Math Portfolio
Type 2

Logan's Logo

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Class: IB07

Introduction

Logan has designed the logo at the right. The diagram shows a square which is divided into three regions by **two curves**. The logo is the shaded region between the two curves. Logan wishes to find mathematical functions that model these curves. Now in order to find this mathematical function, the square needs to be measured by placing it on a graph paper (figure 2) and then the data points must be identified and recorded to represent a model function for each of the curves. I have decided to choose my units in cm.

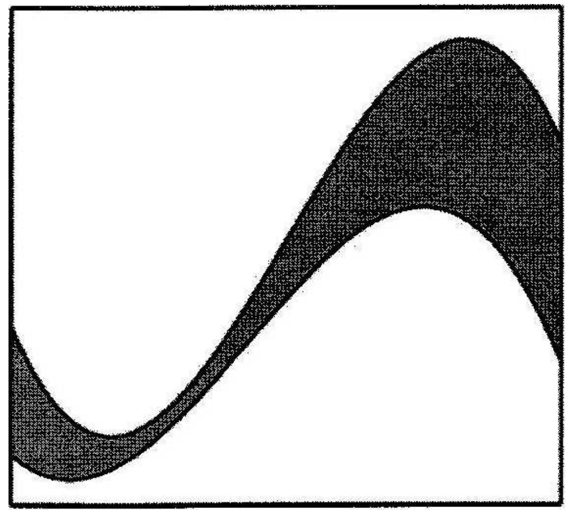


Figure 1

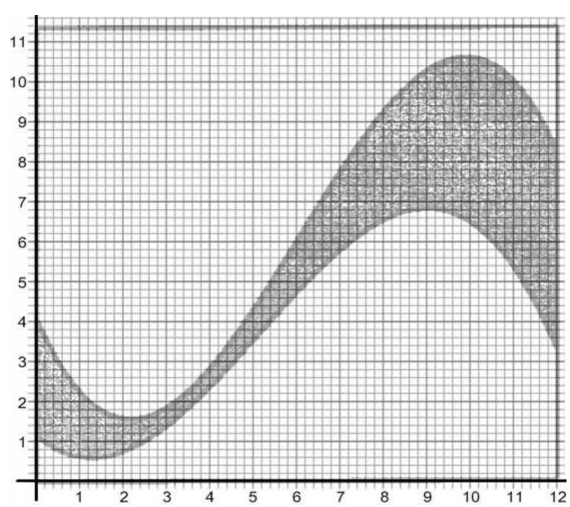


Figure2

From this simple plot we can estimate and examine a proper set of axes and record the coordinates in each curve. Bellow there is table1 to show the data points that the curve lower (inferior) curve and table2 to show the data points that the upper (superior) curve follows.

Table1

X	*f(x)*
0	0.7
0.5	0.4
0.8	0.35
1	0.4
1.5	0.6
2	1
2.5	1.4
3	2
3.5	2.55
4	3.15
4.5	3.65
5	4.05
5.5	4.23
6	4.17
6.5	3.8
7	3.15
7.5	2.1

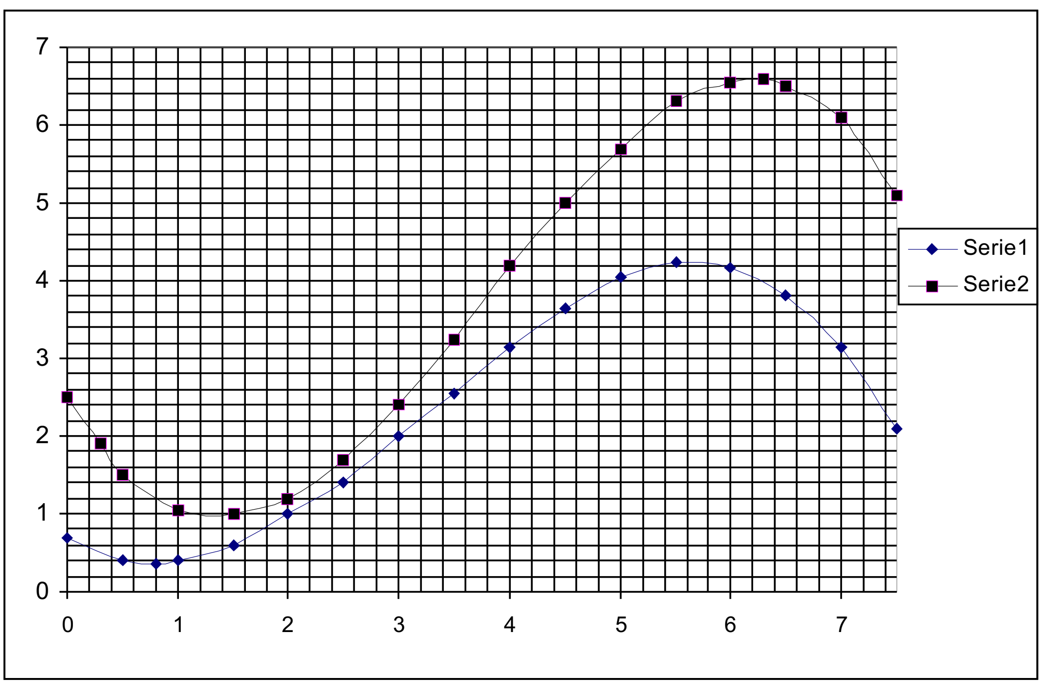
Table2

u	*f(u)*
0	2.5
0.3	1.9
0.5	1.5
1	1.05
1.5	1
2	1.2
2.5	1.7
3	2.4
3.5	3.25
4	4.2
4.5	5
5	5.7
5.5	6.3
6	6.55
6.3	6.6
6.5	6.5
7	6.1
7.5	5.1

Variables:

- *x* represents any data point on the x-axis in the domain (0-7.5) which stand for the lower curve
- *f(x)* is the image for all the values of *x* and is permitted in a range of 0.7- 4.23
- *u* represents any data point on the x-axis in the domain (0-7.5) which stand for the upper curve
- *f(u)* is the image for all the values of *x* and is permitted in a range of 2.5- 6.6

a graph has been drown to display the data points shown in table 1 and 2



The data points in the tables have been applied to model functions that follow the behavior of each curve shown. We can notice from the graph that these two curves are parallel to each other hence they might follow the same type of equation but vary in the parameters values.

Technology (GDC calculator) has been used to plot these curves and many types of functions have been examined to this model. I have come to the conclusion that the functions type that models the data behavior is a quartic equation.

Quartic equation = $ax^4+bx^3+cx^2+dx+e$

A quartic function is a symmetrical polynomial function because it is constructed from number of variables and constants, using only the operations of addition, subtraction, multiplication constant positive whole number exponents.

The reason I chose this function is because it is the most plausible function that fits and follows the behavior of the data points.

The two of the curves follow this function type but they differ in the parameters.
The parameters for the first curve (lower curve) are:

- $a=0.018450285$
- $b=-0.898140257$
- $c=0.7355801435$
- $d=-1.013678057$
- $e=0.7300506358$

Hence the function of the lower curve is:
$f(x)= 0.018450285\,x^4 -0.898140257\,x^3 +0.7355801435\,x^2 -1.013678057x +0.7300506358$

The parameters for the second curve (upper curve) are:

- $a=-5.602082x10^{-4}$
- $b=-0.0918509572$
- $c=1.096110546$
- $d=-2.471844394$
- $e=2.511349345$

Therefore the function of the upper curve is:
$f(u)= -5.602082x10^{-4}u^4 -0.0918509572u^3 + 1.096110546u^2-2.471844394u+2.511349345$

There are some limitations that might not let these functions fit in perfectly. One of these limitation is we don't have a wide range of values which means we are not able to state clearly if this function is symmetrical or not. Second because of the limited set of values and hence the limited domain and range, one of the curves may not be parallel to the other one and this makes on of the function wrong.

Logan wishes to print T-shirt with the logo on the back. She must double the dimensions of the logo. In order to achieve Logan's whishes the loge must be stretched to the double from each of the dimension; the width and the length or height. Since the functions of the curves that surrounds the logo are know, it is now possible to apply function graphing skills on the these graphs or curves to be stretched.

In order to stretch a graph to the double parallel to the *y-axis* the formula must be multiplied by 2 for example:

The original equation:	stretched equation:
$f(x)= x^2$	$2\,f(x)=2x^2$

In order to stretch a graph to the double parallel to the *x-axis* the formula must be written this way

The original equation:	stretched equation:
$f(x)= x^2$	$f(x)= (\frac{1}{2}x)^2$

Applying these techniques to our function:

$f(x)= ax^4+bx^3+cx^2+dx+ e$

$f(x)= 2[a(\frac{1}{2}x)^4+b(\frac{1}{2}x)^3+c(\frac{1}{2}x)^2+d\frac{1}{2}x + e]$

Each equation of the curves must be modified in this way in order to achieve the purpose of doubling the logo.

Logan's also wishes to print business card. A standard business card has the dimension of 9 cm and 5 cm. The Original Square has the dimensions of 7.5cm by 7 cm. To stretch and squeeze it so it can fit in with the business cards dimensions, we need to know the ratio between the business card dimensions and the original square once.

5/7 = 0.7142857143
9/7.5= 1.2

So now if we multiply 7cm by 0.7142857143 the height of the original square will be compressed to 5cm. and if 7.5m is multiplied by 1.2 the length of the original square will be stretched up to 9cm.

With the purpose of extending the logo from one end of the card to the other, each function must be modified in a certain way. At first, we need to know to what extent the logo must be compressed and stretched. Then the function graphing skills can be employed. The amplitude of the logo in the original square is 6.2 cm this means that the logo is occupying 88% height of the square. Hence it should be compressed to 4.4286 cm. if we take the ratio of the two dimensions;

6.2/4.4286 = 0.7142857143 which the same as 5/7

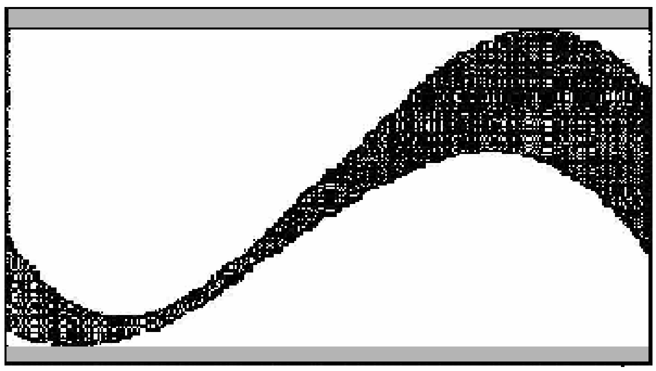
The ratio of the length dimensions to stretch the logo is: 9.7/5 = 1.2

This means:

- the graph of *f(x) and f(u)* must be stretched parallel to the *x-axis* by a scale factor of 9.7/5= 1.2
- the graph of *f(x) and f(u)* must be stretched parallel to the y-axis by a scale factor of 5/7= 0.7142857143

If we indicate 5/7 with a symbol *w,* and to 0.833333 (1/1.2) with a symbol *z*. and apply this to our application $f(x)= ax^4+bx^3+cx^2+dx+ e$:

$w.f(zx)=w[a(zx)^4+b(zx)^3+c(zx)^2+dzx+ e] = wa(zx)^4+wb(zx)^3+wc(zx)^2+wdzx+ we$



Business card with the logo extended on it

The fraction area can be measured by measuring the area under each of the curves and subtracting the area under the upper curve fron the area under the lower curve. This is how the area of the logo can be calculated
To measure the area under each of the curves the definite integral is taken for each of the graphs of the functions *f(x) and f(u)* where there is a lower limit and upper limit.

The equation of the lower curve in the card is:
$f(x)= 0.0013178775(0.83333x)^4 – 0.0637028755(0.83333x)^3 +0.5254143882(0.83333x)^2 – 0.724055755(0.83333x) + 0.5214647399=0$

The equation of the upper curve in the card is:
$f(u) = – 4.00148714x10^{-4}(0.83333u)^4 – 0.656078266(0.83333u)^3 + 0.7829361043(0.83333u)^2 – 1.765631742(0.83333u) + 1.793820961 = 0$

the general equation after integration:

$$A = \int_b^a f(x)dx$$

*Where the constant *b* of the integral in the lower limit
*the constant a of the integral is the upper limit

The area under the lower graph is:

$$A = \int_b^a f(x)dx$$

$$\left[\begin{array}{l} 0.0013178775(0.83333x)^4 - 0.0637028755(0.83333x)^3 + 0.5254143882(0.83333x)^2 \\ - 0.72405575\,5(0.83333x) + 0.521464739\,9 \end{array}\right]_0^{7.5} dx$$

$= 10.74054666\text{cm}^2$

The area under the upper graph is:

$$A = \int_0^{7.5} f(u)du$$

$$\left[\begin{array}{l} - 4.0014871\,4\,x10^{-4}(0.83333u)^4 - 0.6560782\,6\,6(0.83333u)^3 + 0.7829361\,0\,4(0.83333u)^2 \\ - 1.7656317\,4\,2(0.83333u) + 1.7938209\,6\,1 \end{array}\right]_0^{7.5} du$$

= - 174.21259 < there is probably something wrong with the calculating

The accuracy of area of the logo is important in a business card because there are always some code and numbers written within these logos.