03.02.2004
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Portfolio #2 - Type I

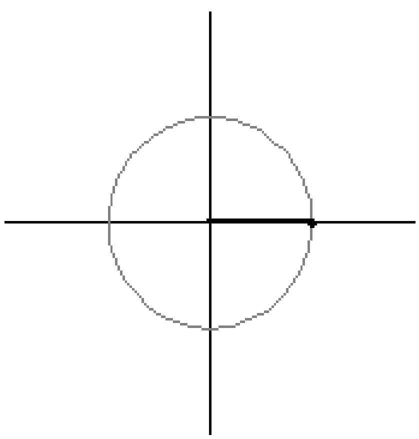
Investigating the Graphs of Sine Functions

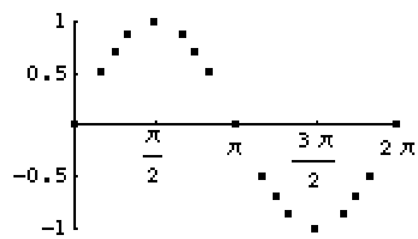
The purpose of this assignment is to obtain general rules for transformations of sine functions from analysing patterns got from examples of these. To justify my conjectures of all of the following functions I used “Magic Graph” - an electronic graphing program which allowed me to present them with a high level of precision. The trigonometric settings and the radian mode were kept constant throughout the whole investigation.

Part 1

Graph of y= sin x

To present this graph properly there are several possibilities: one can use a graphing calculator, a computer program, draw the graph from tabled values or from the unit circle. I chose the unit circle method because it is then more understandable how sine of x gets its shape and position, since sine of an angle is the ratio of the length of the opposite side to the length of the hypotenuse and therefore[1]:





Characteristics of y= sin x

The sine curve is symmetric with the origin, it is an even function, and has infinite intercepts at multiples of π, as well as infinite maximum and minimum points at –1 and 1.

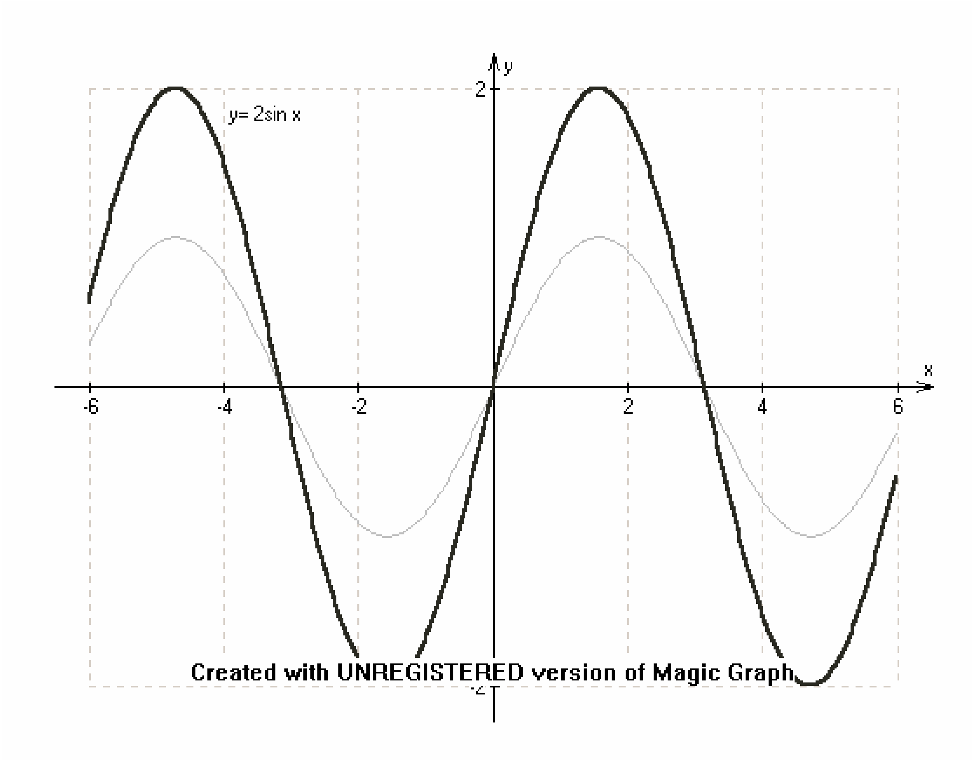
Because the coefficient of sine is 1, the amplitude of y= sin x, the distance from the central value or the height of each peak above the baseline, is 1.

As the period of a function is the length of the time the system takes to go through one cycle of its motion the period of y= sin x is 2π and the complete graph consists of the above graph repeated over and over: the domain of the sine curve is the set of all real numbers and the range is [-1,1].

[1] http://documents.wolfram.com/teachersedition/Teacher/UnitCircleandSine.html

Examples for y= Asin x (black graph)
and comparisons with y= sin x (grey graph)

Graph of y= 2sin x



(a) transformation of the standard curve y= sin x:

y= 2sin x is a dilation along the y-axis by the factor of 2.

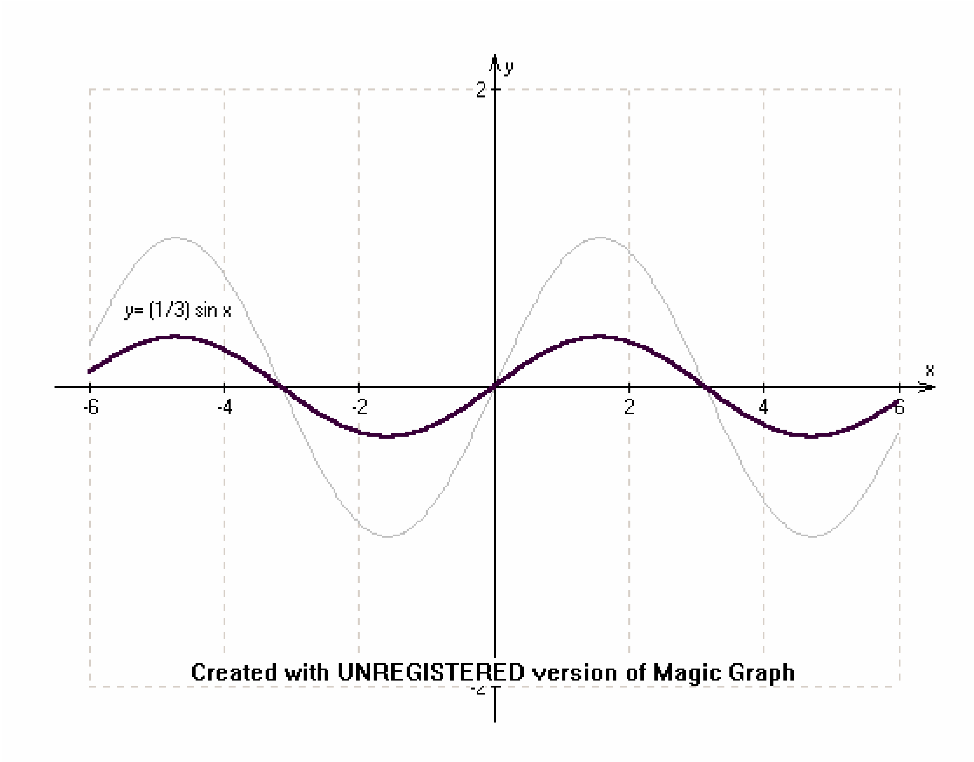
(b) characteristics of y= 2sin x:

the curve is symmetric with the origin, it is an even function, and has infinite intercepts at multiples of π , as well as infinite maximum and minimum points at –2 and 2. Intercepts with the x-axis are invariant points with y=sin x.

Amplitude: the coefficient of sine is 2 and therefore the amplitude of y= 2sin x is 2.

The period is 2π , the complete graph consists of the above graph repeated over and over: the domain of the curve is the set of all real numbers and the range is [-2,2].

Graph of y= $\frac{1}{3}$sin x



(a) transformation of the standard curve y= sin x:

y= $\frac{1}{3}$sin x is a dilation along the y-axis by the factor of $\frac{1}{3}$.

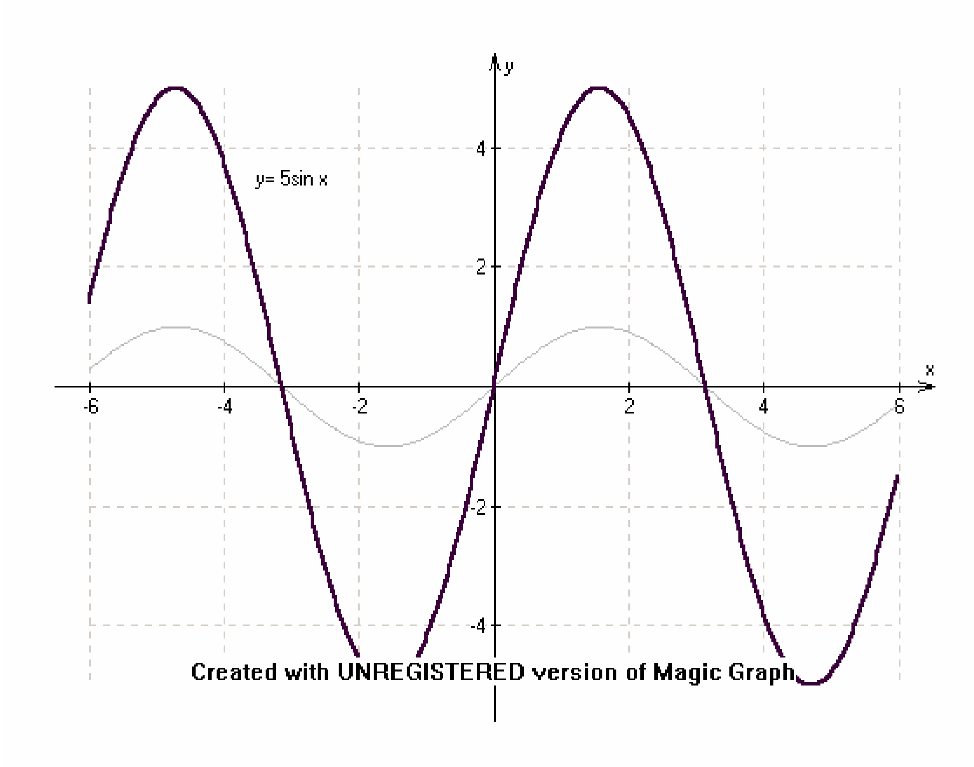
(b) characteristics of y= $\frac{1}{3}$sin x:

the curve is symmetric with the origin, it is an even function, and has infinite intercepts at multiples of π , as well as infinite maximum and minimum points at $-\frac{1}{3}$ and $\frac{1}{3}$. Intercepts with the x-axis are invariant points with y=sin x.

Amplitude: the coefficient of sine is $\frac{1}{3}$ and therefore the amplitude of y= $\frac{1}{3}$sin x is $\frac{1}{3}$.

The period is 2π, the complete graph consists of the above graph repeated over and over: the domain of the curve is the set of all real numbers and the range is [-$\frac{1}{3}$,$\frac{1}{3}$].

Graph of y=5sin x



(a) transformation of the standard curve y= sin x:

y= 5sin x is a dilation along the y-axis by the factor of 5.

(b) characteristics of y= 5sin x:

the curve is symmetric with the origin, it is an even function, and has infinite intercepts at multiples of π , as well as infinite maximum and minimum points at –5 and 5. Intercepts with the x-axis are invariant points with y=sin x.

Amplitude: the coefficient of sine is 5 and therefore the amplitude of y= 5sin x is 5.

The period is 2π , the complete graph consists of the above graph repeated over and over: the domain of the curve is the set of all real numbers and the range is [-5,5].

Graph of y= -2 sin x

Conjecture:

(a) transformation of the standard curve y= sin x:

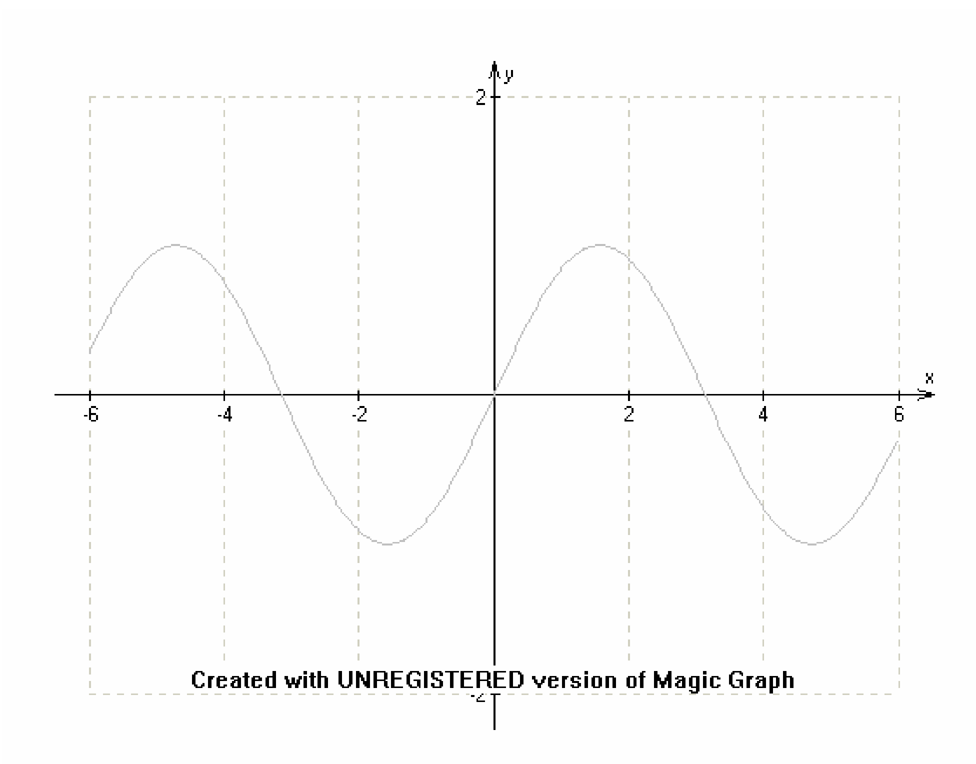
from the examples above I can deduce that the amplitude changes according to the coefficient, therefore y= -2sin x must be a dilation along the y-axis by the factor of two. Because the coefficient is now negative I conjecture that the graph is not the same as y= 2sin x but reflected on the x-axis as –2 affects the whole function.

(b) characteristics of y= 5sin x:

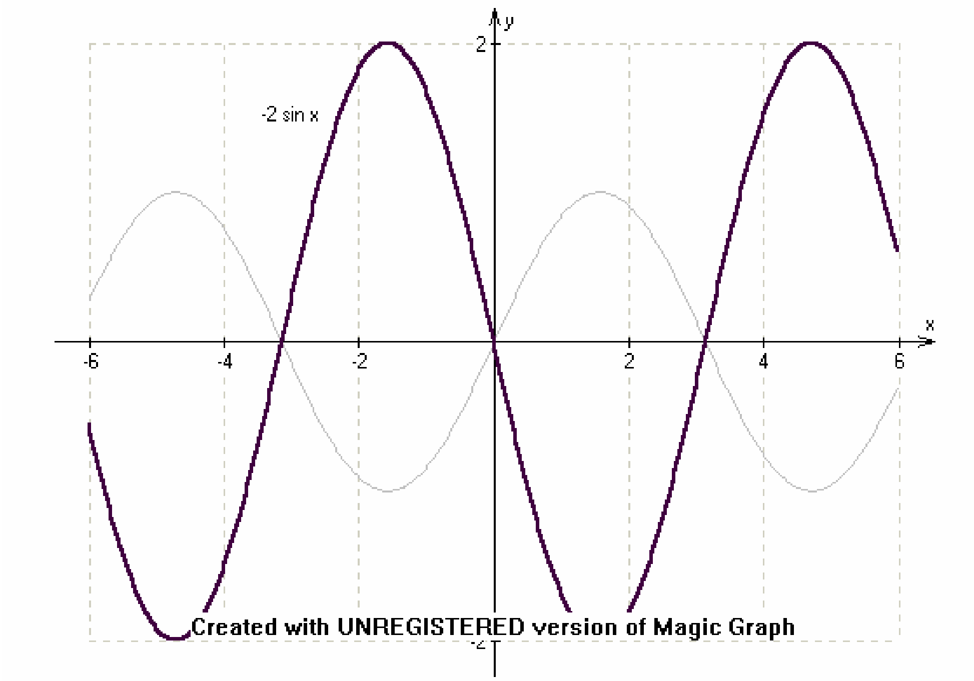
the curve is symmetric with the origin, it is an even function, and has infinite intercepts at multiples of π , as well as infinite maximum and minimum points at –2 and 2. Intercepts with the x-axis are invariant points with y=sin x.

Amplitude: the coefficient of sine is -2 and therefore the amplitude of y= -2sin x is 2.

The period is 2π, the complete graph consists of the above graph repeated over and over: the domain of the curve is the set of all real numbers and the range is [-2,2].



Justifying my conjecture of y= -2 sin x



By comparing the true graph of y= -2 sin x and my conjecture of it, I can conclude that they are equal and therefore my conjecture for its transformations and its characteristics was right.

Graph of y= –1/2 sin x

Conjecture:

(a) transformation of the standard curve y= sin x:

from the examples above I can deduce that the amplitude changes according to the coefficient, therefore y= -1/2sin x must be a dilation along the y-axis by the factor of 1/2. Because the coefficient is now negative I conjecture that the graph is not the same as y= 1/2sin x but reflected on the x-axis as –1/2 affects the whole function.

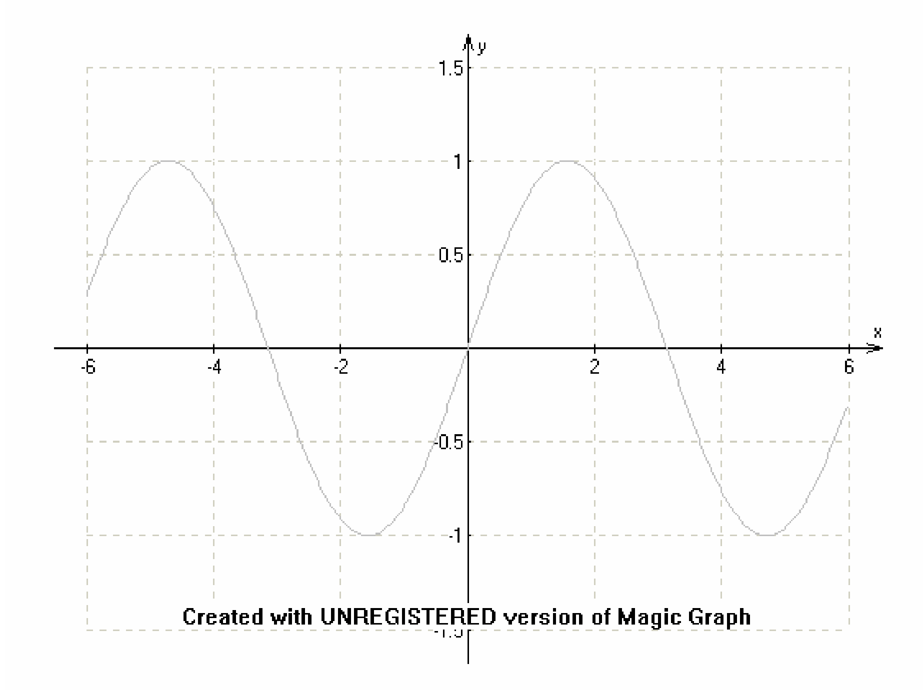
(b) characteristics of y= 5sin x:

the curve is symmetric with the origin, it is an even function, and has infinite intercepts at multiples of π , as well as infinite maximum and minimum points at –1/2 and 1/2. Intercepts with the x-axis are invariant points with y=sin x.

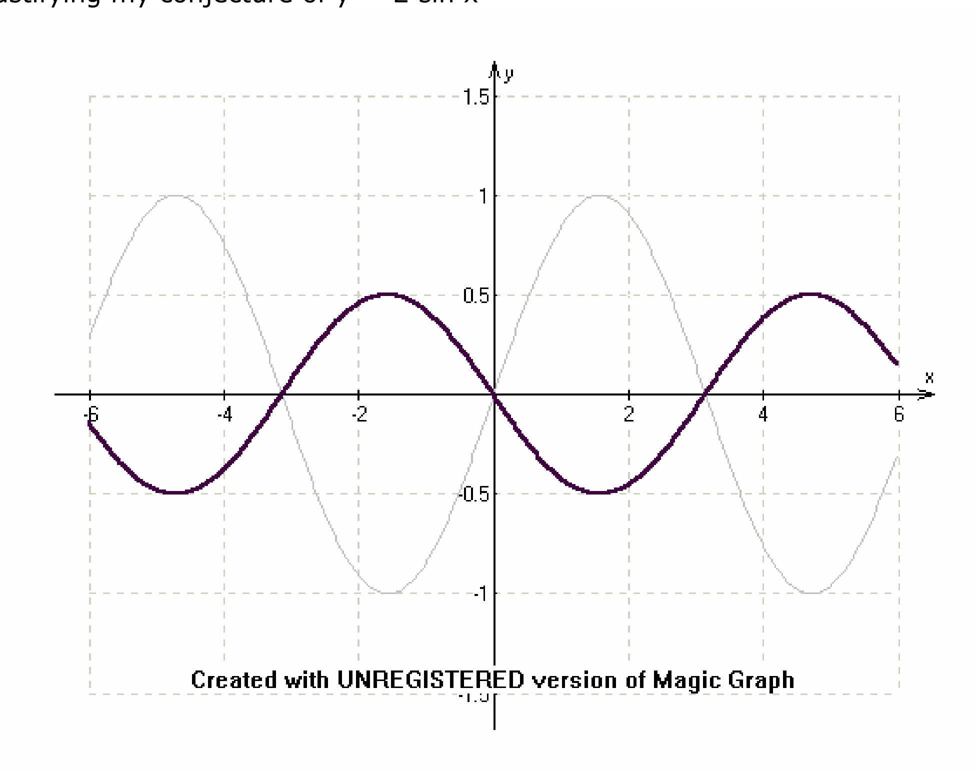
Amplitude: the coefficient of sine is –1/2 and therefore the amplitude of y= -1/2sin x is 1/2.

The period is 2π , the complete graph consists of the above graph repeated over and over: the domain of the curve is the set of all real numbers and the range is [-1/2,1/2].

My conjecture of –1/2 sin x



Justifying my conjecture of y= -2 sin x



By comparing the true graph of y= -1/2 sin x and my conjecture of it, I can conclude that they are equal and therefore my conjecture for its transformations and its characteristics was right.

Graph of y= - sin x

Conjecture:
(a) transformation of the standard curve y= sin x:

from the examples above I can deduce that the amplitude changes according to the coefficient, therefore y= -sin x must be a dilation along the y-axis by the factor of 1. Because the coefficient is now negative I conjecture that the graph is not the same as y= sin x but reflected on the x-axis as –1 affects the whole function.

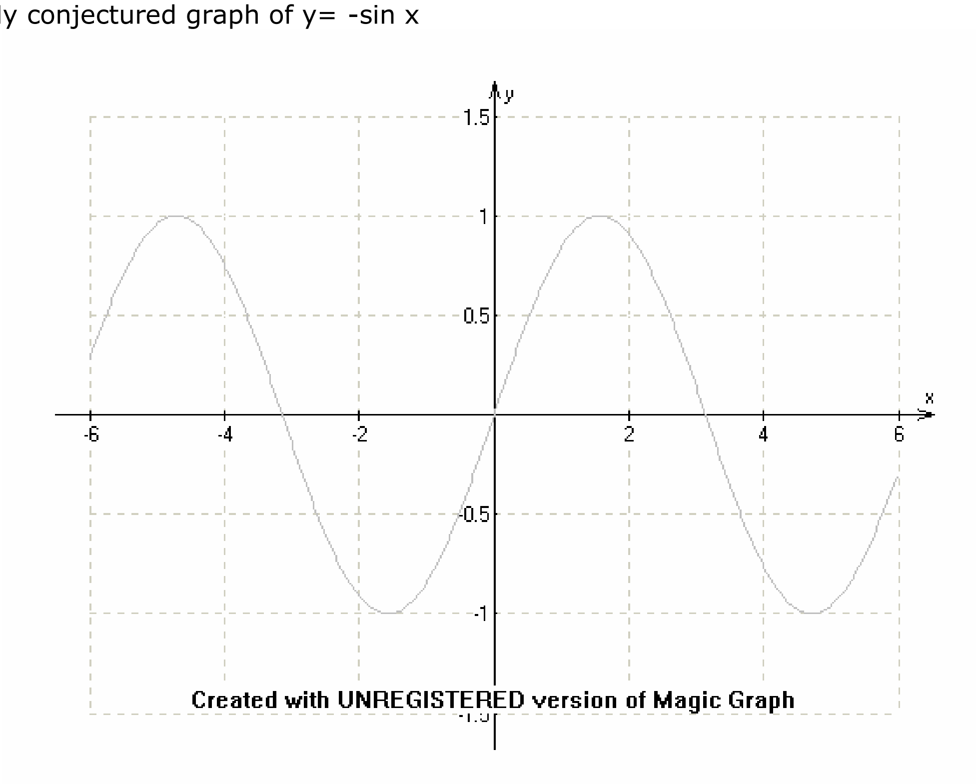
(b) characteristics of y= -sin x:

the curve is symmetric with the origin, it is an even function, and has infinite intercepts at multiples of π, as well as infinite maximum and minimum points at –1 and 1. Intercepts with the x-axis are invariant points with y=sin x.

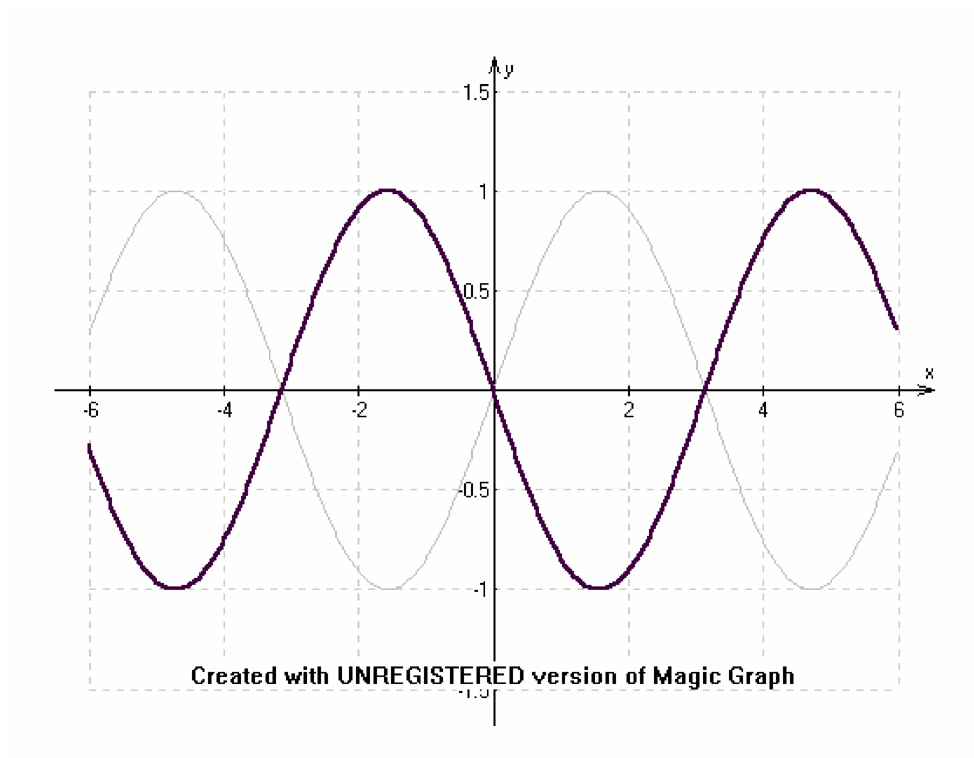
Amplitude: the coefficient of sine is –1 and therefore the amplitude of
y= -1sin x is 1.

The period is 2π, the complete graph consists of the above graph repeated over and over: the domain of the curve is the set of all real numbers and the range is [-1,1].

My conjectured graph of y= -sin x



Justifying my conjectured graph of y= - sin x



By comparing the true graph of y= -sin x and my conjecture of it, I can conclude that they are equal and therefore my conjecture for its transformations and its characteristics was right.

Conclusion for y= Asin x

As A varies in y= Asin x only the shape varies: A is the amplitude of the graph of sine. If A is negative the graph of y= -A sin x is obtained by changing the amplitude by its coefficient and reflecting it along the x axis.

Part 2

Graphs of type y= sin Bx

Graph of y= sin 2x

Conjecture:
(a) transformation of the standard curve y= sin x:

as x is directly affected by the factor of 2 in y= sin 2x, I conjecture that the period of the standard curve, 2π, is now multiplied by the factor of two: 4π. Thus y= sin 2x is a dilation along the x-axis by the factor of 2.

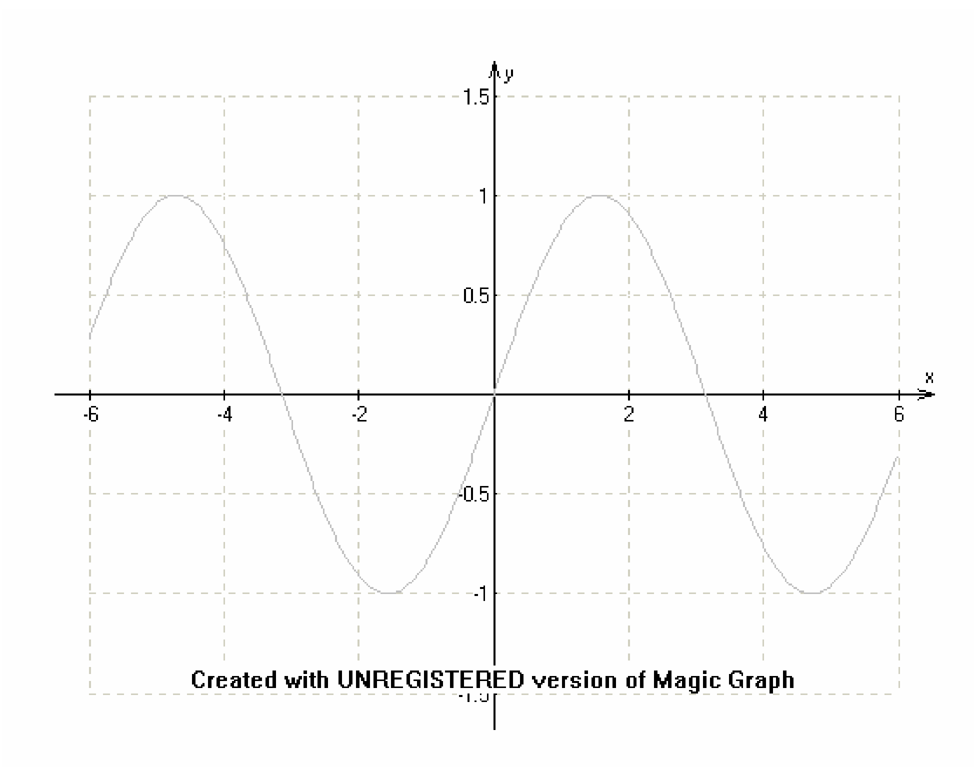
(b) characteristics of y= sin 2x:

the curve is symmetric with the origin, it is an even function, and has infinite intercepts at multiples of 2π, as well as infinite maximum and minimum points at –1 and 1.

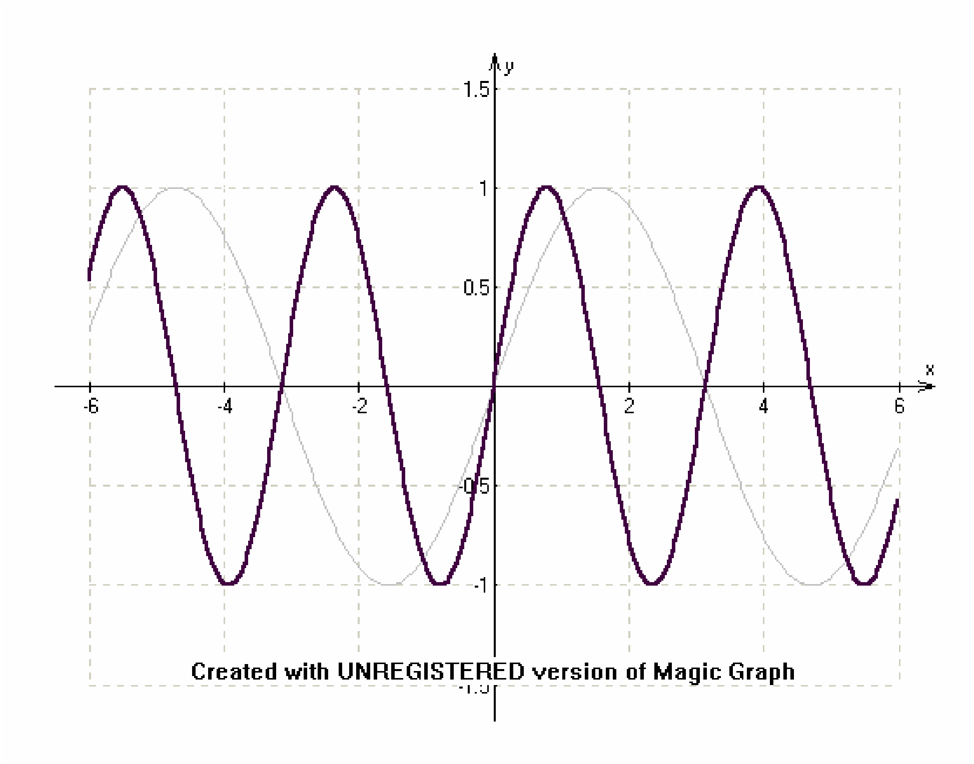
Amplitude: the coefficient of sine is 1 and therefore the amplitude of
y= sin 2x is 1.

The period is 4π, the complete graph consists of the above graph repeated over and over: the domain of the curve is the set of all real numbers and the range is [-1,1].

My conjectured graph of y= sin 2x:



Justifying my conjectured graph of y= sin 2x :

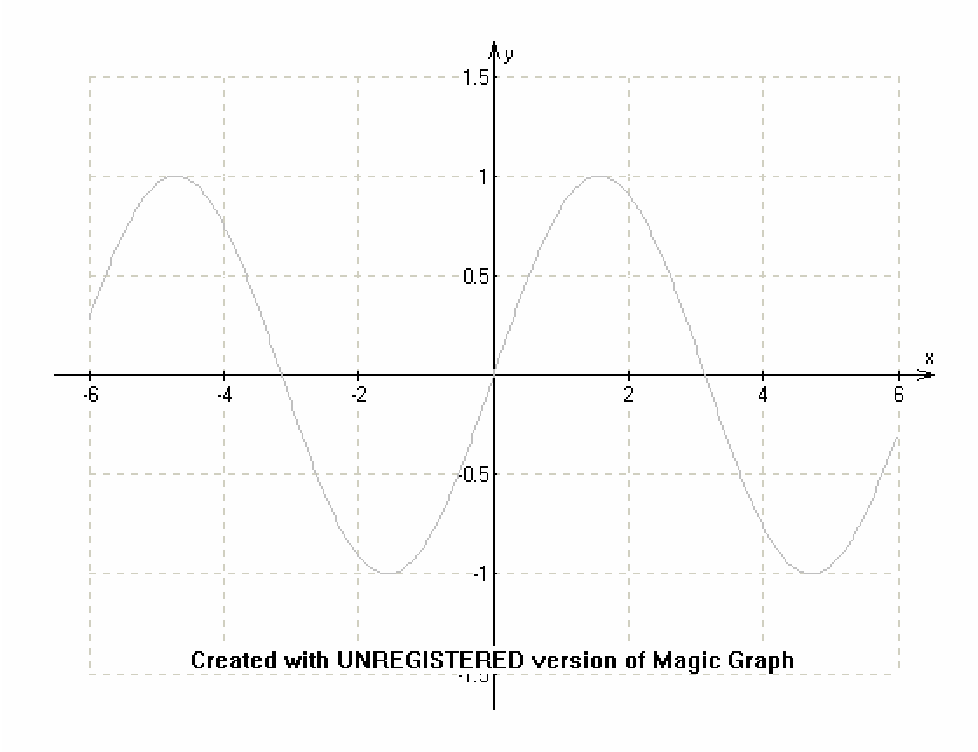


By comparing the true graph of y= sin 2x and my conjecture of it, I can conclude that except my prediction about the period the graphs are equal.
The period of y= sin 2x is not obtained by a dilation along the x-axis by the factor of 2 but of the reciprocal factor of 2 or the factor of ½.

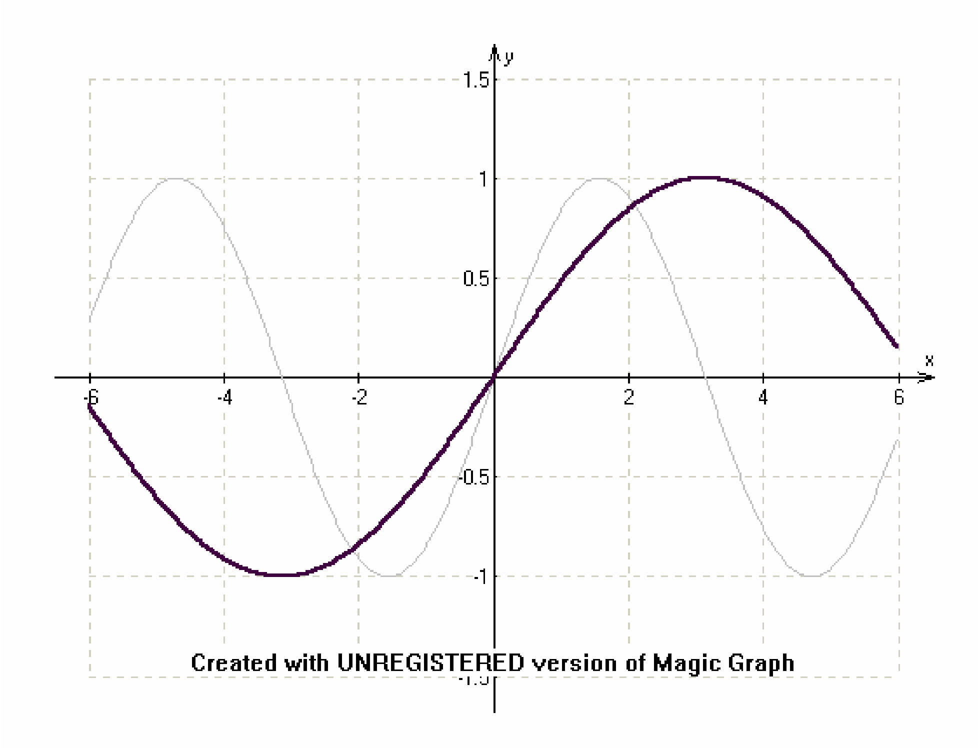
Graph of y= sin ½ x

Conjecture:
(a) transformation of the standard curve y= sin x:
from the finding of my last example I concluded that the period in y= sin Bx is multiplied by the *reciprocal* factor of B along the x-axis.
So as x is *directly* affected by the factor of 1/2 in y= sin 1/2x, I conjecture that the period of the standard curve, 2π, is now multiplied by the reciprocal factor of two: 4π.
Thus y= sin 1/2x is a dilation along the x-axis by the factor of 2.
(b) characteristics of y= sin 2x:
the curve is symmetric with the origin, it is an even function, and has infinite intercepts at multiples of 2π, as well as infinite maximum and minimum points at –1 and 1.
Amplitude: the coefficient of sine is 1 and therefore the amplitude of
y= sin 1/2x is 1.
The period is 4π, the complete graph consists of the above graph repeated over and over: the domain of the curve is the set of all real numbers and the range is [-1,1].

My conjectured graph of y= sin ½ x:



Justifying my conjectured graph of y= sin ½ x



By comparing the true graph of y= sin ½ x and my conjecture of it, I can conclude that they are equal and therefore my conjecture for its transformations and its characteristics was right.

Graph of y= sin –3x

Conjecture:

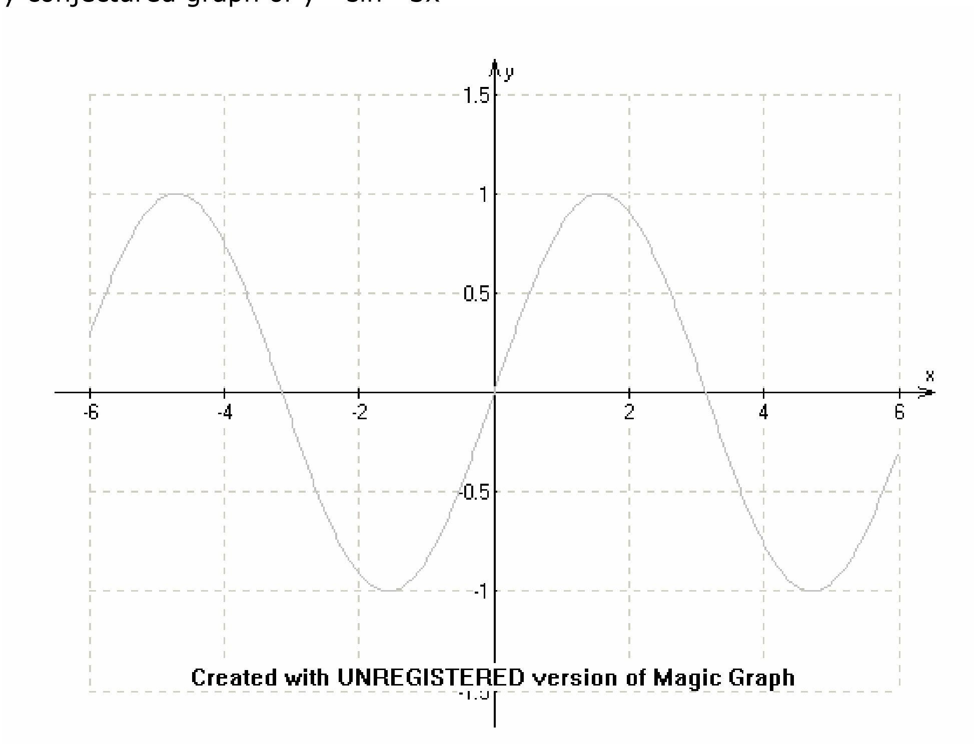
(a) transformation of the standard curve y= sin x:

from the finding of my last examples I concluded that the period in y= sin Bx is multiplied by the *reciprocal* factor of B along the x-axis.
So as x is *directly* affected by the factor of -3 in y= sin -3x, I conjecture that the period of the standard curve, 2π , is now multiplied by the reciprocal factor of three: $2\pi/3$. Thus y= sin –3 x is a dilation along the x-axis by the factor of 1/3 and because the negative three is also only directly affecting the x, y= -3 x will be obtained by reflecting the graph along the y-axis in addition.

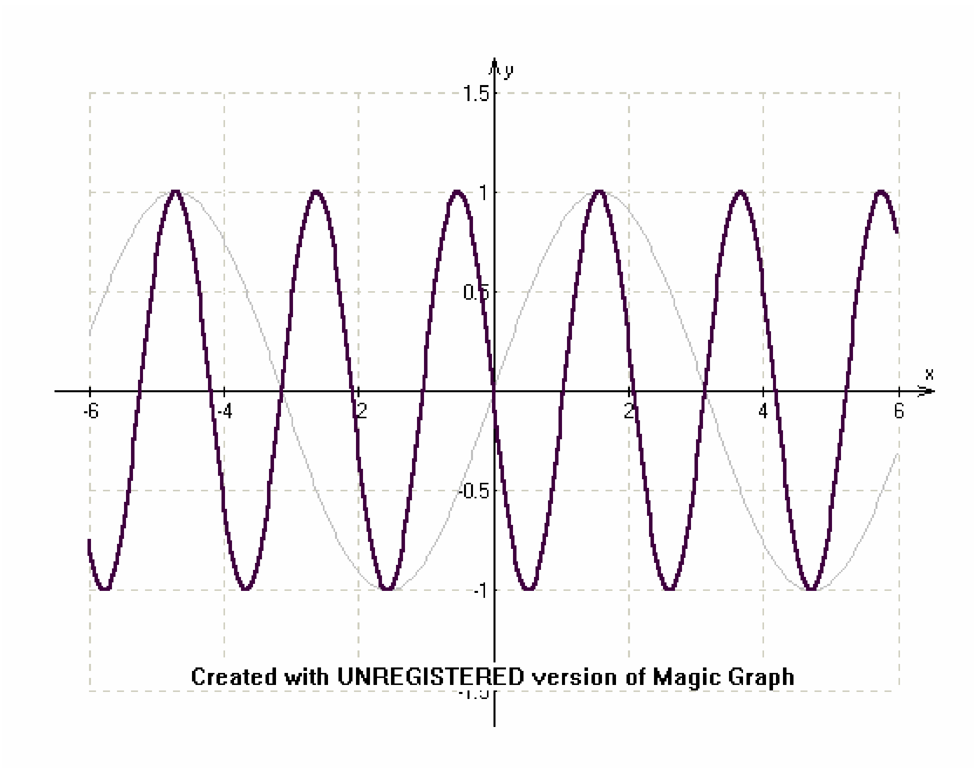
(b) characteristics of y= sin -3x:

the curve is symmetric with the origin, it is an even function, and has infinite intercepts at multiples of $\pi/3$, as well as infinite maximum and minimum points at –1 and 1.
Amplitude: the coefficient of sine is 1 and therefore the amplitude of y= sin -3x is 1.
The period is $2\pi/3$, the complete graph consists of the above graph repeated over and over: the domain of the curve is the set of all real numbers and the range is [-1,1].

My conjectured graph of y= sin –3x



Justifying my conjectured graph of y= sin −3x



By comparing the true graph of y= sin -3x and my conjecture of it, I can conclude that they are equal and therefore my conjecture for its transformations and its characteristics was right.

Graph of y= sin –1/3 x

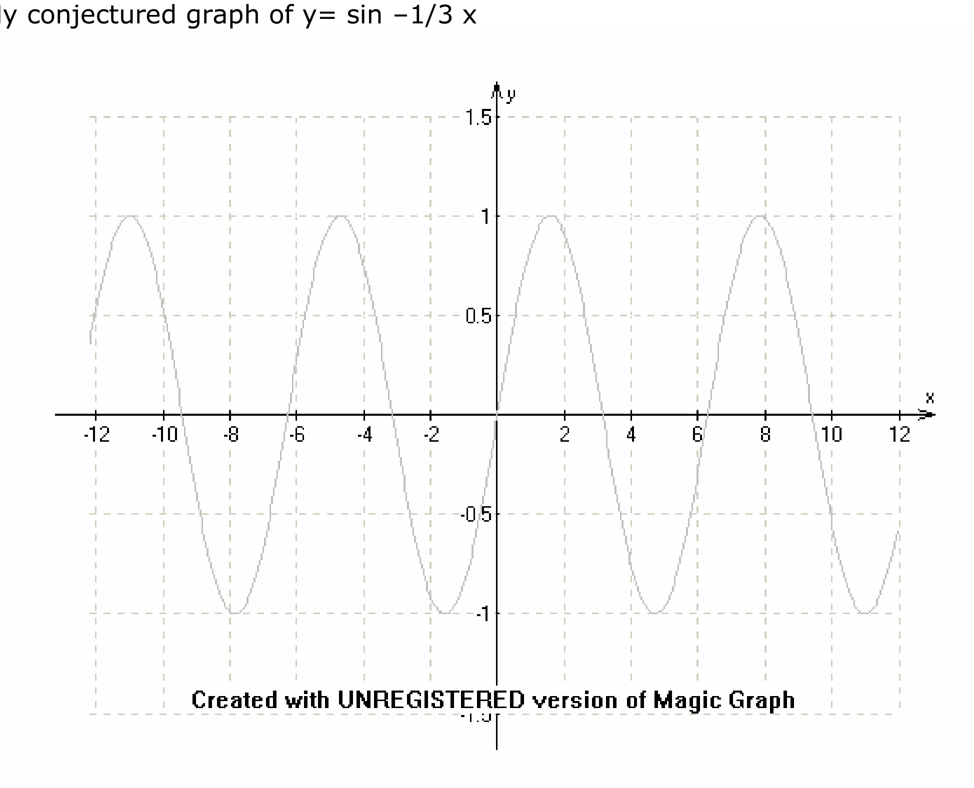
Conjecture:

(a) transformation of the standard curve y= sin x:
from the finding of my last examples I concluded that the period in y= sin Bx is multiplied by the *reciprocal* factor of B along the x-axis.
So as x is *directly* affected by the factor of –3 in y= sin –1/3x, I conjecture that the period of the standard curve, 2π, is now multiplied by the factor of three: 6π. Thus y= sin –1/3 x is a dilation along the x-axis by the factor of 3 and because the negative 1/3 is also only directly affecting the x, y= -1/3 x will be obtained by reflecting the graph along the y-axis in addition.

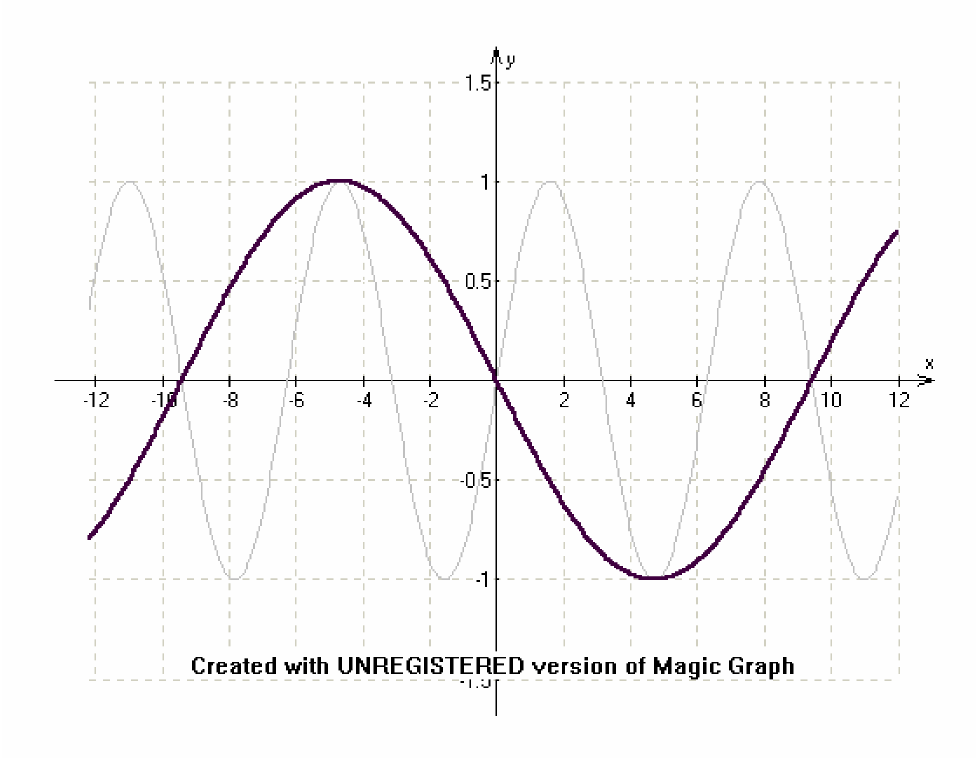
(b) characteristics of y= sin -3x:

the curve is symmetric with the origin, it is an even function, and has infinite intercepts at multiples of 3π, as well as infinite maximum and minimum points at –1 and 1.
Amplitude: the coefficient of sine is 1 and therefore the amplitude of
y= sin –1/3x is 1.
The period is 6π, the complete graph consists of the above graph repeated over and over: the domain of the curve is the set of all real numbers and the range is [-1,1].

My conjectured graph of y= sin –1/3 x



Justifying my conjectured graph of y= sin −1/3 x:



By comparing the true graph of y= sin −1/3x and my conjecture of it, I can conclude that they are equal and therefore my conjecture for its transformations and its characteristics was right.

Conclusion for y= sin Bx

As B varies in y= sin Bx only the shape varies: B is the period of the graph of sine. If B is negative, the graph of y= sin -Bx is obtained by changing the period by its reciprocal factor and reflecting it along the x axis.

Part 3

Graphs of the type y= sin(x+C)

Graph of y= sin(x+1)

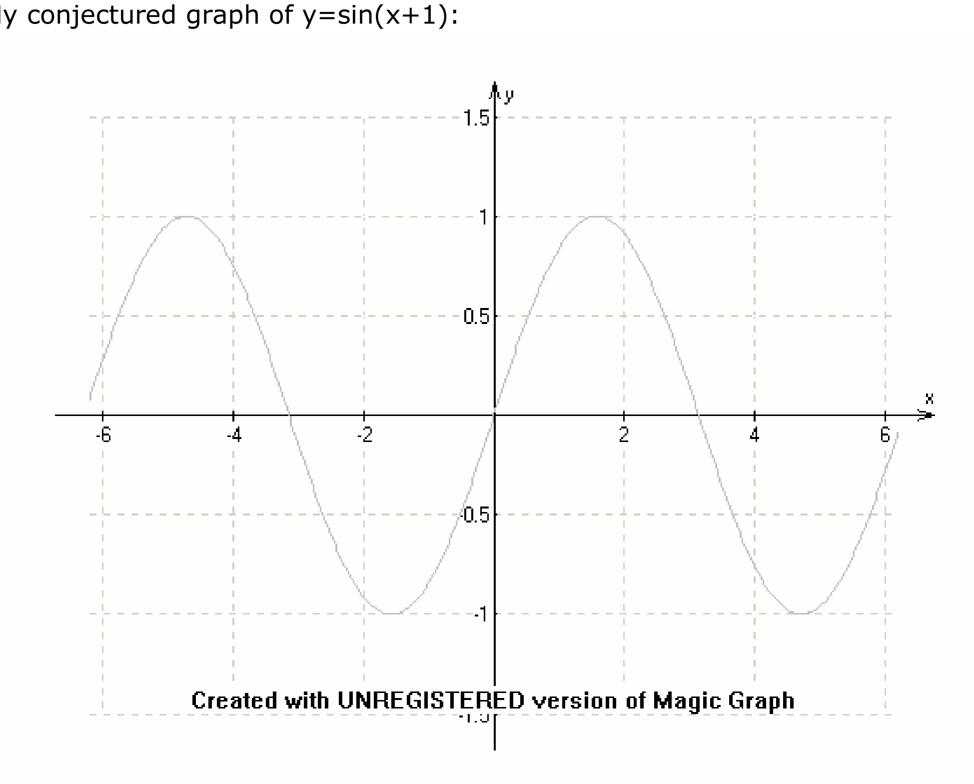
Conjecture:

(a) transformation of the standard curve y= sin x:
Now I am varying x by adding or subtracting C units. There are no negative values in this function so there won't be any reflections of the standard graph. This time only the position will be affected in the way that y= sin x will move 1 unit to the right, because it is positive one.

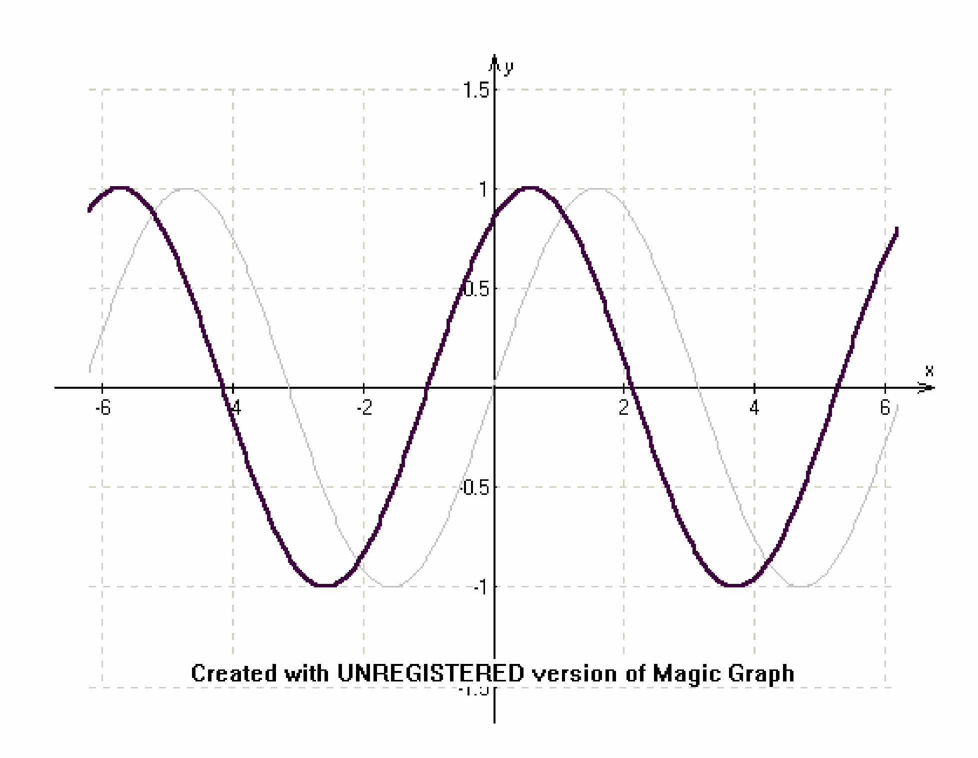
(b) characteristics of y= sin(x+1):

it is an even function, and has infinite intercepts at multiples of 1+ π , as well as infinite maximum and minimum points at -1 and 1.
Amplitude: the coefficient of sine is 1 and therefore the amplitude of
y= sin (x+1) is 1.
The period is equal to the period in the standard curve, the complete graph consists of the above graph repeated over and over: the domain of the curve is the set of all real numbers and the range is [-1,1].

My conjectured graph of y=sin(x+1):



Justifying my conjectured graph of y=sin(x+1):



By comparing the true graph of y= sin(x+1) and my conjecture of it, I can conclude that the x-intercept are multiples of π-1 and that the graph is obtained by translating y= sin x along the x-axis by 1 units to the left.

Graph of y= sin(x+(-1/2))

Conjecture:

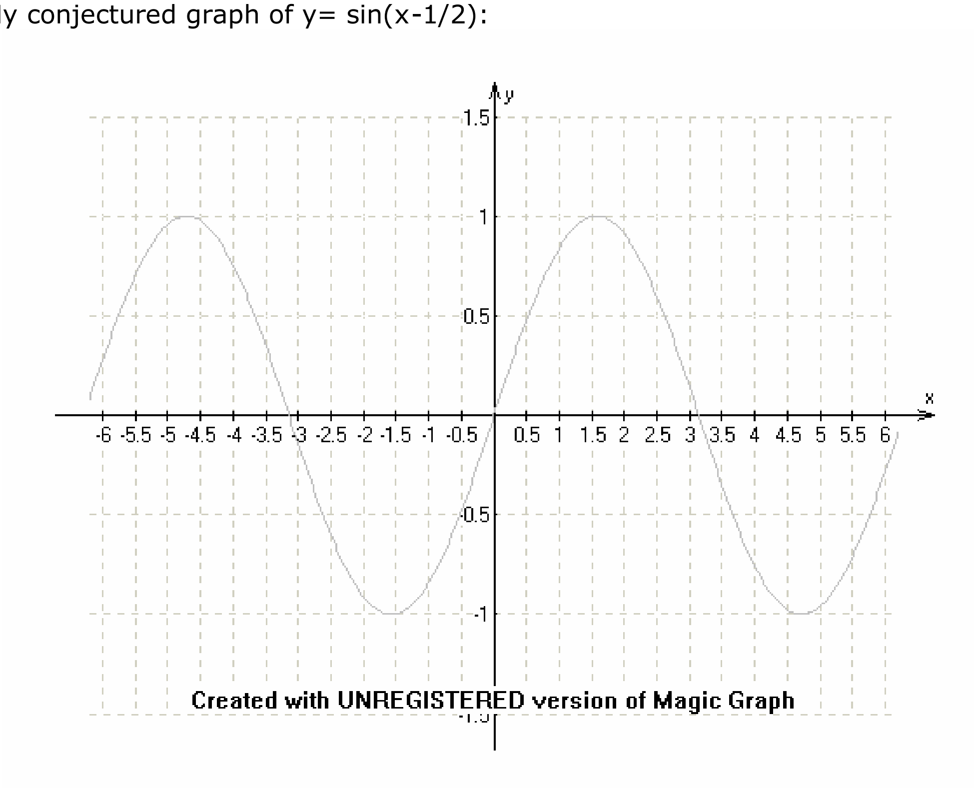
(a) transformation of the standard curve y= sin x:

this function is equal to y= sin(x-1/2). From my last example I found out that if one add C positive units to x, the standard graph moves C units to the left along the x axis, now C is negative and therefore I conjecture that the graph will move ½ units to the right.

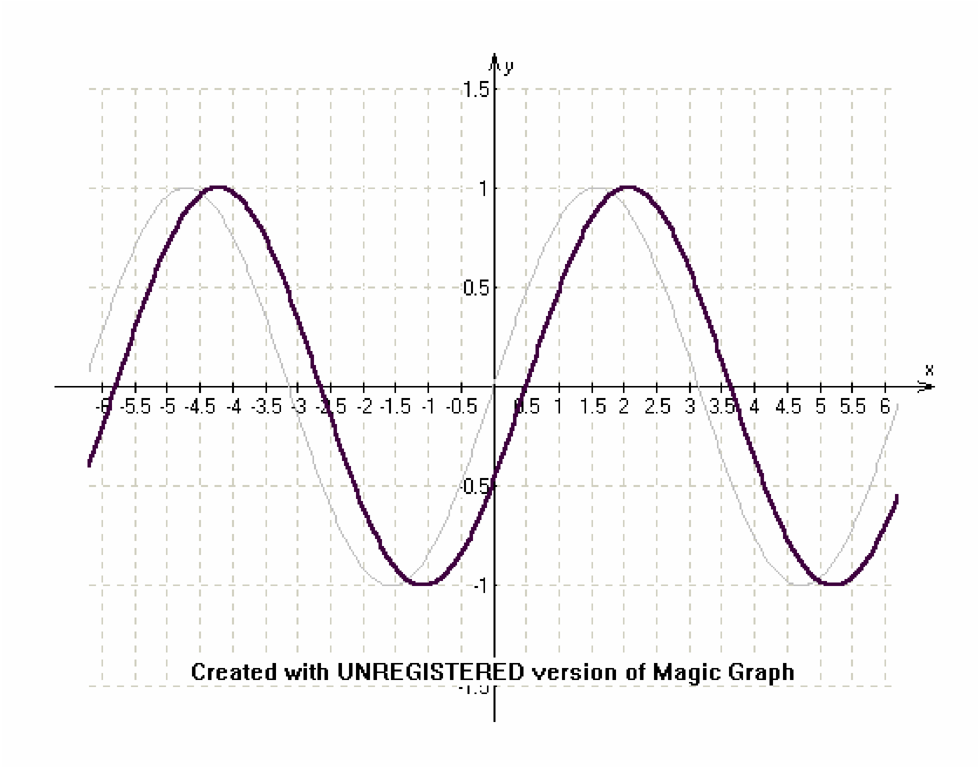
(b) characteristics of y= sin(x-1/2):

it is an even function, and has infinite intercepts at multiples of π - 1/2, as well as infinite maximum and minimum points at −1 and 1.
Amplitude: the coefficient of sine is 1 and therefore the amplitude of
y= sin (x-1/2) is 1.
The period is equal to the period in the standard curve, the complete graph consists of the above graph repeated over and over: the domain of the curve is the set of all real numbers and the range is [-1,1].

My conjectured graph of y= sin(x-1/2):



Justifying my conjectured graph of y= sin(x-1/2):



By comparing the true graph of y= sin (x-1/2) and my conjecture of it, I can conclude that they are equal and therefore my conjecture for its transformations and its characteristics was right.

Conclusion for y= sin(x+C)

As C varies in y= sin (x+C) only the position varies: If C is positive, the graph is translated C units to the left along the x-axis, if C is negative the graph is translated C units to the right along the x-axis.

Part 4

Graphs of the type y= Asin B(x+C)

Using my rules from the parts above I now can predict the shape and position of the following graphs:

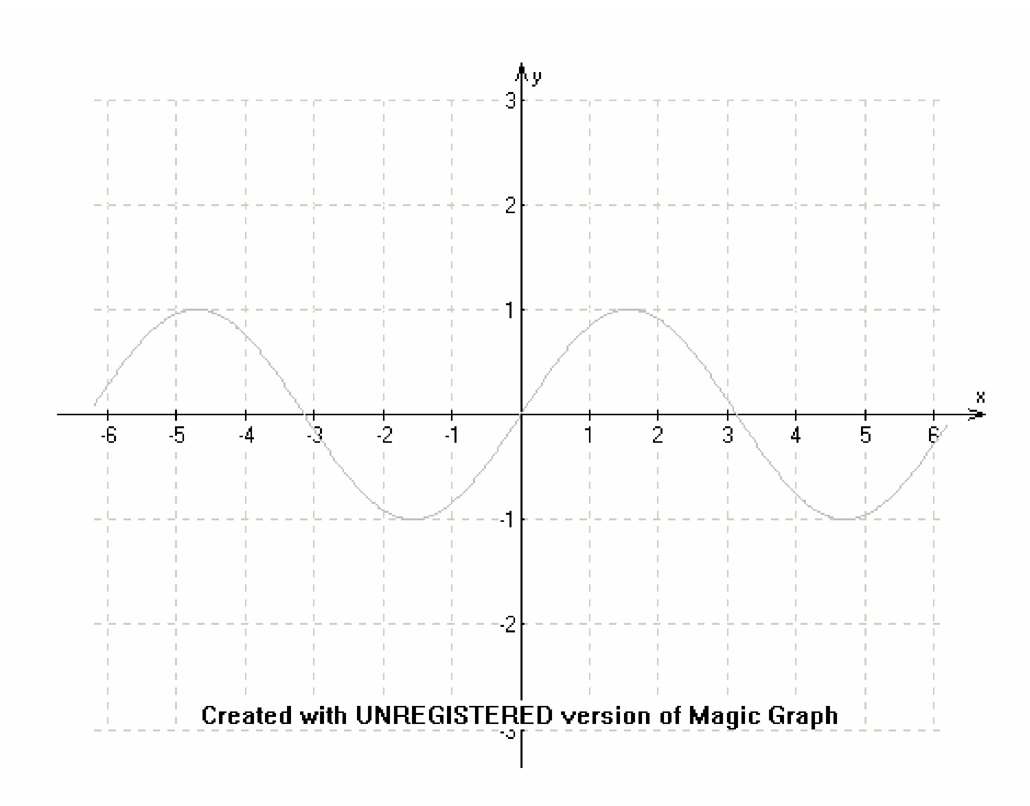
y= 3sin2(x+2)

Conjecture:

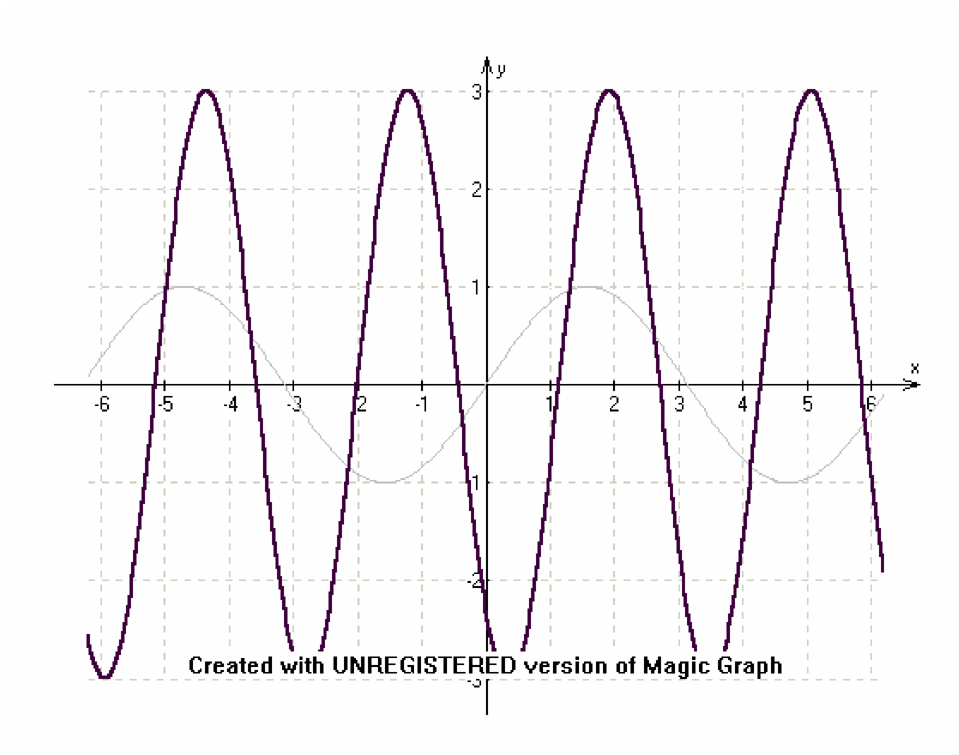
y= 3sin2(x+2) is equal to y= 3sin(2x+4) and is therefore obtained by a dilation along the x-axis by the factor of ½, a translation along the x-axis by 4 units to the left and a dilation along the y-axis by the factor of 3.

Intercepts= multiples of π/2 - 2
Amplitude= 3
Period= π

My conjectured graph of y= 3sin2(x+2):



Justifying my conjectured graph of y= 3sin2(x+2):



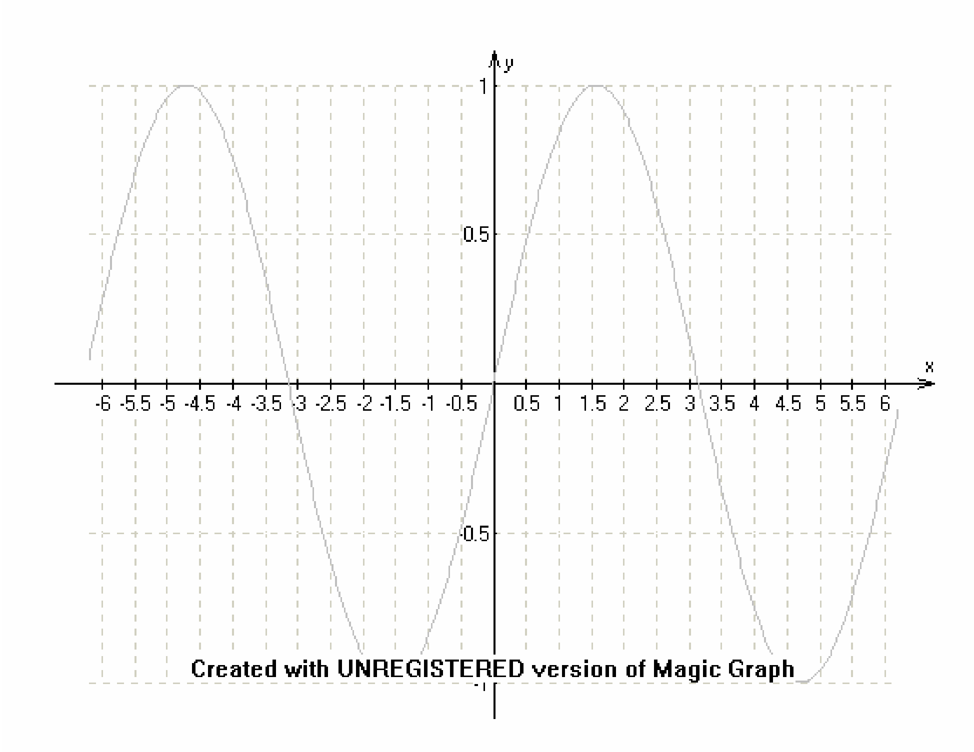
By comparing the true graph of y= 3sin2(x+2) and my conjecture of it, I can conclude that they are equal and therefore my conjecture for its transformations and its characteristics was right.

Graph of y= $\frac{1}{2}$sin3(x+1)

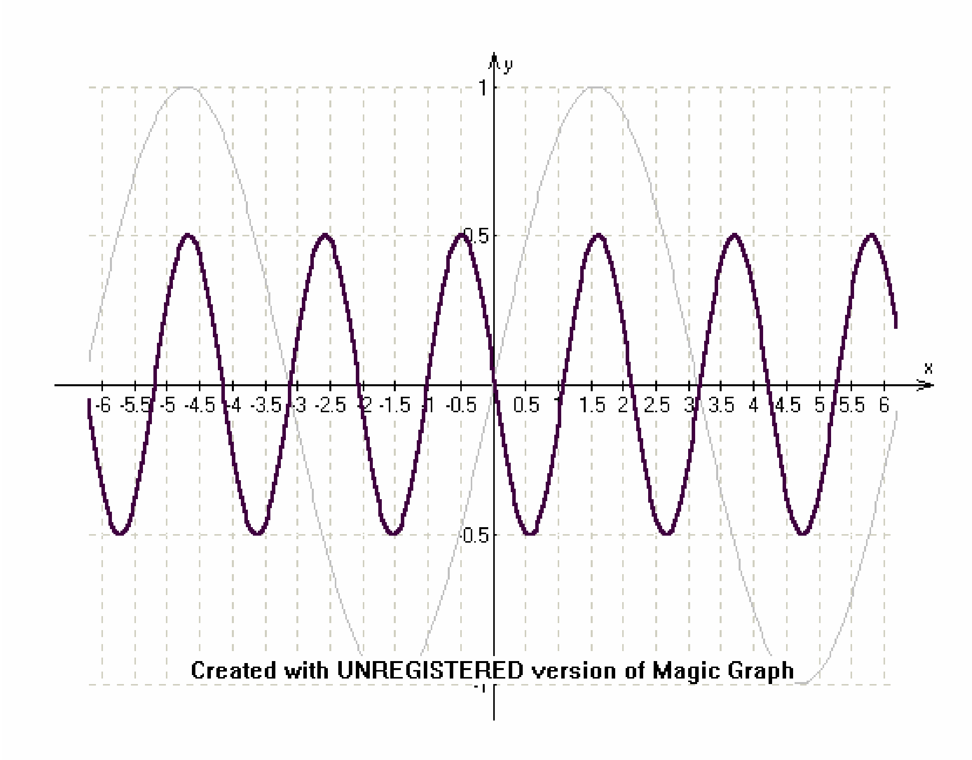
Conjecture:

y= $\frac{1}{2}$sin3(x+1) is equal to y= $\frac{1}{2}$sin(3x+3) and is therefore y= $\frac{1}{2}$sin3(x+1) obtained by a dilation along the x-axis by the factor of 3, a translation along the x-axis by 3 units to the left and a dilation along the y-axis by the factor of ½.

My conjectured graph of y= $\frac{1}{2}$sin3(x+1):



Justifying my conjectured graph of y= $\frac{1}{2}$sin3(x+1):

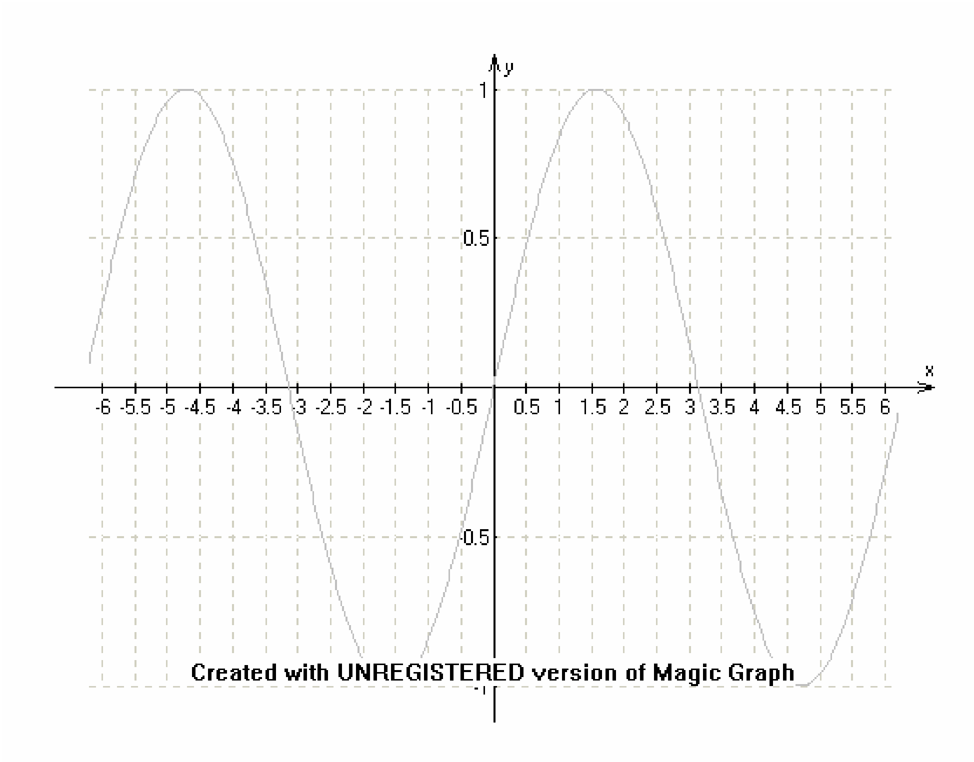


By comparing the true graph of y= $\frac{1}{2}$sin3(x+1) and my conjecture of it, I can conclude that they are equal and therefore my conjecture for its transformations and its characteristics was right.

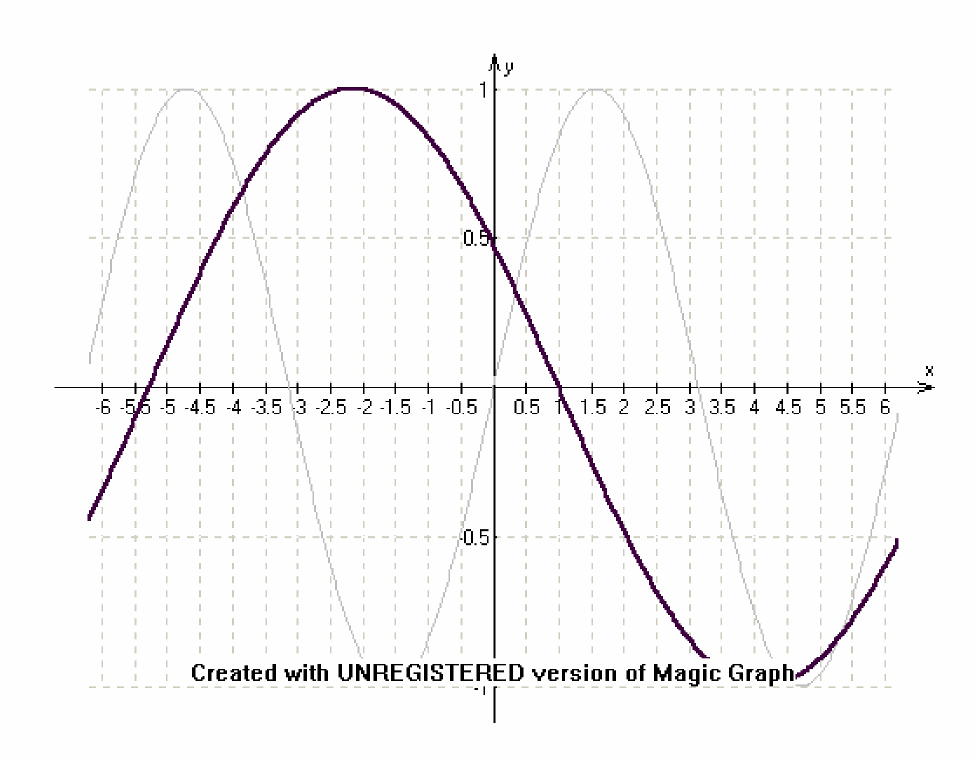
Graph of y= -sin$\frac{1}{2}$(x-1)

Conjecture: y= -sin$\frac{1}{2}$(x-1) is equal to y= -sin($\frac{1}{2}$x-1/2) and is therefore obtained by dilation along the x-axis by the factor of 2, a translation of ½ units to the right along the x-axis and a reflection on the x-axis.

My conjectured graph of y=-sin$\frac{1}{2}$(x-1):



Justifying my conjectured graph of y= $-\sin\frac{1}{2}(x-1)$:



By comparing the true graph of y= $-\sin\frac{1}{2}(x-1)$ and my conjecture of it, I can conclude that they are equal and therefore my conjecture for its transformations and its characteristics was right.

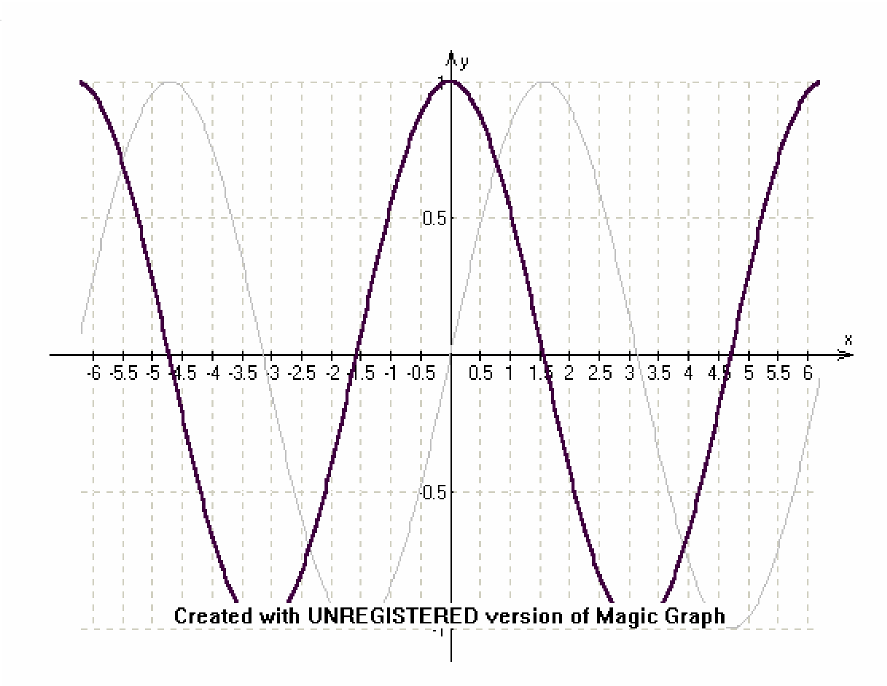
Predicting the shape and position of y= Asin B(x+C)

The shape and position of y= Asin B(x+C) can be predicted since A is the amplitude of the graph, B the period of the gr aph and c determines the position of the graph.

	A	B	C
Positive	dilation: stretch along y-axis	dilation: shrink along the x-axis	translation: shift along the x-axis C units to the left
Negative	dilation + reflection along x-axis	dilation: shift to the right along the x-axis	translation: shift along the x-axis C units to the right
Less than 1 **Bigger than 0**	dilation: shrink along the y-axis	dilation: stretch along the x-axis	translation: shift along the x-axis C units to the left

Part 5

Graph of y= cos x (black curve) and y= sin x (grey curve)



Both graphs have the same period (2π) and the same amplitude (1) and therefore if y= A sin Bx and y= A cos Bx ($A \neq 0$, B>0) is $|A|$ the amplitude and p= 2π / B the period.

Just their position is different - but if y= cos x is translated π/2 units to the right along the x-axis y= cos x is equal to y= sin x and vice versa.

To justify all of the above a last example:
y= -sin(½x – ½) and y= -cos(½x –½)