Mathematics

Portfolio Type 2

LOGAN'S LOGO

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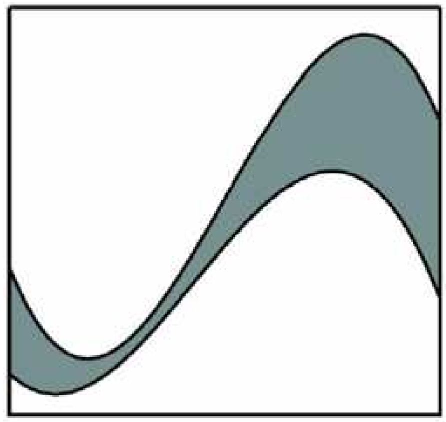


This diagram shows a rectangle which is divided into three regions by two curves. The shaded region between the two curves is a logo which can be represented by two mathematical functions. The rectangle is 6.5cm by 6.1cm with a 0.5mm thick frame and the lines of the curves are 0.5mm thick. The aim for this portfolio is to manage to find mathematical functions to represent the logo, be able to modify the logo so that it fits other types of dimensions, and in the end to be able to calculate the area fraction of the logo on the business card. What we can ask ourselves is if the area fraction of the business card will be of the same value as the original logo.

1. Developing mathematical functions for the logo

1.1. Finding the coordinates

The problem is to find the two functions that model these two curves. One type of function that can represent these curves might be sine functions. To check this thesis out, it is first necessary to know the coordinates of the curves.

A parameter is a quantity that defines characteristics of functions. There are several parameters relevant to this portfolio:

- The thickness of the lines: 0.5 mm
- The length of the logo: 6.5 cm
- The height of the logo: 6.1 cm
- The type of function used

A variable is a value that may vary. There are several variables relevant to this portfolio:

- The x-value
- The measurements, as others may have measured differently
- The size of the card

It is hard to find the coordinates of curves at a logo, so what I did was that I glued a millimetre paper to the logo. As the lines of the curves are 0.5mm thick, the most accurate way to find the coordinates were to find the coordinates in the middle of the line, if I had found the coordinates either at the top or the bottom of the line, the coordinates would not have represented the curves correctly. The points measured are not equally distributed; they are measured according to which ones were most accurate according to the millimetre paper.

Coordinates for the lower curve:

X	Y	X	Y
0	0,6	3,6	2,9
0,4	0,35	4	3,3
0,6	0,3	4,3	3,5
1	0,35	4,9	3,7
1,3	0,5	5,2	3,6
1,7	0,8	5,6	3,4
2	1,1	6	2,9
2,2	1,3	6,2	2,5
2,6	1,8	6,4	2,1
2,8	2	6,5	1,9
3,1	2,4		

Coordinates for the upper curve:

X	Y	X	Y
0	2	3,5	3,6
0,2	1,6	3,6	3,8
0,4	1,3	3,8	4,1
0,8	0,9	4	4,4
1,1	0,8	4,2	4,7
1,6	0,9	4,5	5,1
1,7	1	4,9	5,5
2,1	1,4	5,2	5,7
2,5	1,9	5,6	5,7
2,7	2,3	6,1	5,2
3	2,7	6,4	4,6
3,2	3,1	6,5	4,4

1.2. Sine regression

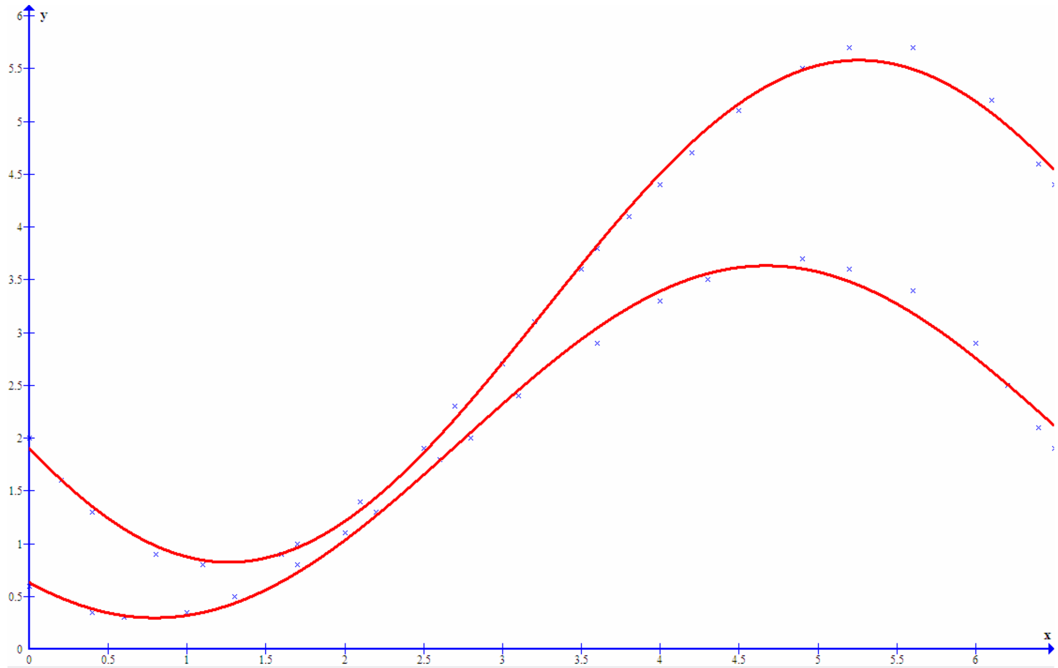
After listing these coordinates into the list function on my GDC, a Texas TI-84 Plus, I can use the sine regression to find an appropriate function for the points.

The lower curve (g(x)):
$y = a \cdot \sin(bx + c) + d$
$a = 1.668240274$
$b = 0.8100728241$
$c = -2.216031762$
$d = 1.96525663$

$g(x) = 1.668240274 \sin(0.8100728241x - 2.216031762) + 1.96525663$

The upper curve (f(x)):
$y = a \cdot \sin(bx + c) + d$
$a = 2.378686945$
$b = 0.7859285241$
$c = -2.565579771$
$d = 3.202574671$

$f(x) = 2.378686945 \sin(0.7859285241x - 2.565579771) + 3.202574671$



By comparing the two functions to the points of the logo, it can be noticed that these functions do not represent the two curves in the logo very precisely. This can be explained by that the curves are not sine functions. Evidence for this is that for a function to be a sine function it has to be symmetric, and the curves of the logo do not have that property.

1.3. Cubic regression

Another type of function that may represent the curves can be polynomial function of 3rd order, a cubic function. Using the graphing package, "Graph 4.3" by Ivan Johansen downloaded at http://www.padowan.dk/graph/, I can find the polynomial function that best represent the curves.

The lower curve (g(x)):

$y = ax^3 + bx^2 + cx + d$

$a = -0.089515466$

$b = 0.74847837$

$c = -0.88572438$

$d = 0.58866081$

$R^2 = 0.9995$

$g(x) = -0.089515466x^3 + 0.74847837x^2 - 0.88572438x + 0.58866081$

The upper curve (f(x)):

$y = ax^3 + bx^2 + cx + d$

$a = -0.13020708$

$b = 1.2745508$

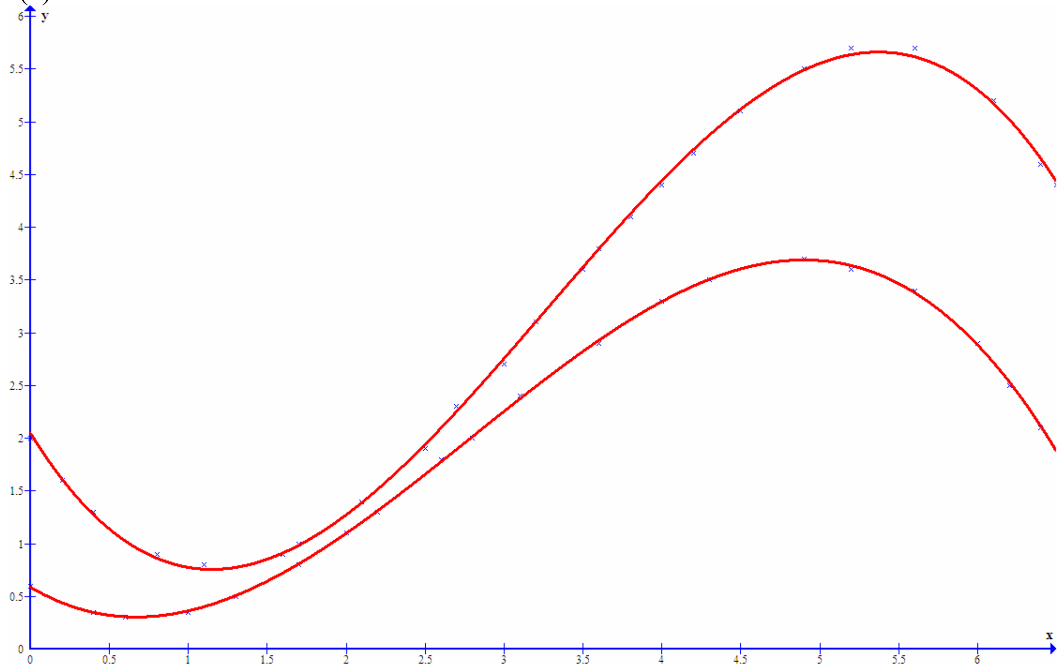
$c = -2.4169654$

$d = 2.0480192$

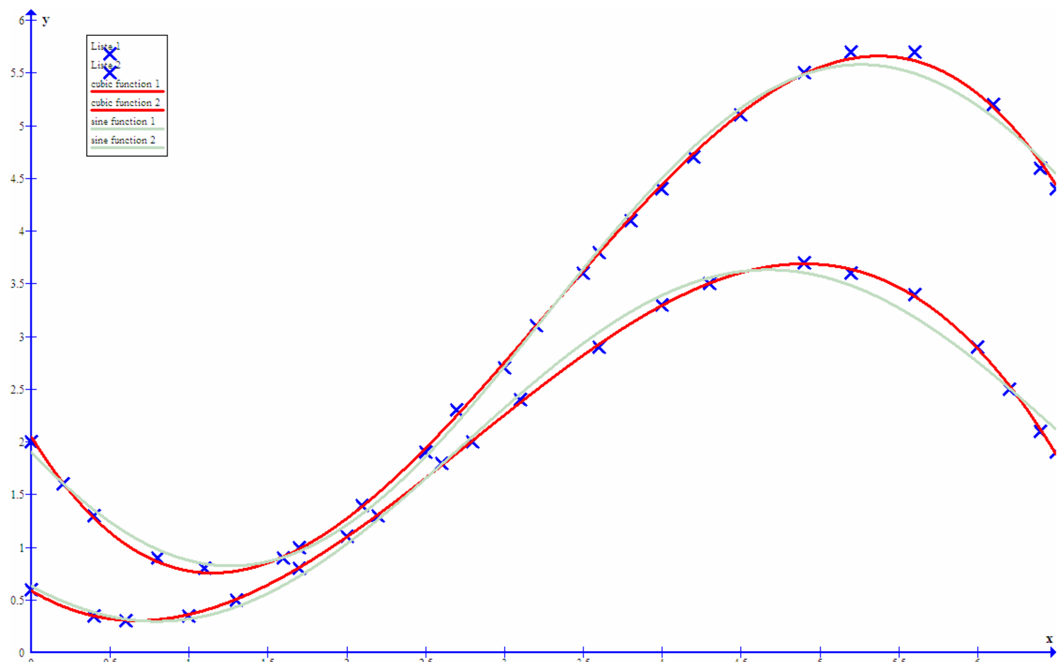
$R^2 = 0.9995$

$f(x) = -0.13020708x^3 + 1.2745508x^2 - 2.4169654x + 2.0480192$

By trial and error, a more simple form of the functions can be discovered. By changing the level of accuracy to 4 decimal places, a simpler form of the functions can be found.
$g(x) = -0.0895x^3 + 0.7485x^2 - 0.8857x + 0.5887$
$f(x) = -0.1302x^3 + 1.2746x^2 - 2.4170x + 2.0480$



1.4. Comparing the sine and the cubic regression
To show graphically that the cubic functions represent the logo better than the sine functions, we can place them both in the same coordinate diagram and compare them.



We can see that the cubic functions represent the points of the logo much better than the sine functions, so the best way to represent the logo with mathematical functions is to use cubic functions.
There are limitations to these calculations; the main limitation may be inaccuracy of measurements of the points.

In addition, a regression line appears to fit the central portion of the scatter plot well, so there will always be an uncertainty to how well the line represents the plots.
The R^2 value is statistic variables that represent the coefficient of determination, and it represents the accuracy of the graph according to the points. Both of the R^2 values given from my calculations were the same, 0.9995, this means that the limitations for both graphs are the same, and by that it can be concluded that the mathematical model represent the logo very well.

2. Modifying the logo to double dimensions for t-shirts

2.1. Modifying the logo to double dimensions

The functions have to be modified for the curves to fit into double dimensions. In double dimensions, the axes are twice the length as in normal dimensions.

x-axis = 13.0
y-axis = 12.2

For the graphs to fit in these new dimension they have to be stretched both vertically and horizontally.

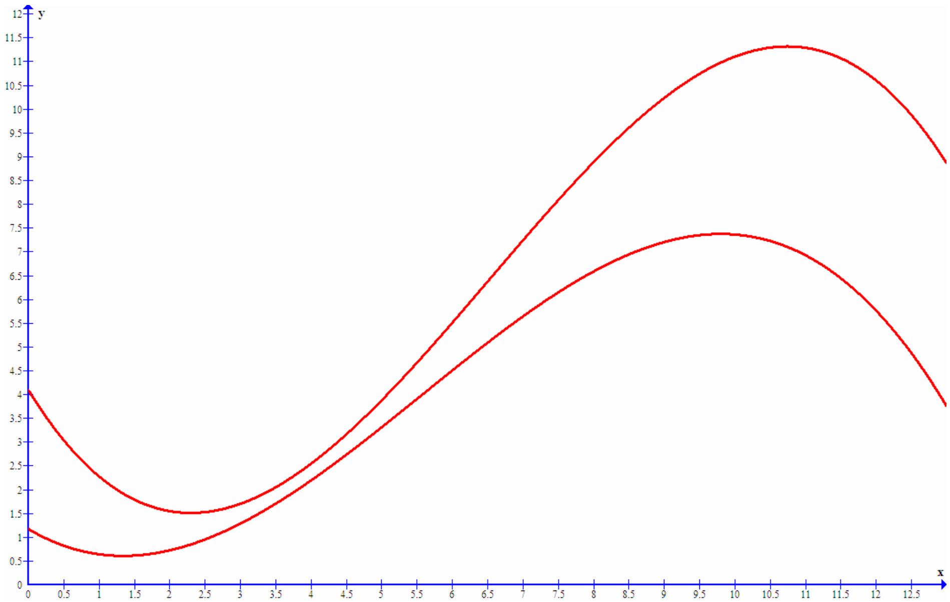
For a function to be vertically stretched, it has to be multiplied by a factor of r. The expression for a vertical stretch of the function will be:
$rf(x) = r(ax^3 + bx^2 + cx + d)$

For a function to be horizontally stretched, the x-value has to be multiplied by $\frac{1}{r}$

The expression for a horizontal stretch of the function will be:
$f\left(\frac{1}{r}\right)x = a\left(\frac{1}{r}\right)x^3 + b\left(\frac{1}{r}\right)x^2 + c\left(\frac{1}{r}\right)x + d$

As the dimensions are doubled, the value of r will be 2, and the expression of the modified function will be:
$2\ f(½x) = 2(\ a(½)x^3 + b(½)x^2 + c(½)x + d)$

$g(x) = 2(–0.0895(½)x^3 + 0.7484(½)x^2 – 0.8857(½)x + 0.5886)$
$f(x) = 2(–0.1302(½)x^3 + 1.2745(½)x^2 – 2.4169(½)x + 2.0480)$



3. Modifying the logo for a buisines card

3.1. Modifying the logo

If this logo was to be printed to a standard business card of 9 by 5 cm the functions also have to be modified.

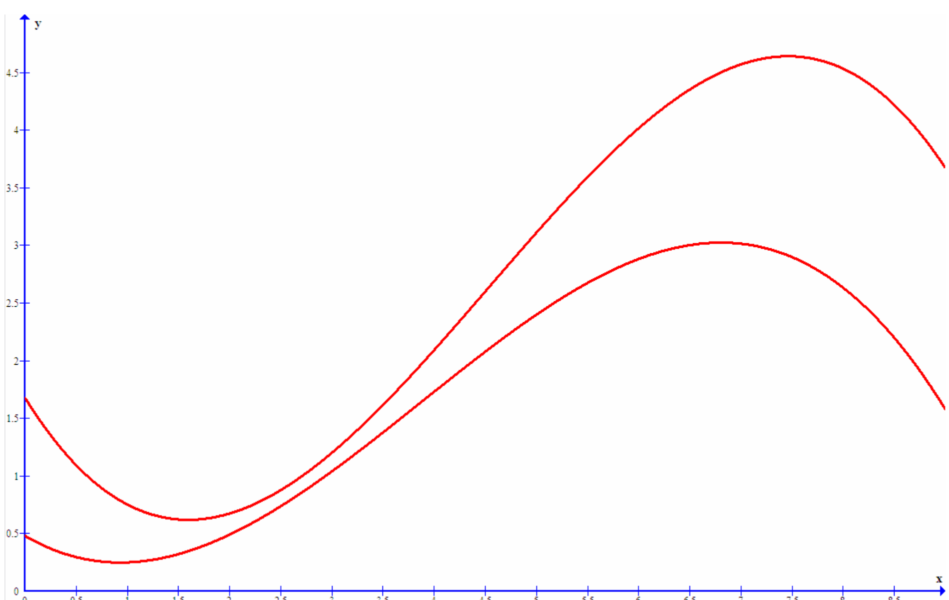
To calculate the value for the vertical and horizontal stretch, the new dimensions have to be divided by the original dimensions. The values calculated will represent the stretches.

Vertical stretch: $5/6.1 = 0.82$

Horizontal stretch: $9/6.5 = 1.38$ $1/1.38 = 0.72$

$g(x) = 0.82\,(-0.0895(0.72)x^3 + 0.7484(0.72)x^2 - 0.8857\,(0.72)x + 0.5886)$

$f(x) = 0.82\,(-0.1302(0.72)x^3 + 1.2745(0.72)x^2 - 2.4169\,(0.72)x + 2.0480)$



3.2. The area fraction of the logo

To find out what fraction of the area of the card the logo occupy, the integral of the lower curve has to be subtracted from the integral of the upper curve. The result of the integral calculations then has to be divided by the total area of the card.

$$\int_0^9 f(x)dx - \int_0^9 g(x)dx$$

$$\int_0^9 f(x)dx = \int_0^9 0.82\,(-0.13020(0.72x)^3 + 1.2745(0.72x)^2 - 2.4169(0.72x) + 2.0480)\,dx$$

$$\int_0^9 g(x)dx = \int_0^9 0.82\,(-0.08950(0.72x)^3 + 0.7484(0.72x)^2 \; 0.8857(0.72x) + 0.5886)\,dx$$

An accurate way to calculate this is to use the integral function on the GDC.

This gives the area(A) beneath the two curves to be:

A(g(x)) = 15.5419

A(f(x)) = 23.6114

area = 23.6114 – 15.5419
= 8.0695

Total area of the card: 9·5 = 45

$$\frac{8.0695}{45} = 0.179322$$

It is impossible to simplify this fraction, but if we first simplify the answer to fewer decimals it is possibe to get a simplified fraction.

$0.18 = \frac{9}{50}$

So the logo occupy $\frac{9}{50}$ of the card surface.

This might be an important aspect of a business card because it is important for a logo to be noticed but at the same time not take up to much space.

4. Conclusion

In the introduction I asked if the area of the logo on the business card was the same as the area of the original logo, and to answer this the area of the initial logo has to be calculated.

Using the GDC the area of the initial logo is given to be:

$0.18 = \frac{9}{50}$

So the answer is yes, the area of the logo on the business card is the same as the area of the original logo. This means that all the calculations and modifications are correct and that the logo is precicely modified into other dimensions.