The Koch snowflake

1. Using an initial side length of 1, create a table that shows the values of N_n, I_n, P_n and A_n for n=0, 1, 2 and 3. Use exact values in your results. Explain the relationship between successive terms in the table for each quantity N_n, I_n, P_n and A_n.

Let Nn= the length of sides.
When the n values are 0, 1, 2, and3, the number of sides will be;

n	number of sides
0	3
1	12
2	48
3	192

As coming of new triangle each side, the number of side increase. Each of one side will be separated into 4. Therefore the function to find the number of sides will be $[N_n = 4 N_{n-1}]$. Don't forget the first triangle's number of side is 3.

Let ln= the length of single side
When the n values are 0, 1, 2, and3, the length of sides will be;

n	length of side
0	1
1	1/3
2	1/9

3	1/27

As coming of new triangle each side, the length of a single side will be decrease because it will be separated 3 part of it. In other words, each of side will be separated 1/3 each time. Therefore the function of to find the length of a single side will be [$\ln = \frac{1}{3} l_{n-1}$]. First, the length of a single side is 1.

Let Pn= the length of the perimeter

When the n values are 0, 1, 2, and 3, the length of the perimeter will be;

n	length of the perimeter
0	3
1	4
2	16/3
3	64/9

The length of perimeter can represent a function what is [$P_n = N_n \times l_n$] because perimeter means the total length of the figure. The first perimeter of triangle is 3.

Let An= the area of the snowflake

When the n values are 0, 1, 2, and 3, the length of the perimeter will be;

n	area of the

	snowflake
0	$\sqrt{3}/4$
1	$\sqrt{3}/3$
2	$10\sqrt{3}/27$
3	$94\sqrt{3}/243$

This is not constant to increase their area because the area of coming triangle will be decreasing according to increase the number of n. So, it needs to find how to change the area of coming triangle.

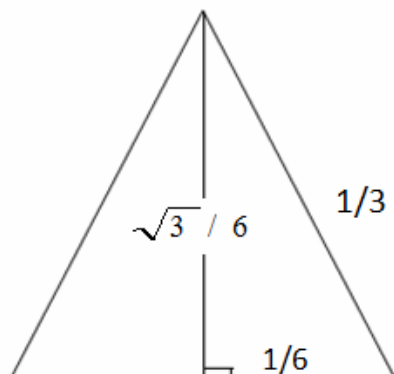The area of triangles will be found as using the Pythagorean Theorem [$a^2 = b + c^2$].

$$a^2 = b + c^2$$
$$1^2 = (1/2)^2 + c^2$$
$$c^2 = 1^2 - (1/2)^2$$
$$c^2 = 3/4$$
$$c = \sqrt{3}/2$$
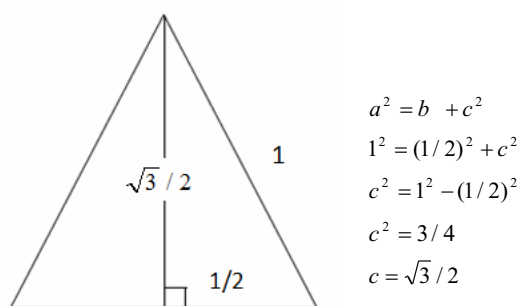
Therefore the area of first triangle will be

$$\frac{\sqrt{3}}{2} \times 1 \times \frac{1}{2}$$

$$= \frac{\sqrt{3}}{4} \cdots ①$$

The second triangle is able to find as calculate of the coming triangle's area add to the first triangle which indicated ①. So, area of coming triangle will be

$$a^2 = b^2 + c^2$$
$$(1/3)^2 = (1/6)^2 + c^2$$
$$c^2 = (1/3)^2 - (1/6)^2$$
$$c^2 = 3/36$$
$$c = \sqrt{3}/\sqrt{36} = \sqrt{3}/6$$

Area of one of the coming triangle is

$$A = \frac{\sqrt{3}}{6} \times \frac{1}{6} \times \frac{1}{2}$$

$$= \frac{\sqrt{3}}{36} \cdots ②$$

Thus the total area of second figure will be

$$①@ + ①A \times R□ \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{36} \times 3$$

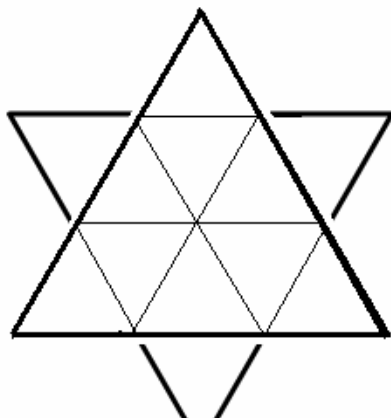$$= \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{12}$$

$$= \frac{4\sqrt{3}}{12} = \frac{\sqrt{3}}{3}$$

The coming triangle's area has characteristic which is 1/9 of the first triangle.

$$② \times 9 = ①$$

$$\frac{\sqrt{3}}{36} \times 9 = ①$$

$$\frac{\sqrt{3}}{4} = ① \qquad [① = \frac{\sqrt{3}}{4}]$$

Therefore, the coming triangle's area will be 1/9 of the first triangle.

Moreover, the triangle can be separated into 9. The one of the small area of triangle will equal with coming triangle's areas. That's why the coming triangle's area will be 1/9 of the first

triangle area.

It is considerable that can say others area of coming triangle due to the same theory.

So the function of A_n will be [$A_n = A_0 \times (\frac{1}{9})^n \times N_{n-1} + A_{n-1}$]
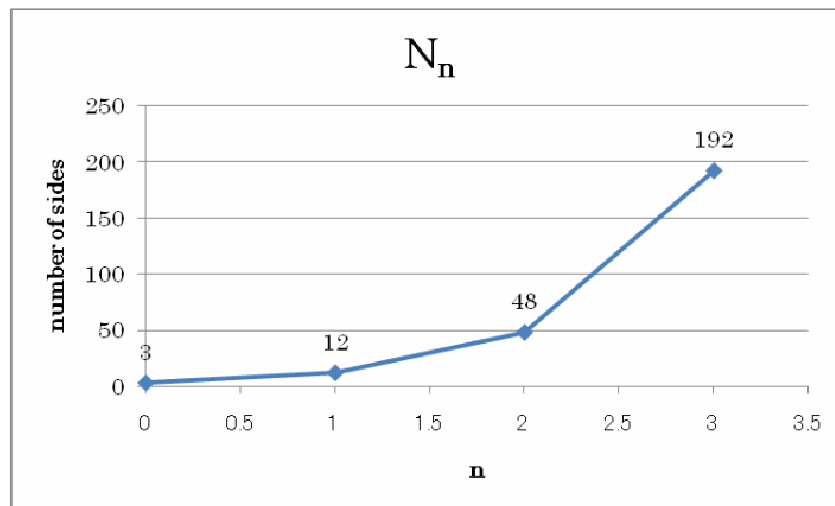
$A_0 \times (\frac{1}{9})^n$ = the area of coming triangle.

N_{n-1} = the number of coming triangles.

A_{n-1} = the area of figure which was found before of it.

2. Using a GDC or a suitable graphing software package, create graphs of the four sets of values plotted against the value of n. Provide separate printed output for each graph.
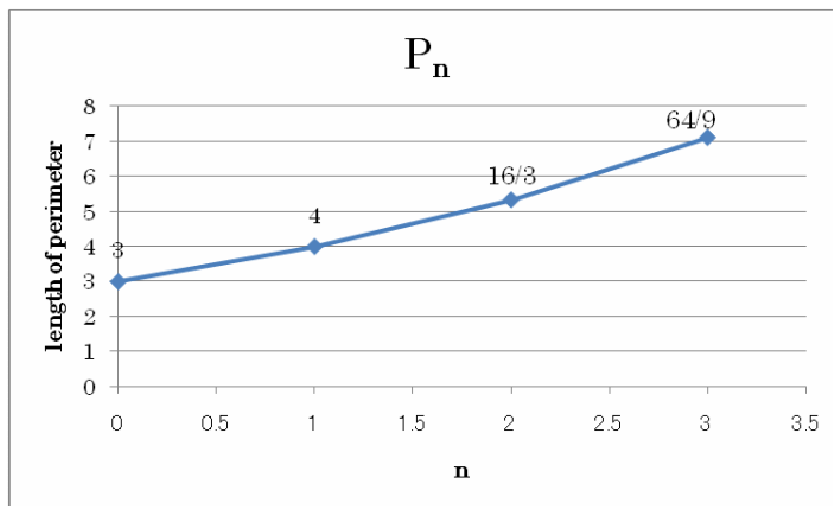
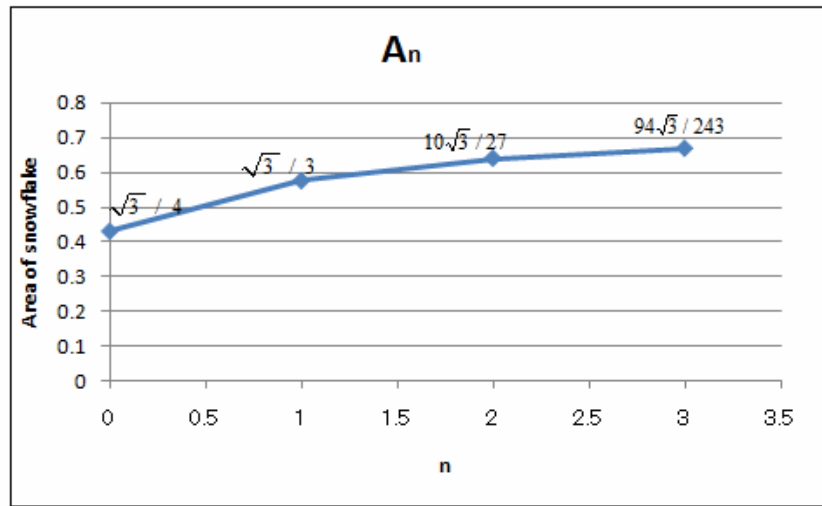The relationship between n and the number of side

The relationship between n and the number of a single side
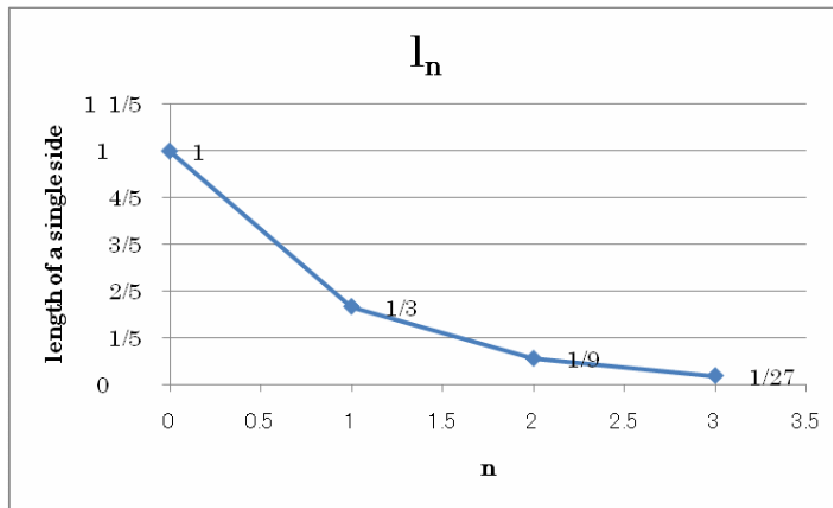
The relationship between n and the length of perimeter

The relationship between n and area of snowflake

3. For each of the graphs above, develop a statement in terms of n that generalizes the behavior shown in its graph. Explain how you arrived at your generalizations. Verify that your generalizations apply consistently to the sets of values produced in the table.

I got these functions below which have already indicated the reasons.

$[N_n = 4 N_{n-1}]$

$[\text{l}n = \frac{1}{3} l_{n-1}]$

$[P_n = N_n \times l_n]$

$[A_n = A_0 \times (\frac{1}{9})^n \times N_{n-1} + A_{n-1}]$

These functions are fit with graphs which had already done no2.

$[N_n = 4 N_{n-1}]$

Due to the n increase, the number of side also increases because each time the one side will be separated into 4.

Now we check that function is correct or not.

It has to be the same values in table below.

n	number of sides
0	3
1	12
2	48
3	192

$[N_n = 4 N_{n-1}]$

$N_1 = 4 \times 3 = 12$

$N_2 = 4 \times 12 = 48$

$N_3 = 4 \times 48 = 192$

Therefore, this function is correct.

$[\ln = \dfrac{1}{3} l_{n-1}]$

Due to the n increase, the number of side decreases because a single side will be separated each time, so the length of a single side has to decrease.

Now, check the functions.

n	length of side
0	1
1	1/3
2	1/9
3	1/27

$[\ln = \dfrac{1}{3} l_{n-1}]$

$$l_1 = \frac{1}{3} \times 1 = \frac{1}{3}$$

$$l_2 = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$$

$$l_3 = \frac{1}{3} \times \frac{1}{9} = \frac{1}{27}$$

So, this function is correct either.

$$[P_n = N_n \times l_n]$$

Due to increase the value of n, the length of the perimeter also increases because of the number of sides are increasing.

Now, check the function

n	length of the perimeter
0	3
1	4
2	16/3
3	64/9

$$[P_n = N_n \times l_n]$$

$$P_0 = 1 \times 3 = 3$$

$$P_1 = 12 \times \frac{1}{3} = 4$$

$$P_2 = 48 \times \frac{1}{9} = \frac{16}{3}$$

$$P_3 = 192 \times \frac{1}{27} = \frac{64}{9}$$

Thus, this function is correct as well.

$$[A_n = A_0 \times (\frac{1}{9})^n \times N_{n-1} + A_{n-1}]$$

Due to value of n increase, the area also increases because it keeps expanding. The changing of area is going to a little because of the area of coming triangle is smaller and smaller.

Now, check the function.

n	area of the snowflake
0	$\sqrt{3}/4$
1	$\sqrt{3}/3$
2	$10\sqrt{3}/27$
3	$94\sqrt{3}/243$

$$[A_n = A_0 \times (\frac{1}{9})^n \times N_{n-1} + A_{n-1}]$$

$$A_1 = \frac{\sqrt{3}}{4} \times (\frac{1}{9})^1 \times 3 + \frac{\sqrt{3}}{4} = \frac{3\sqrt{3}}{36} + \frac{9\sqrt{3}}{36} = \frac{\sqrt{3}}{3}$$

$$A_2 = \frac{\sqrt{3}}{4} \times (\frac{1}{9})^2 \times 12 + \frac{\sqrt{3}}{3} = \frac{12\sqrt{3}}{324} + \frac{108\sqrt{3}}{324} = \frac{120\sqrt{3}}{324} = \frac{10\sqrt{3}}{27}$$

$$A_3 = \frac{\sqrt{3}}{4} \times (\frac{1}{9})^3 \times 48 + \frac{10\sqrt{3}}{27} = \frac{48\sqrt{3}}{2916} + \frac{1080\sqrt{3}}{2916} = \frac{1128\sqrt{3}}{2916} = \frac{94\sqrt{3}}{243}$$

The function is correct as well.

4. Investigate what happens at n=4. Use your conjectures from step3 to obtains

for N_4, I_4, P_4 and A_4. Now draw a large diagram of one "side" (that is, one side of the original triangle that has been transformed) of the fractal at stage 4 and clearly verify your predictions.

$[N_n = 4 N_{n-1}]$

$N_4 = 4 N_3 = 4 \times 192 = \underline{768}$

$[\text{l}n = \dfrac{1}{3} l_{n-1}]$

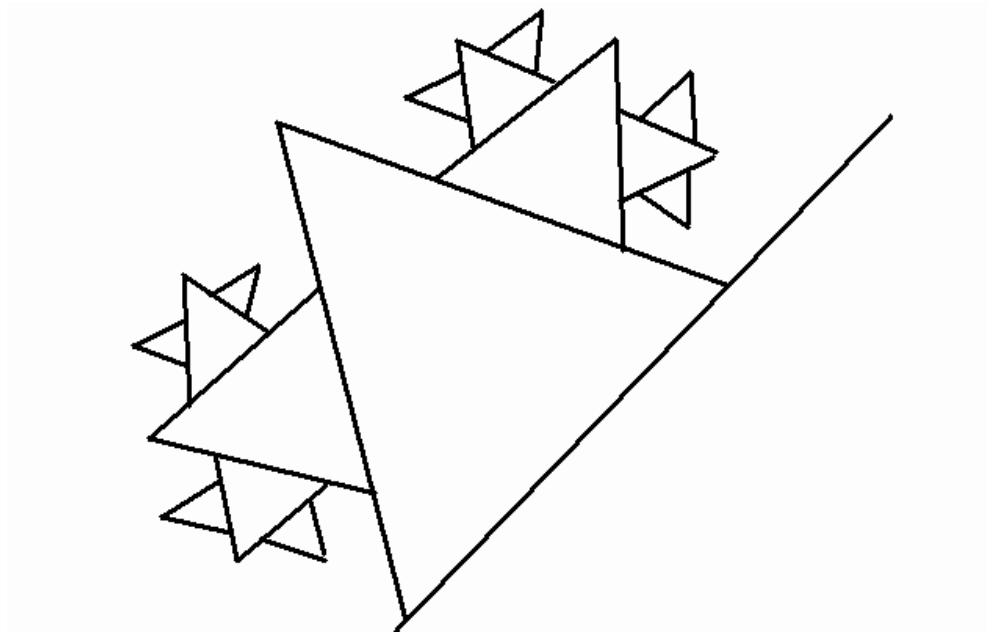$l_4 = \dfrac{1}{3} l_3 = \dfrac{1}{3} \times \dfrac{1}{27} = \dfrac{1}{\underline{81}}$

$[P_n = N_n \times l_n]$

$P_4 = N_4 \times l_4 = 768 \times \dfrac{1}{81} = \dfrac{26}{\underline{27}}$

$[A_n = A_0 \times (\dfrac{1}{9})^n \times N_{n-1} + A_{n-1}]$

$A_4 = \dfrac{\sqrt{3}}{4} \times (\dfrac{1}{9})^4 \times N_3 + A_3 = \dfrac{192 \ \sqrt{3}}{6561} + \dfrac{94 \ \sqrt{3}}{243} = \dfrac{192 \ \sqrt{3} + 2538 \ \sqrt{3}}{6561} = \dfrac{862 \ \sqrt{3}}{\underline{2187}}$

Diagram of one of side when n=4.

5. Calculate values for N_6, I_6, P_6 and A_6. You need not verify these answers.

$[N_n = 4 N_{n-1}]$

$N_6 = 4 N_5 = 4 \times (4N_4) = \underline{12288}$

$[\ln = \frac{1}{3} l_{n-1}]$

$l_6 = \frac{1}{3} l_5 = \frac{1}{3} \times (\frac{1}{3} l_4) = \frac{1}{3} \times \frac{1}{243} = \frac{1}{\underline{729}}$

$[P_n = N_n \times l_n]$

$P_6 = N_6 \times l_6 = 768 \times \frac{1}{81} = \frac{256}{\underline{27}}$

$$[A_n = A_0 \times (\frac{1}{9})^n \times N_{n-1} + A_{n-1}]$$

$$A_6 = \frac{\sqrt{3}}{4} \times (\frac{1}{9})^6 \times N_5 + A_5 = \frac{3072 \quad \sqrt{3}}{21257684} + A_5$$

$$[A_5 = \frac{\sqrt{3}}{4} \times (\frac{1}{9})^5 \times N_4 + A_3 = \frac{768 \quad \sqrt{3}}{262966} + \frac{832 \quad \sqrt{3}}{2187} = \frac{98664 \quad \sqrt{3}}{236966} = \frac{7822 \quad \sqrt{3}}{19683}]$$

$$A_6 = \frac{3072 \quad \sqrt{3}}{21257684} + \frac{7822 \quad \sqrt{3}}{19683} = \frac{54987 \quad \sqrt{3}}{19683}$$

6. Write down successive values of A_n in term of A_0. What pattern emerges?

$[A_n = A_0 \times (\frac{1}{9})^n \times N_{n-1} + A_{n-1}]$ have been proved. However, the value of A_0 can't get

from this function because of N_{-1} and A_{-1} does not exist in this report. Thus find

another characteristic as transform the function, the values of 1, 2, 3, and 4.

$$A_1 = A_0 \times (\frac{1}{9})^1 \times N_0 + A_0$$

$$A_2 = A_0 \times (\frac{1}{9})^2 \times N_1 + A_0 \times (\frac{1}{9})^1 \times N_0 + A_0$$

$$A_3 = A_0 \times (\frac{1}{9})^3 \times N_2 + A_0 \times (\frac{1}{9})^2 \times N_1 + A_0 \times (\frac{1}{9})^1 \times N_0 + A_0$$

$$A_4 = A_0 \times (\frac{1}{9})^4 \times N_3 + A_0 \times (\frac{1}{9})^3 \times N_2 + A_0 \times (\frac{1}{9})^2 \times N_1 + A_0 \times (\frac{1}{9})^1 \times N_0 + A_0$$

These function can be more simply thus

$$A_1 = A_0[(\frac{1}{9})^1 \times N_0 + 1]$$

$$A_2 = A_0[(\frac{1}{9})^2 \times N_1 + (\frac{1}{9})^1 \times N_0 + 1]$$

$$A_3 = A_0[(\frac{1}{9})^3 \times N_2 + (\frac{1}{9})^2 \times N_1 + (\frac{1}{9})^1 \times N_0 + 1]$$

$$A_4 = A_0[(\frac{1}{9})^4 \times N_3 + (\frac{1}{9})^3 \times N_2 + (\frac{1}{9})^2 \times N_1 + (\frac{1}{9})^1 \times N_0 + 1]$$

It is obvious that the function has characteristic thus it can define.

Let $F_n = (\frac{1}{9})^n \times N_{n-1}$

$$A_1 = A_0(1 + F_1)$$

$$A_2 = A_0(1 + F_1 + F_2)$$

$$A_3 = A_0(1 + F_1 + F_2 + F_3)$$

$$A_4 = A_0(1 + F_1 + F_2 + F_3 + F_4)$$

Therefore the value of A_n will be

$$A_n = A_0(\text{sum of } F_1 \sim F_n)$$

7. Explain what happens to the perimeter and area as n gets very large. What conclusion can you make about the area as n $\rightarrow \infty$? Comment on your results.

The perimeter and area will be infinite. By the solution of area of figure could represent [$A_n = A_0(\text{sum of } F_1 \sim F_n)$]. That means when value of n is infinite, area also infinite as well. Therefore, the value of perimeter and area of snowflake are infinite when the n value is infinite either.