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SL Maths Portfolio I
18th August 2008.

Logarithm Bases.

Portfolio Type I is based primarily on logarithms, sequences and series. It first presents 3 sequences of logarithms of approximately 4 -5 terms. I was first asked to determine the next two terms of each sequence (namely, terms 5 and 6, or 6 and 7 in the sequence ones case). Looking at each sequence, I first noticed that the base of each logarithm was simply the first base, which I called m_1, to the power of the term trying to be determined. I concluded that the next two terms of the sequences could be found through $m_n = m_1^n$, where m_n is the unknown base, n is the term and m_1 is the first base of the sequence.

E.g. For the first sequence, to find the 6th term:
$m_6 = 2^{(6)}$
$m_6 = 64$

The following table shows the 3 sequences, including the last two terms that needed to be determined.

Term	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Logarithm	$\log_2 8$	$\log_4 8$	$\log_8 8$	$\log_{16} 8$	$\log_{32} 8$	$\log_{64} 8$	$\log_{128} 8$
	$\log_3 81$	$\log_9 81$	$\log_{27} 81$	$\log_{81} 81$	$\log_{243} 81$	$\log_{729} 81$	
	$\log_5 25$	$\log_{25} 25$	$\log_{125} 25$	$\log_{625} 25$	$\log_{3125} 25$	$\log_{15625} 25$	

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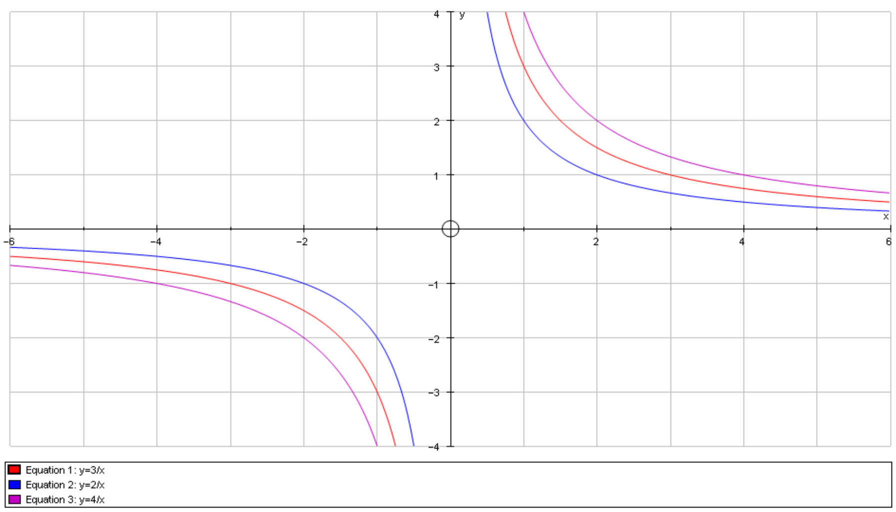
My next step was to find an expression of the n^{th} term. To find an expression of the n^{th} term, I firstly converted the logarithms into base 10 so it may be sol ved on my graphics display calculator – a TI-84 Plus. I then converted the answers into fractions as shown below:

Term	1	2	3	4	5	6	7
	$\frac{\log 8}{\log 2} = \frac{3}{1}$	$\frac{\log 8}{\log 4} = \frac{3}{2}$	$\frac{\log 8}{\log 8} = \frac{3}{3}$	$\frac{\log 8}{\log 16} = \frac{3}{4}$	$\frac{\log 8}{\log 32} = \frac{3}{5}$	$\frac{\log 8}{\log 64} = \frac{3}{6}$	$\frac{\log 8}{\log 128} = \frac{3}{7}$
	$\frac{\log 81}{\log 3} = \frac{4}{1}$	$\frac{\log 81}{\log 9} = \frac{4}{2}$	$\frac{\log 81}{\log 27} = \frac{4}{3}$	$\frac{\log 81}{\log 81} = \frac{4}{4}$	$\frac{\log 81}{\log 243} = \frac{4}{5}$	$\frac{\log 81}{\log 729} = \frac{4}{6}$	
	$\frac{\log 25}{\log 5} = \frac{2}{1}$	$\frac{\log 25}{\log 25} = \frac{2}{2}$	$\frac{\log 25}{\log 125} = \frac{2}{3}$	$\frac{\log 25}{\log 625} = \frac{2}{4}$	$\frac{\log 25}{\log 3125} = \frac{2}{5}$	$\frac{\log 25}{\log 15625} = \frac{2}{6}$	

For each sequence a pattern can be seen. In the 1st sequence, the numerator is seen to be a constant integer for each term. The denominator on the other hand is seen to be the number of the term. This can be seen in the following two sequences. Using this information I was able to determine that the formula for the 1 st sequence, in terms of $\frac{p}{q}$, is $y=\frac{3}{n}$, where p is U_1 , or first number of each sequence and q is represented by n which is the term.



The formulas for the 2nd and 3rd sequences are $y=\frac{4}{n}$ and $y=\frac{2}{n}$ respectively. To justify my claim through technology, I used a program called Autograph 3.20.

I first entered the three expressions I had found and got the above graph.

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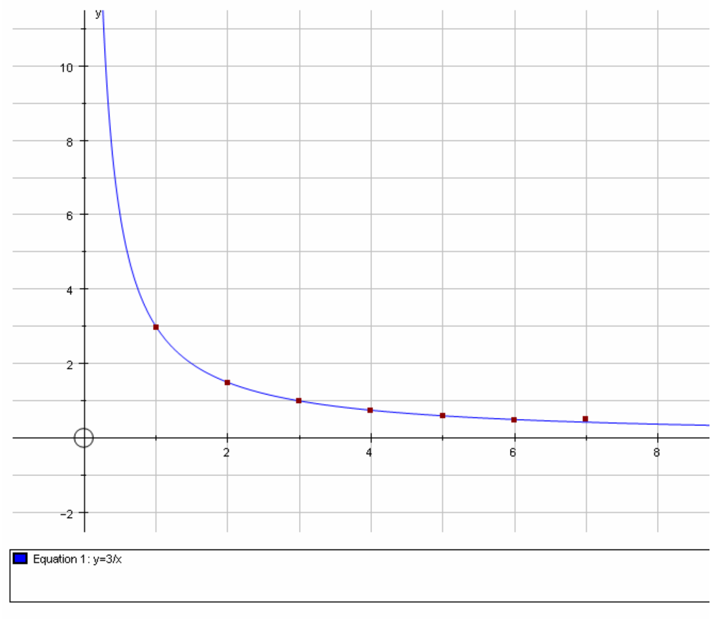
I next entered the information in the previous table for the first sequence as well as including $y=\frac{3}{n}$ in the graph. I obtained the graph seen on the left.



The dots are the plot points of the information I manually entered into the XY data set. As seen, the equation $y=\frac{3}{n}$ cuts through each plot point, which means that it is possible to also find the 6 and 7 th terms of sequence 1, as well as any other term.

Following this, I was then asked to convert the following logarithm sequences into the form of $\frac{p}{q}$.

	1	**2**	**3**
Logarithm 1	$\log_4 64$	$\log_8 64$	$\log_{32} 64$
$\frac{p}{q}$ form	$\frac{3}{1}$	$\frac{2}{1}$	$\frac{6}{5}$
Logarithm 2	$\log_7 49$	$\log_{49} 49$	$\log_{343} 49$
$\frac{p}{q}$ form	$\frac{2}{1}$	$\frac{1}{1}$	$\frac{2}{3}$
Logarithm 3	$\log_{\frac{1}{5}} 125$	$\log_{\frac{1}{125}} 125$	$\log_{\frac{1}{625}} 125$
$\frac{p}{q}$ form	$\frac{-3}{1}$	$\frac{-1}{1}$	$-\frac{3}{4}$
Logarithm 4	$\log_8 512$	$\log_2 512$	$\log_{16} 512$
$\frac{p}{q}$ form	$\frac{3}{1}$	$\frac{9}{1}$	$\frac{27}{12}$

I was next told to describe how to obtain the third answer in each row from the first two answers and then to create two more examples that showed the pattern as well as to find the general statement that expresses $\log_{ab}x$ in terms of c and d when $\log_a x = c$ and $\log_b x = d$.

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Firstly, obtaining the third answer, in logarithm form, can be done by multiplying the bases of the first two logarithms. For example, the bases of $\log_4 64$ and $\log_8 64$ will give the base of the third logarithm which is 32.

On the other hand, obtaining the third answer when the first two answers are in $\frac{p}{q}$ form can be better explained through the general statement that expresses $\log_{ab}x$.

When looking at the $\frac{p}{q}$ form, I noticed a pattern could be seen – the product of the numerators of the first two logarithms is equal to the numerator of the third. As well as this, the sum of these two numerators is equivalent to the denominator of the third. Through this information I was able to come to the conclusion that the general statement is:

$$log_{ab}x = \frac{cd}{c+d}$$

Next I created two more examples that fit the described pattern and also used these test the validity of my general statement.

Two Examples that fit the pattern:

Logarithm	$\log_3 9$	$\log_9 9$	$\log_{27} 9$
$\frac{p}{q}$ form	$\frac{2}{1}$	$\frac{1}{1}$	$\frac{2}{3}$
Logarithm	$\log_2 16$	$\log_{16} 16$	$\log_{32} 16$
$\frac{p}{q}$ form	$\frac{4}{1}$	$\frac{1}{1}$	$\frac{4}{5}$

The next step was to discuss the scope and/or the limitations of *a, b*, and *x* in my general statement. To do this I tested my general statement by using different sets of values for *a, b*, and *x*.

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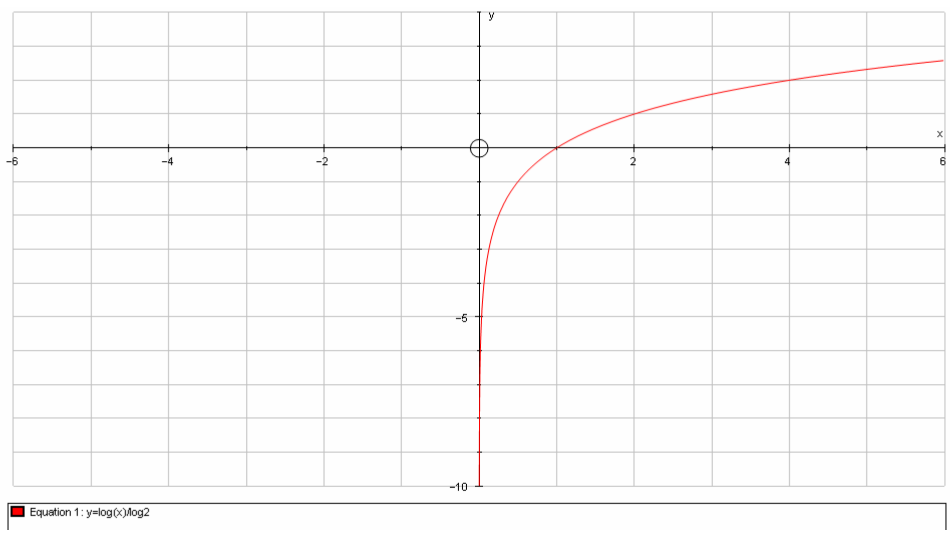
I decided that I will first start with *x*, first choosing negative values before moving to positive integers whilst having different values for *a* and *b*. This is shown in the following table, showing logarithm form and calculated figu re on the GDC:

x-value	**$\log_a x$**	**$\log_b x$**	**$\log_{ab} x$**
-2	$\log_2(-2)$	$\log_3(-2)$	$\text{Log}_6(-2)$
	ERROR	ERROR	ERROR
-1	$\log_2(-1)$	$\log_3(-1)$	$\log_6(-1)$
	ERROR	ERROR	ERROR
0	$\log_2 0$	$\log_3 0$	$\log_6 0$
	ERROR	ERROR	ERROR
1	$\log_2 1$	$\log_3 1$	$\log_6 1$
	0	0	0
2	$\log_2 2$	$\log_3 2$	$\log_6 2$
	1	0.63	0.39

As you can see, *x* cannot be equal to or less than 0. This is further reinforced by the following graph. As seen, the graph does not cut the y -axis, forming a vertical asymptote and so *x* is never equal to 0 or any negative integer.



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a-value	$\log_a x$	$\log_b x$	$\log_{ab} x$
-2	$\log_{-2}2$	$\log_3 2$	$\log_{-6}2$
	ERROR	0.63	ERROR
-1	$\log_{-1}2$	$\log_3 2$	$\log_{-3}2$
	ERROR	0.63	ERROR
0	$\log_0 2$	$\log_3 2$	$\log_0 2$
	ERROR	0.63	ERROR
1	$\log_1 2$	$\log_3 2$	$\log_3 2$
	ERROR	0.63	0.63
2	$\log_2 2$	$\log_3 2$	$\log_6 2$
	1	0.63	0.39

I repeated the above table for the *a*-values. With values of *a*, it can be seen that it cannot be equal to or less than 0, much like *x*. This is because the base of the third logarithm is the product of the first two bases and therefore if zero were the base for one of the first two logarithms, the third logarithm would have a base of zero. This also applies to negative numbers and so therefore $a>0$.

The findings for *a* values could also be applied to the *b* values as the base of the third logarithm would be zero or a negative number if either of the first two logarithms were to have a base of zero or negative integer. Therefore $b>0$.

As well as this, I also considered "what if $c+d=0$?", since 0 as the denominator of the formula would then make it undefined. For a denominator of 0 to be possible, it would mean that *c* and *d* would have to be opposites, for example $c=8$ and $d=-8$ hence $c+d=0$.

Logarithm	$\log_3 9$	$\log_{\frac{1}{3}} 9$	$\log_1 9$
	2	-2	ERROR

As you can see, an error occurred whilst trying to calculate the third logarithm. This is due to the fact that it could not divide by 0. From this it can be determined that $a \neq \frac{1}{b}$ and $b \neq \frac{1}{a}$.

Overall, the limitations of my general statement, $log_{ab}x = \frac{cd}{c+d}$, are as follows:

$x>0$ $a>0$ $b>0$ $a \neq \frac{1}{b}$ $b \neq \frac{1}{a}$.

As stated previously, my general statement is $log_{ab}x = \frac{cd}{c+d}$ and has been proven to work under the conditions of the limitations stated before. I obtained this statement by first looking out for patterns created by sequences in both their logarithm and $\frac{p}{q}$ form, testing to

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see if the patterns were shared with the other sequences and then creating a statement that would allow me to express it.