + d$

$3.93 = a(5)^3 + b(5)^2 + c(5) + d$

$5.45 = a(7)^3 + b(7)^2 + c(7) + d$

$$2.27 = a(10)^3 + b(10)^2 + c(10) + d$$

$$\begin{bmatrix} 27 & 9 & 3 & 1 \\ 125 & 25 & 5 & 1 \\ 343 & 49 & 7 & 1 \\ 1000 & 100 & 10 & 1 \end{bmatrix}^{-1} \times \begin{bmatrix} 1.6 \\ 3.93 \\ 5.45 \\ 2.27 \end{bmatrix} = \begin{bmatrix} -0.0375 \\ 0.4617 \\ -0.6900 \\ 0.5275 \end{bmatrix}$$

This gives the function: $f(x) = -0.0375x^3+0.4617x^2-0.6900x+0.5275$

However if more points are incorporated, a better fit line is obtained. This cannot be attained by using matrices because in order to obtain the inverse of a matrix it must be a square. In order to calculate this we must use a different method, a method that incorporates all these points, we will now use the cubic regression tool on a Ti-84 Plus Graphical Display Calculator (GDC). To calculate this we must upload the 11 points observed, this is done by first pressing the "STAT" button followed by "1:Edit". A table is then brought to the screen, this will be filled with the x and y values observed. The L1 table corresponds to the x values and the L2 for the y values. Once completed the table is as follows:

L1	L2	L3 1
0.0000	.9000	------
1.0000	.4100	
2.0000	.7300	
3.0000	1.6000	
4.0000	2.7000	
5.0000	3.9300	
6.0000	4.9100	

L1(1) =0

Fig 1: table of points on lower curve

Using the values of L1 and L2, we can now calculate the values for a,b,c and d using the cubic regression tool of the GDC. To do this we must return to the Statistics menu on the GDC by pressing the "STAT" button once again and then scrolling to the "CALC" tab, a menu is presented then select "6:CubicReg". We are brought back to the home screen with "CubicReg" displayed on the screen. Now we select the L1 and L2 lists with a comma in between them. The cubic regression tool gives the following values:

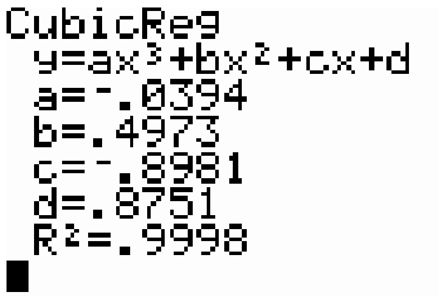
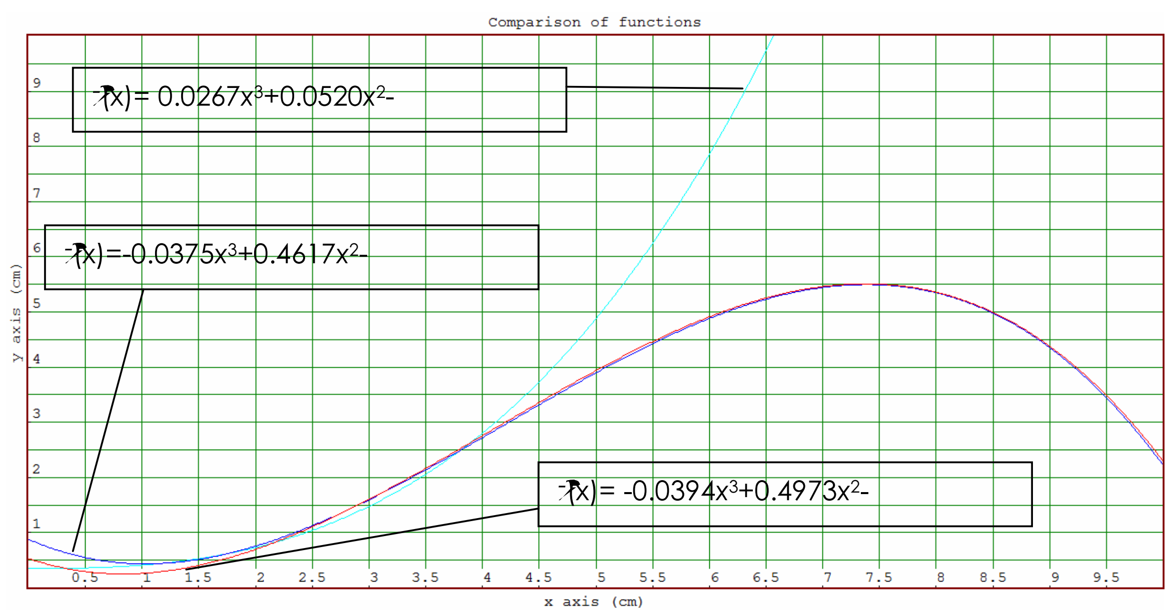


Fig 2: Cubic Regression of lower curve

The cubic regression tool gives us the function:

$f(x) = -0.0394x^3+0.4973x^2-0.8981x+0.8751$

The R^2 value is a measure of accuracy. The closer the value it is to 1, the more accurate the values of the parameters are. In this case, an R2 value of 0.9998 means that the graph 99.98% meets the measured points. Considering the accuracy of this function, this function will be used as a cubic function model of the lower curve. A comparative graph is listed below, illustrating the function formulated using the cubic regression tool and matrices, of which one set has a wider spread of points chosen.



Graph 3: Comparison of function attained

Another way of measuring accuracy is through the Mean Absolute error (MAE) of the functions, and how much the function matches the observed points. To calculate this, the absolute value of the difference between the y observed and y expected values, where y expected is the value that was calculated by the function. The values for y-expected were found using Graphmatica. Microsoft Excel was used to calculate the absolute error. This is summarized in the table below:

x	y observed	y expected	Absolute Error
0	0.9	0.88	0.02
1	0.41	0.4349	0.0249
2	0.73	0.7529	0.0229
3	1.6	1.5927	0.0073
4	2.7	2.7179	0.0179
5	3.93	3.8921	0.0379
6	4.91	4.8789	0.0311
7	5.45	5.4419	0.0081
8	5.32	5.3447	0.0247
9	4.31	4.3509	0.0409
10	2.27	2.2241	0.0459
Total			0.2816

Table 3: Absolute Error values for lower curve using cubic function

The MAE is calculated by adding the absolute error values together and dividing it by the number of the points chosen:

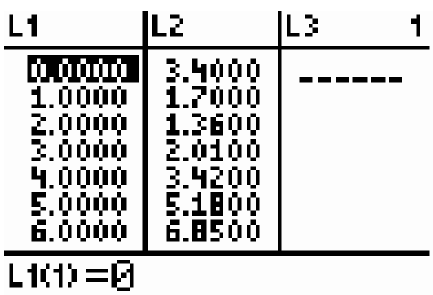
$$MAE = \frac{0.2816}{11} = 0.0256$$

This implies that on average the generated y value from the cubic function model misses the observed points by 0.0256 units

<u>Upper curve</u>

The same process used to calculate the lower curve will be used to calculate the upper curve. Recall that the function for best fit curve for a cubic function could be calculated by matrices or the cubic regression. Seeing as how the cubic regression gave a more accurate function we shall use this method and disregard the matrices method.

Recall that in order to calculate a more accurate function we must use a method that incorporates all the points observed, therefore we will use the cubic regression tool on a Ti-84 Plus Graphical Display Calculator (GDC). Recall that o calculate this we must upload the 11 points observed, this is done by first pressing the "STAT" button followed by "1:Edit". A table is then brought to the screen, this will be filled with the x and y values observed. The L1 table corresponds to the x values and the L2 for the y values. Once completed the table is as follows:



L1	L2	L3 1
0.0000	3.4000	------
1.0000	1.7000	
2.0000	1.3600	
3.0000	2.0100	
4.0000	3.4200	
5.0000	5.1800	
6.0000	6.8500	

L1(1)=0

Fig 3: table of points on upper curve

Using the values of L1 and L2, we can now calculate the values for a,b,c and d using the cubic regression tool of the GDC. To do this we must return to the Statistics menu on the GDC by pressing the "STAT" button once again and then scrolling to the "CALC" tab, a menu is presented then select "6:CubicReg". We are brought back to the home screen with "CubicReg" displayed on the screen. Now we select the L1 and L2 lists with a comma in between them. The cubic regression tool gives the following values:

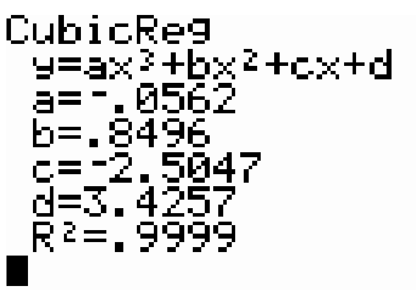
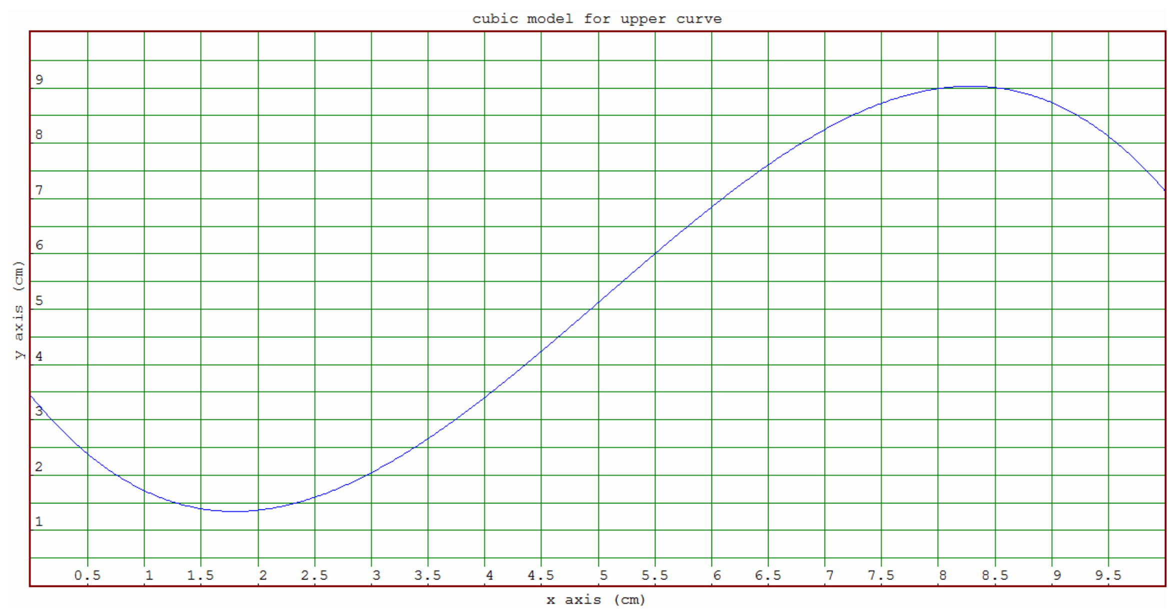


Fig 4: Cubic Regression of upper curve

The cubic regression tool gives us the function:

$g(x) = -0.0562x^3+0.8496x^2-2.5047x+3.4257$

The R^2 value is a measure of accuracy. The closer the value it is to 1, the more accurate the values of the parameters are. In this case, an R2 value of 0.9999 means that the graph 99.99% meets the measured points. Considering the accuracy of this function, this function will be used as a cubic function model of the lower curve. A comparative graph is listed below, illustrating the function formulated using the cubic regression tool and matrices, of which one set has a wider spread of points chosen.



Graph 4: Cubic model for upper curve

Another way of measuring accuracy is through the Mean Absolute error (MAE) of the functions, and how much the function matches the observed points. To calculate this, the absolute value of the difference between the y observed and y expected values, where y expected is the value that was calculated by the function. The values for y-expected were found using Graphmatica. Microsoft Excel was used to calculate the absolute error. This is summarized in the table below:

x	y observed	y expected	Absolute Error
0	3.45	3.44	0.01
1	1.69	1.7144	0.0244
2	1.35	1.3651	0.0151
3	2.00	2.0406	0.0406
4	3.47	3.4037	0.0663
5	5.12	5.1172	0.0028
6	6.85	6.0000	0.8500
7	8.25	6.8439	1.4061
8	8.95	8.2466	0.7034
9	8.71	9.7312	1.0212
10	7.15	7.0700	0.0800
Total			4.2199

Table 4: Absolute Error values for upper curve using cubic function

The MAE is calculated by adding the absolute error values together and dividing it by the number of the points chosen:

$$MAE = \frac{4.2199}{11} = 0.3836$$

This implies that on average the generated y value from the cubic function model misses the observed points by 0.3836 units

Sine Model

Another possible family of function that can be used to describe the behaviour of the curve is the sine function. The generic function: $y = a\sin(bx + c) + d$,

Where x and y are the horizontal distance the vertical distance from the origin, respectively. The parameter a represents the amplitude of the curve, which is the distance between a maximum (or minimum) point and the main axis. The parameter b represents the period of the curve, which is defined as the length of 1 repetition of a cycle. It should be noted that the original sine curve has a period of 2π, thus the period of a transformed sine function is, $\frac{2\pi}{b}$. The parameter c represents the horizontal shift of the curve, and d represents for the vertical shift of the curve.

Lower curve

In order to formulate a sine model for the lower curve, the parameters (a.b,c,d) must be calculated.

The minimum point of the lower curve was measured with a ruler, this came out to be (1.13, 0.45) and the maximum point was (7.41, 5.52). Since the amplitude is the distance between a maximum (or minimum) point and the main axis, a is calculated as follows:

$$a = \frac{5.52 - 0.45}{2} = 2.5350$$

The period is the horizontal distance between two identical stationary points. However, in the diagram we are restricted by the domain and range $0 \le x, y \le 10$ .According to the diagram only a minimum and a maximum point can be seen. Thus, the difference between the two stationary points must be multiplied by 2 in order to obtain b. b is calculated as follows:

$$b = \frac{2\pi}{(7.41 - 1.13) \cdot 2} = 0.5003$$

Recall that c is the horizontal translation of the graph. The $\sin(x)$ curve has its minimum point at $x = -\frac{\pi}{2} = -1.5708$, however the lower curve has its first minimum point at x=1.13. It should be noted that since the period has been altered, the place of the minimum points have been changed as well, therefore the difference of the original minimum and the recent minimum point has to be multiplied by the period, b.

$$c = -\frac{\pi}{2} - ((1.13 \times 0.5003)) = -2.1361$$

Recall that d is the vertical translation of the curve. An original $\sin(x)$ function has its first maximum point at $y = 1$ and the lower curve has its maximum point at y=5.52. The value of d is the difference of the lower curve's maximum point and the original sine curve's maximum point multiplied by the amplitude.

$$d = 5.52 - ((1 \times 2.5350)) = 2.9850$$

Having found the values of the parameters, it is now possible to come up with a function that characterizes the lower curve using the sine model:

$$h(x) = 2.5350\sin(0.5003(x - 2.1361)) + 2.9850$$

Just as there is a cubic regression tool there is also a sin regression tool on the GDC. Using the sin regression tool gives us a more accurate sin function to fit the curve based on the points observed. To calculate this we must upload the 11 points observed, this is done by first pressing the “STAT” button followed by “1:Edit”. A table is then brought to the screen, this will be filled with the x and y values observed. The L1 table corresponds to the x values and the L2 for the y values. Once completed the table is as follows:

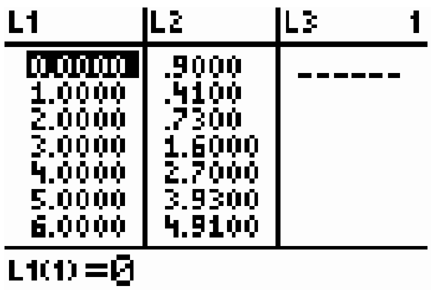


Fig 5: table of points on lower curve

Using the values of L1 and L2, we can now calculate the values for a,b,c and d using the sin regression tool of the GDC. To do this we must return to the Statistics menu on the GDC by pressing the “STAT” button once again and then scrolling to the “CALC” tab, a menu is presented then select “C:SinReg”. We are brought back to the home screen with “SinReg” displayed on the screen. Now we select the L1 and L2 lists with a comma in between them. The sin regression tool gives the following values:

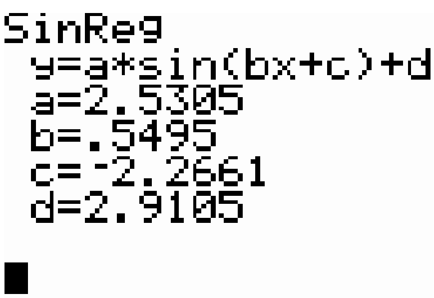
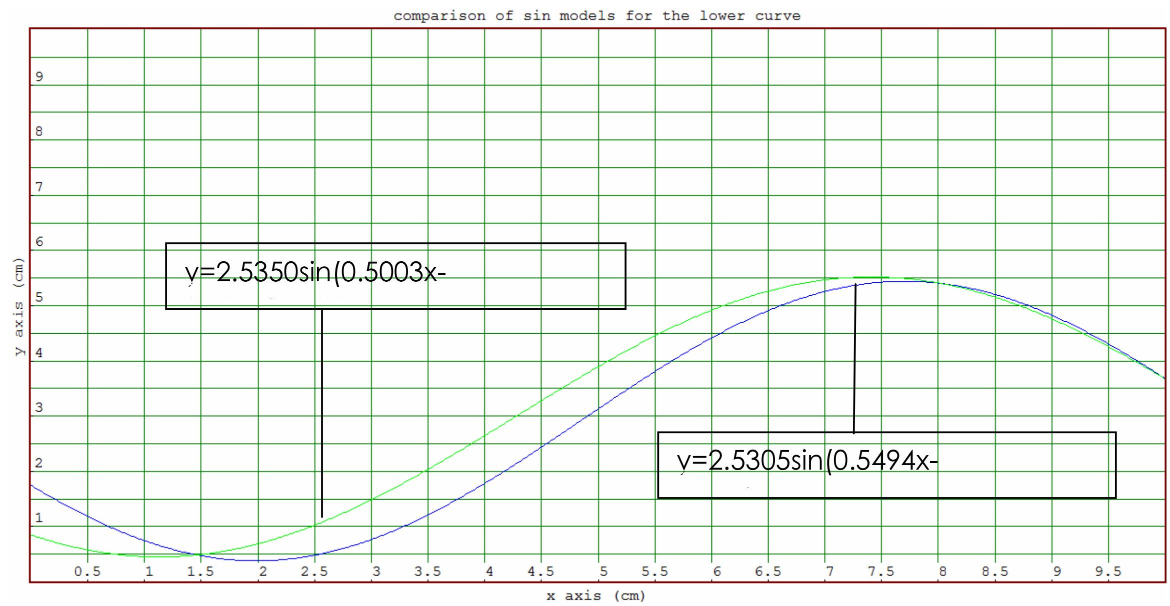


Fig 6: Sin Regression of lower curve

The sin regression tool gives us the function:

$$h(x) = 2.5305\sin(0.5495(x - 2.661)) + 2.9105$$

A comparison of the two functions is given below:



Graph 5: Comparison of sin models for the lower curve

This graph shows that sin regression tool is not very accurate.

We shall calculate the MAE of both functions to see which is better.

Recall, another way of measuring accuracy is through the Mean Absolute error (MAE) of the functions, and how much the function matches the observed points. To calculate this, the absolute value of the difference between the y observed and y expected values, where y expected is the value that was calculated by the function. The values for y-expected were found using Graphmatica. Microsoft Excel was used to calculate the absolute error. This is summarized in the table below:

x	y observed	y1 expected regression function	y2 expected measured function	Absolute Error Y1	Absolute Error Y2
0	0.9	1.73	0.9	0.83	0
1	0.41	0.7411	0.4554	0.3311	0.0454

2	0.73	0.3801	0.6864	0.3499	0.0436
3	1.6	0.7638	1.4809	0.8362	0.1191
4	2.7	1.7794	2.6441	0.9206	0.0559
5	3.93	3.1279	3.8908	0.8021	0.0392
6	4.91	4.4124	4.9155	0.4976	0.0055
7	5.45	5.2548	5.467	0.1952	0.017
8	5.32	5.4073	5.4101	0.0873	0.0901
9	4.31	4.8249	4.7588	0.5149	0.4488
10	2.27	3.6791	3.6726	1.4091	1.4026
Total				6.774	2.2672

Table 5: Absolute Error values for lower curve using sin function

The MAE is calculated by adding the absolute error values together and dividing it by the number of the points chosen:

$$MAE\ of\ Y1 = \frac{6.774}{11} = 0.6158 \qquad MAE\ of\ Y2 = \frac{2.2672}{11} = 0.2061$$

This implies that on average the generated y value from the sin function model formulated by sin regression misses the observed points by 0.6.158 units, whereas the sin function model formulated by measuring the parameters misses the observed points by 0.2061 units. Showing that the sin regression isn't as accurate

<u>Upper curve</u>

Seeing as how the sin regression was not as accurate as the measured value, we shall disregard the sin regression method and go forth with measuring the parameters

Recall that in order to formulate a sine model for the lower curve, the parameters (a.b,c,d) must be calculated.

The minimum point of the upper curve was measured with a ruler, this came out to be (1.76,1.32) and the maximum point was (8.26,9.00). Since the amplitude is the distance between a maximum (or minimum) point and the main axis, a is calculated as follows:

$$a = \frac{9.00 - 1.32}{2} = 3.8400$$

The period is the horizontal distance between two identical stationary points. However, in the diagram we are restricted by the domain and range $0 \le x, y \le 10$ .According to the diagram only a minimum and a maximum point can be seen. Thus, the difference between the two stationary points must be multiplied by 2 in order to obtain b. b is calculated as follows:

$$b = \frac{2\pi}{(8.26 - 1.76) \times 2} = 0.4833$$

Recall that c is the horizontal translation of the graph. The $\sin(x)$ curve has its minimum point at $x = -\frac{\pi}{2} = -1.5708$, however the upper curve has its first minimum point at x=1.76. It should be noted that since the period has been altered, the place of the minimum points have been changed as well, therefore the difference of the original minimum and the recent minimum point has to be multiplied by the period, b.

$$c = -\frac{\pi}{2} - (1.76 \times 0.4833) = -2.4214$$

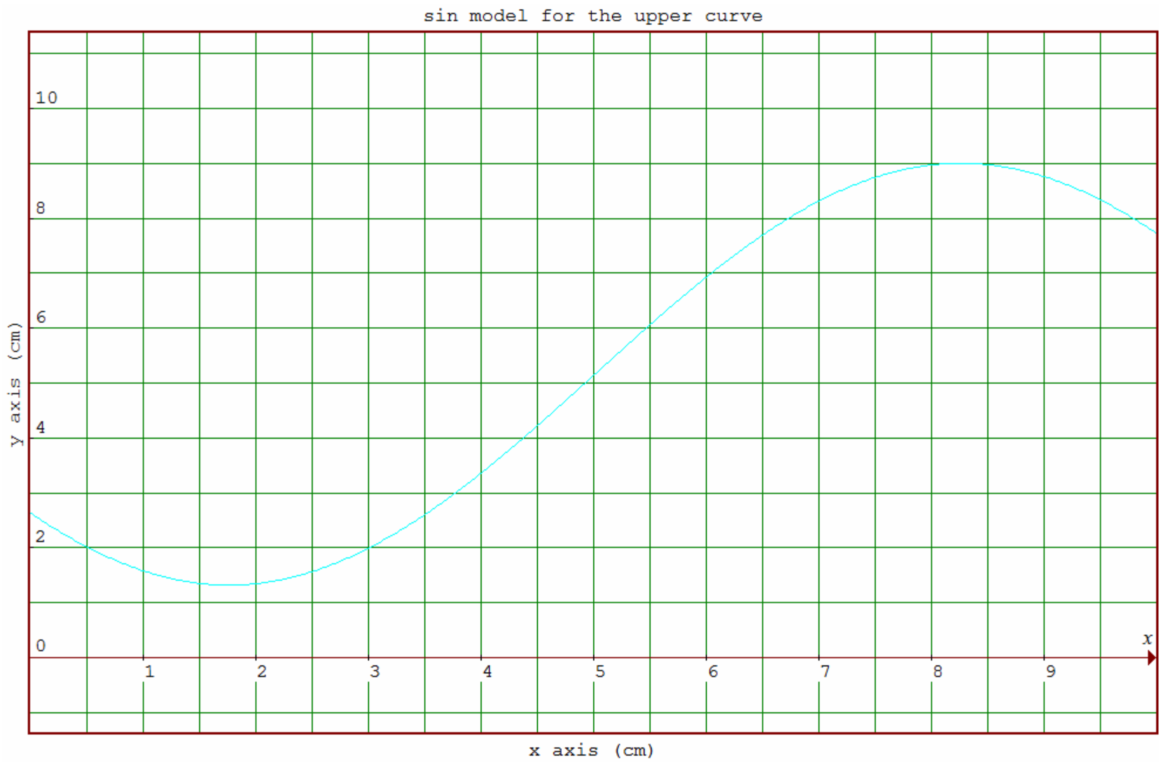
Recall that d is the vertical translation of the curve. An original $\sin(x)$ function has its first maximum point at $y = 1$ and the upper curve has its maximum point at y=9.00. The value of d is the difference of the upper curve's maximum point and the original sine curve's maximum point multiplied by the amplitude.

$$d = 9.00 - (1 \times 3.8400) = 5.1600$$

Having found the values of the parameters, it is now possible to come up with a function that characterizes the lower curve using the sine model:

$$h(x) = 3.84 \sin(0.4833(x - 2.4214)) + 5.16$$

The graph of this function is as follows:



Graph 6: sin model for the upper curve

Recall, to measure accuracy of the function the Mean Absolute error (MAE) of the functions must be calculated. This is a measure of how well the function matches the observed points. To calculate this, the absolute value of the difference between the y observed and y expected values, where y expected is the value that was calculated by the function. The values for y-expected were found using Graphmatica. Microsoft Excel was used to calculate the absolute error. This is summarized in the table below:

x	y observed	Y expected	Absolute Error
0	3.45	2.63	0.82
1	1.69	1.5761	0.1139
2	1.35	1.3458	0.0042
3	2	1.9892	0.0108
4	3.47	3.3589	0.1111
5	5.12	5.1412	0.0212

6	6.85	6.9278	0.0778
7	8.25	8.3094	0.0594
8	8.95	8.9697	0.0197
9	8.71	8.7572	0.0472
10	7.15	7.7208	0.5708
Total			1.8561

Table 5: Absolute Error values for upper curve using sin function

The MAE is calculated by adding the absolute error values together and dividing it by the number of the points chosen:

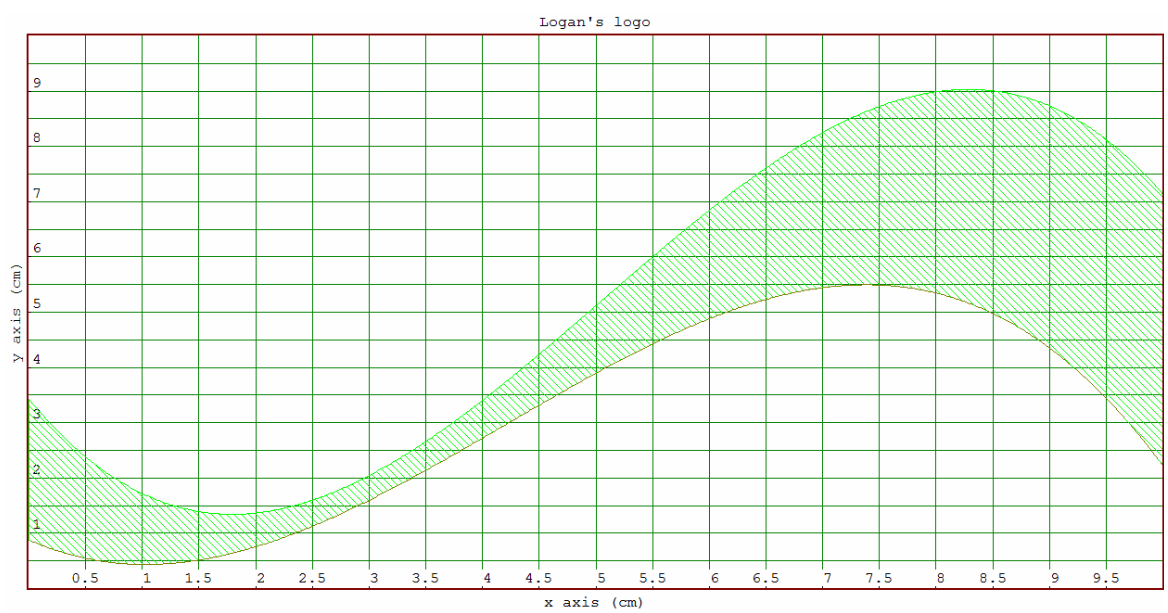
$$MAE = \frac{1.8561}{11} = 0.1687$$

This implies that on average the generated y value from the sin function model is 0.1687 units

	Upper	Lower
Sin	0.1687	0.2061
Cubic	0.3836	0.0256

Table 6: comparison of MAE of both models for both curves

From the data listed above the cubic function seems to work better than the sin function overall. From here on out the cubic function will be the one used.



Graph 7: Logan's logo using the cubic functions for the curve

Transformations of the Logan's Logo – doubling the logo

Now Logan wishes to double the dimensions of the logo to print it on a T-shirt. If Logan wishes to double the dimensions of the logo, the new curves have to be sketched in a new coordinate system, with a domain of $0 \le x \le 20$ and a range of $0 \le y \le 20$. However, if the functions were just sketched in the new coordinate-system, they would not keep their shape and then the logo would be altered. Thus, the functions themselves have to be transformed. The dimensions of the functions have to be doubled as well, so that they will fit into the new coordinate-system. Recall the functions of the lower and upper curves:

Lower curve being:

$f(x)= -0.0394x^3+0.4973x^2-0.8981x+0.8751$

and the upper curve being:

$g(x)=-0.0562x^3+0.8496x^2-2.5047x+3.4257$

The transformation involves a horizontal and vertical stretch of the functions

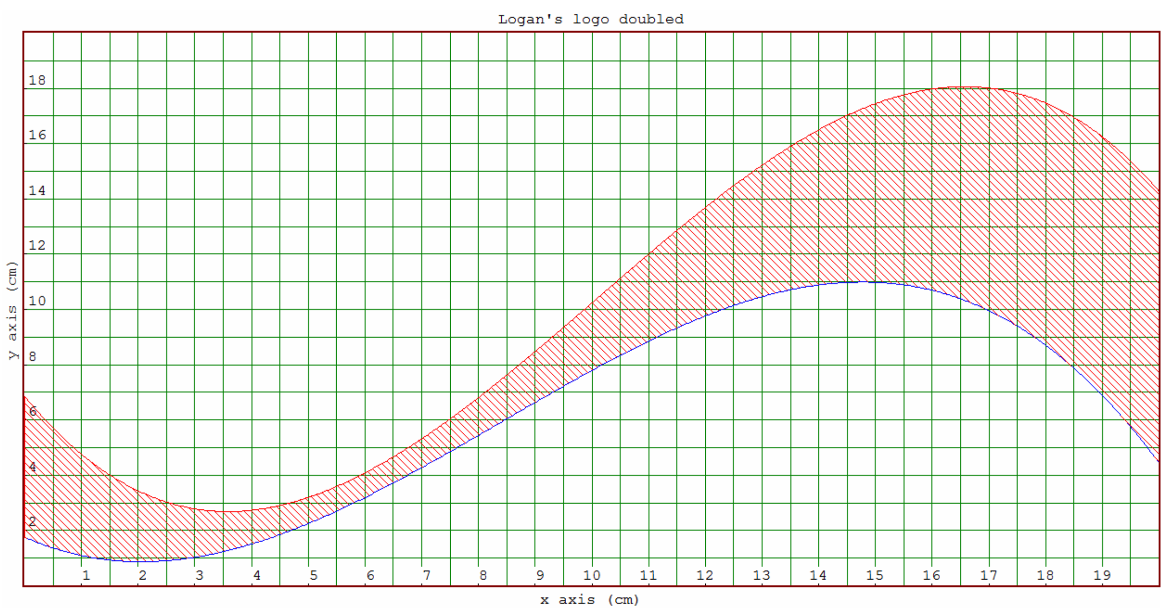
If there is a function such as y=a(x) and if it were to be stretched vertically by a factor of q, the function must be multiplied by q: $y=q \times a(x)$. If 0< q<1, it moves the points of as y=a(x) closer to the x-axis. Functions can be transformed horizontally as well. If the function y=a(x) is to be stretched horizontally, then the function has to be multiplied by a factor k, to demonstrate: y=a(k × x). [illegible] $\frac{1}{k}$.

In Logan's case, she would like to double the size of the logo, so the functions have to be stretched both horizontally and vertically as well, where q=2 and k=2. The functions of the upper and lower curves need to be altered:

$$y = q \times f\left(x \cdot \frac{1}{k}\right) = 2f\left(\frac{x}{2}\right) = -0.0394\left(\frac{x}{2}\right)^3 + 0.4973\left(\frac{x}{2}\right)^2 - 0.8981\left(\frac{x}{2}\right) + 0.8751$$

$$y = q \times g\left(x \cdot \frac{1}{k}\right) = 2g\left(\frac{x}{2}\right) = -0.0562\left(\frac{x}{2}\right)^3 + 0.8496\left(\frac{x}{2}\right)^2 - 2.5047\left(\frac{x}{2}\right) + 3.4257$$

The modified functions were inputted into the program Graphmatica and the following graph was made:



Graph 8: Logan's logo doubled in dimensions.

It should be noted that these new function have the limitations of $0 \le x, y \le 20$.

<u>Business card</u>

Now Logan wishes to print her company's logo onto business cards. In order to do so, she has to change the size of the logo, to fit a 9 cm by 5cm rectangle. To maintain the shape of the logo, it is required to modify the function again; this time they have to shrink horizontally and vertically. With a horizontal factor of $\frac{9}{10}$ and a vertical factor of $\frac{1}{2}$. The functions have to be modified as follows:

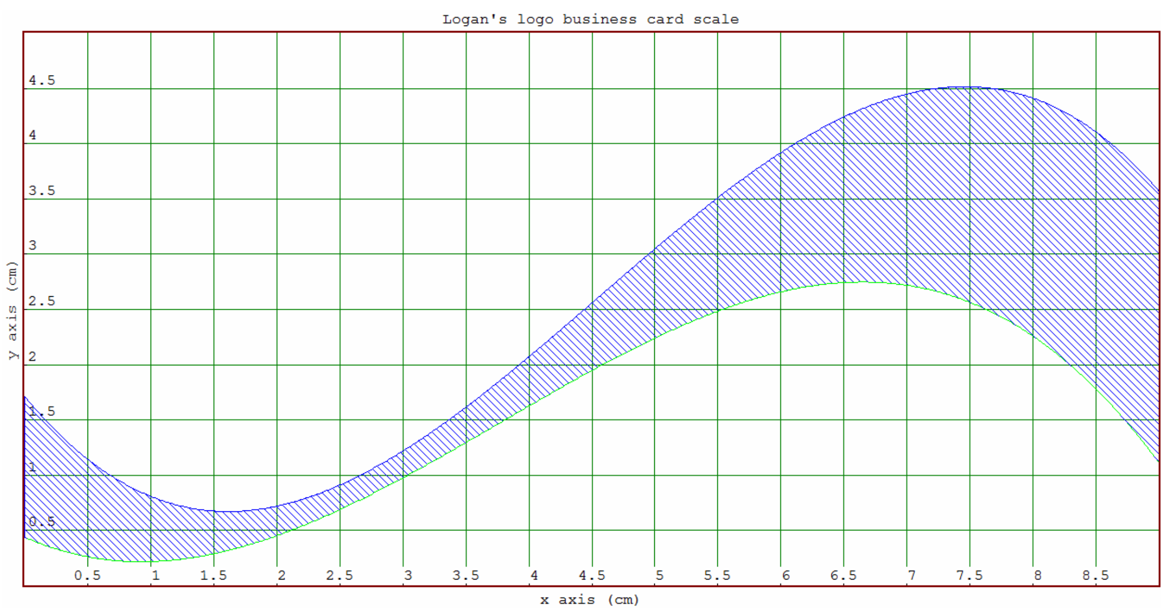
$$\frac{1}{2}\cdot f\left(\frac{10\,x}{9}\right)=-0.0394\left(\frac{10\,x}{9}\right)^3+0.4973\left(\frac{10\,x}{9}\right)^2-0.8981\left(\frac{10\,x}{9}\right)+0.8751$$

$$\frac{1}{2}\cdot g\left(\frac{10\,x}{9}\right)=-0.0562\left(\frac{10\,x}{9}\right)^3+0.8496\left(\frac{10\,x}{9}\right)^2-2.5047\left(\frac{10\,x}{9}\right)+3.4257$$

It should be noted that the newly modified functions of $f(x)$ and $g(x)$ have the limitations

$0 \le x \le 9, 0 \le y \le 5$.

The modified functions were inputted into the program Graphmatica and the following graph was made:



Graph 9: Logon's logo shrunken to the size of a business card

Area between two curves:

To find what fraction of the business card the logo occupies, it is necessary to find the integral of the two curves. The integral is calculated by subtracting the integral of the lower curve from the integral of the upper curve. The lower limit is going to be 0, while the upper limit is going to be 9. The area between the two curves is calculated as follows. Recall the two functions of the business card:

$$\frac{1}{2}\cdot f\left(\frac{10x}{9}\right) = -0.0394\left(\frac{10x}{9}\right)^3 + 0.4973\left(\frac{10x}{9}\right)^2 - 0.8881\left(\frac{10x}{9}\right) + 0.8751$$ (lower curve)

$$\frac{1}{2}\cdot g\left(\frac{10x}{9}\right) = -0.0562\left(\frac{10x}{9}\right)^3 + 0.8496\left(\frac{10x}{9}\right)^2 - 2.5047\left(\frac{10x}{9}\right) + 3.4257$$ (upper curve)

The area of the logo is calculated as follows: $\int_0^9 \left(g(x) - f(x)\right)dx = \int_0^9 g(x)\,dx - \int_0^9 f(x)\,dx$

The function should be written in a simpler format to help to integrate the functions:

$$f(x) = -0.0394\cdot\left(\frac{1000x^3}{729}\right) + 0.4973\cdot\left(\frac{100x^2}{81}\right) - 0.8881\cdot\left(\frac{10x}{9}\right) + 0.4667$$

$$f(x) = -0.054x^3 + 0.6140x^2 - 0.9979x + 0.4667$$

Then we find the definite integral of the function $f(x)$:

$$\left(\int_0^9 f(x)\,dx = \int_0^9 -0.054x^3 + 0.6140x^2 - 0.9979x + 0.4667\,dx\right)/4 =$$

$$\left(\int_0^9 -0.054x^3dx + \int_0^9 0.614x^2dx - \int_0^9 0.9979xdx + \int_0^9 0.4667\,dx\right)/4 =$$

$$\left(-0.054\int_0^9 x^3dx + 0.614\int_0^9 x^2dx - 0.9979\int_0^9 xdx + \int_0^9 0.4667\,dx\right)/4 =$$

$$\left(\left[-0.054\cdot\frac{x^4}{4} + 0.614\cdot\frac{x^3}{3} - 0.9979\frac{x^2}{2} + 0.4667x\right]_0^9\right)/4 =$$

$$\left(\left[-0.054\cdot\frac{9^4}{4} + 0.614\cdot\frac{9^3}{3} - 0.9979\cdot\frac{9^2}{2} + 0.4667x\right]\right)/4 =$$

$$\left(-88.5735 + 149.2020 - 8.9811 + 4.667\right)/4 = 56.3144/4 = 14.0786 \text{ cm}^2$$

Now we need to find the integral of the $g(x)$ function as well. Note that the function can be expressed in a simpler form as well:

$$\frac{1}{2}\cdot g\left(\frac{10x}{9}\right) = -0.0562\left(\frac{10x}{9}\right)^3 + 0.8496\left(\frac{10x}{9}\right)^2 - 2.5047\left(\frac{10x}{9}\right) + 3.4257$$

$$g(x) = -0.0562\cdot\left(\frac{1000x^3}{729}\right) + 0.8496\cdot\left(\frac{100x^2}{81}\right) - 2{,}5047\left(\frac{10x}{9}\right) + 3.4257$$

$$g(x) = -0.0771x^3 + 1.0489x^2 - 2.7830x + 3.4257$$

$$\int_0^9 g(x)\,dx = \left(\int_0^9 -0.0771x^3 + 1.0489x^2 - 2.7830x + 3.4257\,dx\right)/4 =$$

$$\left(\int_0^9 -0.0771x^3dx + \int_0^9 1.0489x^2dx - \int_0^9 2.7830xdx + \int_0^9 3.4257\,dx\right)/4 =$$

$$\left(-0.0771\int_0^9 x^3dx + 1.0489\int_0^9 x^2dx - 2.7830\int_0^9 xdx + \int_0^9 3.4257\,dx\right)/4 =$$

$$\left(\left[-0.0771\cdot\frac{x^4}{4} + 1.0489\cdot\frac{x^3}{3} - 2.7830\cdot\frac{x^2}{2} + 3.257x\right]_0^9\right)/4 =$$

$$\left(\frac{-0.0771\cdot 9^4}{4} + \frac{1.04897\cdot 9^3}{3} - \frac{2.7830\cdot 9^2}{2} + 3.257\cdot 9\right)/4 =$$

$$(-126.4633 + 254.8997 - 55.7806 + 25.0470)/4 = 97.7028/4 = 24.4257 \text{ cm}^2$$

Recall that the area of the logo is the area between the two curves, which can be calculated as follows:

$$\int_0^9 (g(x) - f(x))\,dx = \int_0^9 g(x)\,dx - \int_0^9 f(x)\,dx = 24.4257 - 14.0786 = 10.3471 \text{ cm}^2$$

To be more accurate the area between the two curves will be calculated using graphmatica.

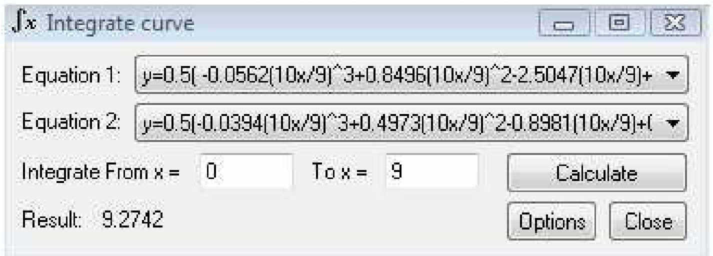


Fig 7: Area between 2 curves using graphmatica

According to graphmatica the area between the two curves is 9.2742 cm^2. The total surface area of the business card is $5 \times 9 = 45$ cm^2 . Using the area between the two curves on the business card (9.2742 cm^2), thus the logo occupies $\frac{9.2742}{45}$ part of the entire business card, which is 20.6093% of the total surface area of the business card.

It is important to know the area of the logo using in a business card because if the logo appears to small on the business card the message of the company will not get spread and resulting in lower rate of customers and a loss in profits due to the making of the business cards. On the other hand if the logo is too big on the card there is no room for text and therefore the message of the company is inhibited once again.

Conclusion

To find an appropriate model for the two curves on Logan's logo to showcase their behaviours, sine and cubic models were tested. A best fit curve for a cubic function was calculated using the cubic regression of the 11 points measured by ruler. It was through this calculation that the parameters of the function were discovered as well. In the end the cubic function seemed to fit better than the sin function. Logan wished to reshape her logo to fit either a t-shirt of a business card, in order to keep the nature of the curve the same, the function had to be altered so that it would be able to fit in the new dimension while still displaying the same characteristics as the original curve for the t-shirts the logo had to be doubled along with the function. When it came to printing the logo on business cards, the functions had to be altered to fit the new 9 x 5 cm dimensions