Math Portfolio Assignment **Type 2**

Investigating the Graphs of Sine Function

The task of this assignment is to investigate various sine graphs and recognise patterns and generalisations.

Part 1

Look at the graph of y = sin x.

Compare the graphs of y = 2 sin x ; y = 1/3 sin x ; y = 5 sin x.

Investigate other graphs of the type y = A sin x.

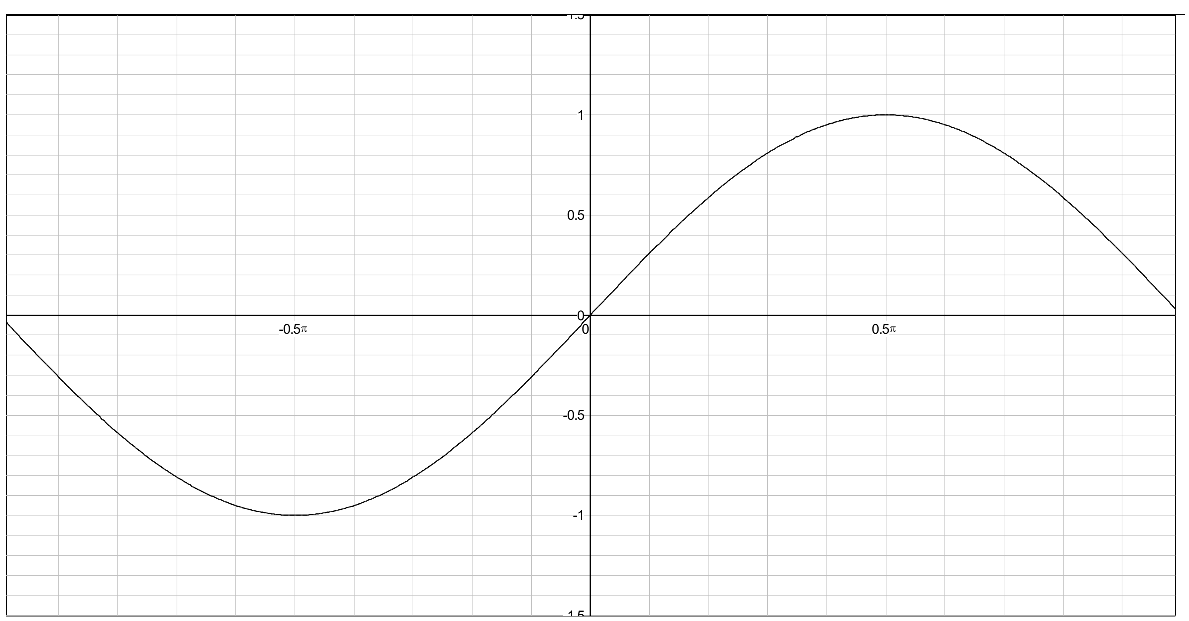
How does the shape of the graph vary as A varies?

Express your conjecture in terms of

a) **transformation(s) of the standard curve y = sin x.**
b) **characteristic(s) of the wave form.**

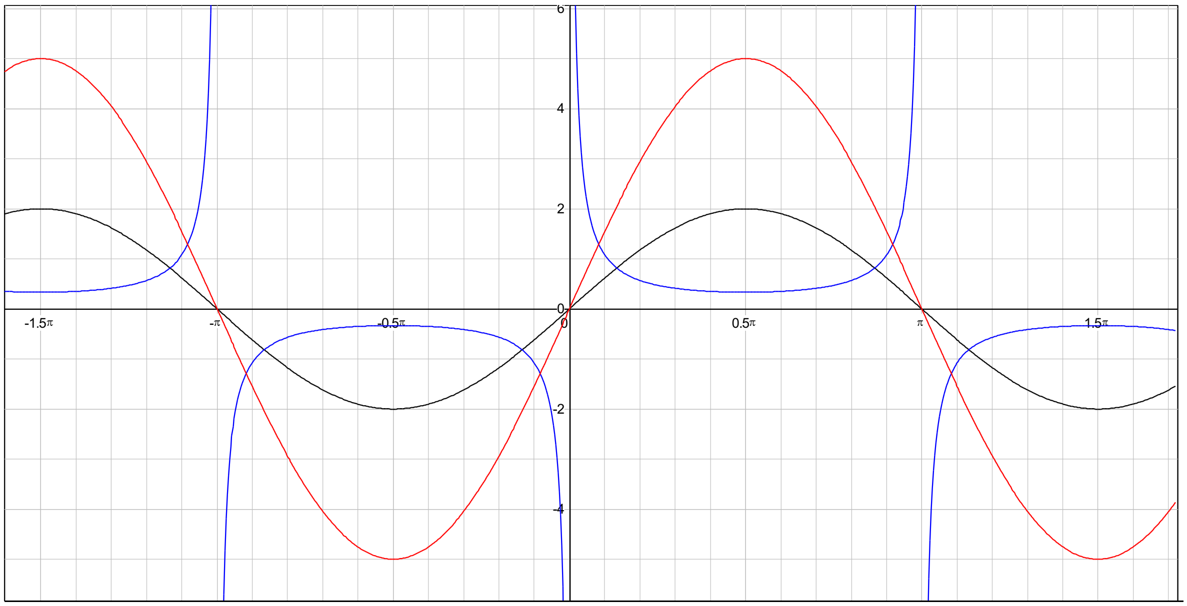
To do the graphs including graph $y = \sin x$, first I used my TI-83 Graph Calculator to have an idea of how the sketch should look. Then, for the real graph I used in the computer, a program called "Omnigraph" that is ideal for drawing graphs in the Cartesian set of axis, and that could be adjusted to draw trigonometric graphs as well with a suitable scale. This program provides all the facilities needed for them to be clear and accurate.

Graph to show the curve of y = sin x



This is the graph of $y = \sin x$ that would be the base curve used to compare all the other curves I would be drawing in this assignment in order to investigate how does different coefficients affect the position and shape of the sine graph. I am asked to compare $y = \sin x$ with three other graphs where sine has a different coefficient.

Graph to show the curves of y = 2 sin x; y = $^1/_3$ sin x; y = 5 sin x.



Using this graph that show 3 waves, and comparing them with the graph on the page before, I can notice how the wave has change in position and shape by using different coefficients of sine.

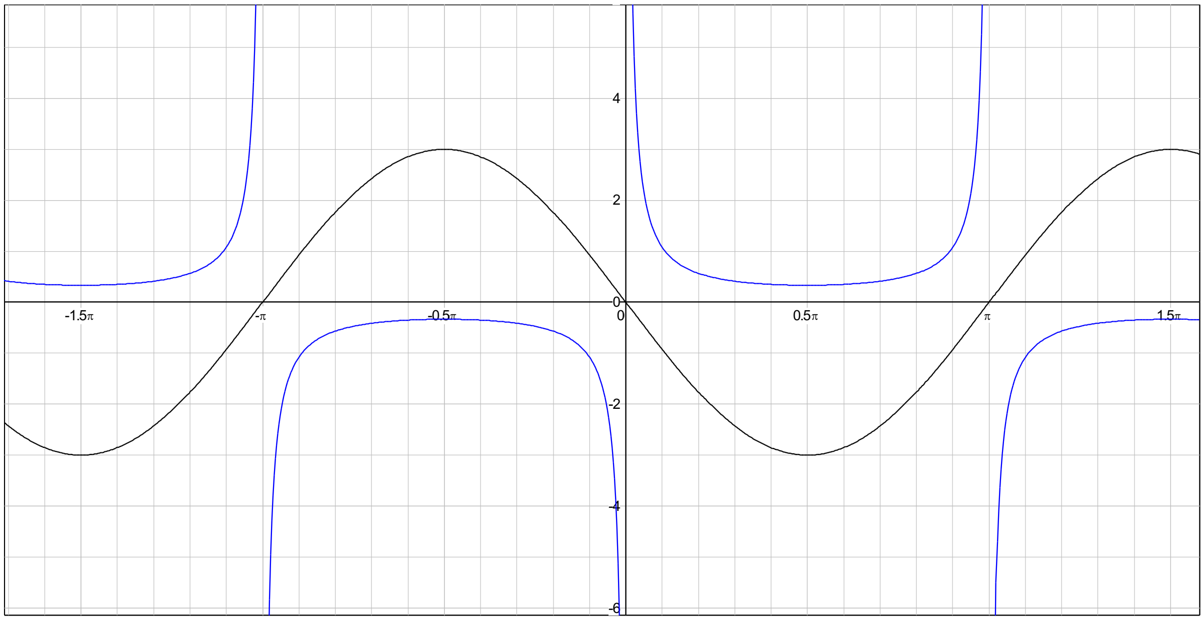
As the equation $y = \sin x$ has *1* as the sine's coefficient, the wave's amplitude is *1*. The same happens in all the other equations *($y = 2 \sin x$; $y = 1/3 \sin x$; $y = 5 \sin x$)* because the amplitude of their waves have changed according to their sine's coefficient. This is the position of the wave that does change when varying the sine's coefficient.

The position doesn't change and this can be noticed when comparing the two graphs shown above and on page 1. All four waves pass through the origin and also cut just in 360º and all its multiples. This means that they their period has not change when varying sine's coefficient.

To extend my investigations, I was asked to make sure that when changing the sine's coefficient it would affect the wave in the same way. The graph on page 3 shows what happens when in the equation $y = A \sin x$, A is changed.

I am going to use two examples to prove that A is also the amplitude of the wave. It is shown in the graph on the following page.

Graph to show the curves of y = - 3 sin x; y = 1/3 sin x



The graph above proves that the shape of the wave varies as *A* varies. *A* represents the amplitude of the wave. This makes the wave to be either smaller or bigger, but in either case its period remains equal. For example, if *A* is 4, then the amplitude of the wave will be 4, but the period will stay the same as in the curve $y = sin\ x$.

This conjecture can be expressed in terms of transformations because it can be noticed by looking at all the graphs shown above, that all of them are transformations of the base graph $y = sin\ x$.

For example, the transformation of the graph $y = sin\ x$ into curve $y = 2\ sin\ x$ is a one way stretch transformation, away from x axis (i.e. invariant line: x axis) with stretch factor + 2. This statement can be proven by looking at the graph on page 2.

Another example that can be proven on the graph shown above is the transformation of the graph of $y = sin\ x$ into the graph of $y = 1/3\ sin\ x$ which is a one way stretch transformation away from the x axis (i.e. invariant line: x axis) with stretch factor + $^1/_3$.

The same happens in every curve when changing *A* (the sine's coefficient) because in all cases it would be a one way stretch transformation, away from x axis and with a stretch factor +*A*. When *A* is a negative number, then the stretch factor will be negative as well.

This conjecture can also be expressed in terms of characteristics of the wave form. *A* represents the wave's amplitude, as it has been said before, but does not affect the wave's period as it remains constant in all curves. This can be noticed in all graphs shown before as it is easy to notice that the waves are stretched vertically according to their sine's coefficient. The wave has not changed its position in any of the cases; it always passes through the origin.

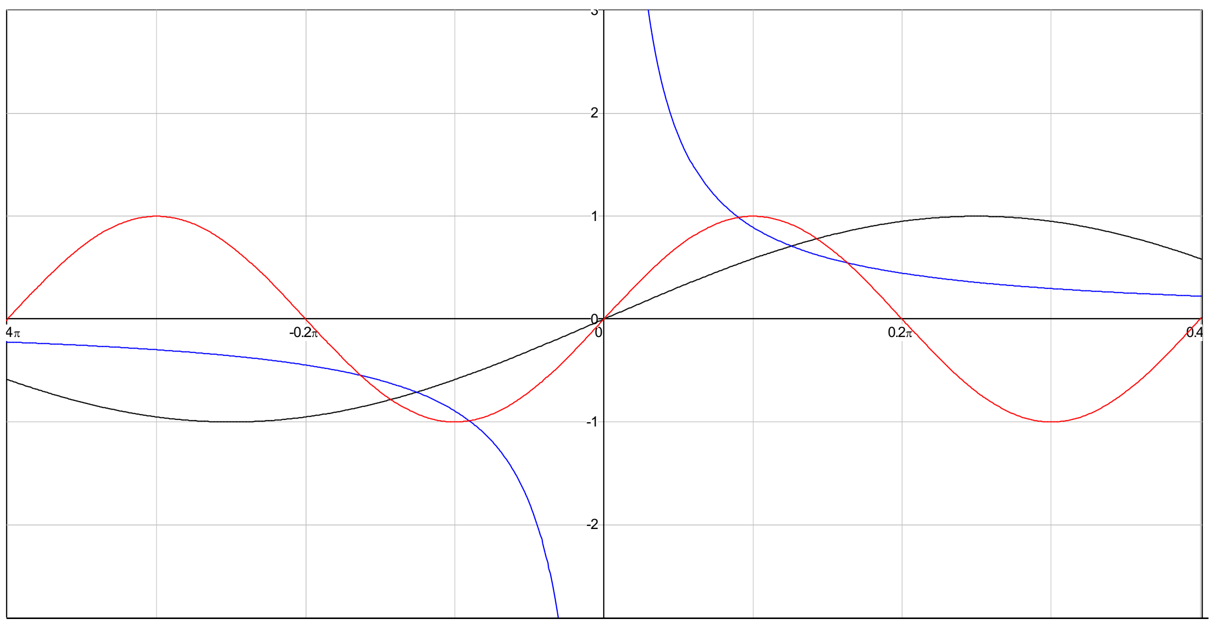
Part 2

Investigate graphs of the type y = sin B x in a similar way.

Part 2 is very similar to part 1, only that now instead of varying the sine's coefficient, I am going to vary *x*'s coefficient. *x* represents an angle.

The graph below shows three waves where *B* has been varied in order to investigate how this change affects the wave's position and shape.

Graph to show y = sin 2 x ; y = sin $^1/_3$ x and y = sin 5 x



In order to investigate what is the effect of *B* on the *y = sin x* curve, the graph above has to be compared with the first graph of this assignment, the base graph of *y = sin x*. It can be noticed that B affects the period of the wave. When varying B, it can be noticed that the wave's period is B times as short as the base wave's period. In other words, the inverse of B ($^1/_B$) is equal to the base wave talking in period terms.

For example, in the graph of *y = sin 2 x* shown above, the period is two times as short as the period of the base graph of *y = sin x*. Now, the period is only 180º which is half of the base or original period.

Another example, where it happens exactly the same thing, is in the graph of *y = sin 1/3 x* that is also shown above. The period is 1/3 times short, or in other words, 3 times as long as the base wave's period. In this case, the period is 1080° because 360° * 3 = 1080°.

This conjecture can also be expressed in terms of transformations because it can be noticed by looking at all the graphs shown above, that all of them are transformations of the base graph of *y = sin x*. In this case the transformation is a one way stretch away from the y axis with stretch factor + $^1/_B$.

For example, the transformation of the graph of *y = sin x* into the graph of *y = sin 5 x* is a one way stretch transformation, away from the y axis (invariant line: y axis), with a stretch factor +$^1/_5$. Exactly the same happens in every graph where B has been changed, with both positive and negative numbers. The stretch factor of the transformation is always the inverse number of the coefficient of x, in other words, the inverse of B.

This conjecture can be also expressed in terms of characteristics of the wave form, exactly as I have done in Part 1. B represents by how many times the wave's period has been shortened. Varying B does not affect the wave's amplitude as it remains constant in all graphs. This can be noticed in all graphs shown before as its easy to notice that the waves are stretched horizontally according to their angle's coefficient, or in other words, according to B. The wave has not changed its position in any of the cases; it always passes through the origin.

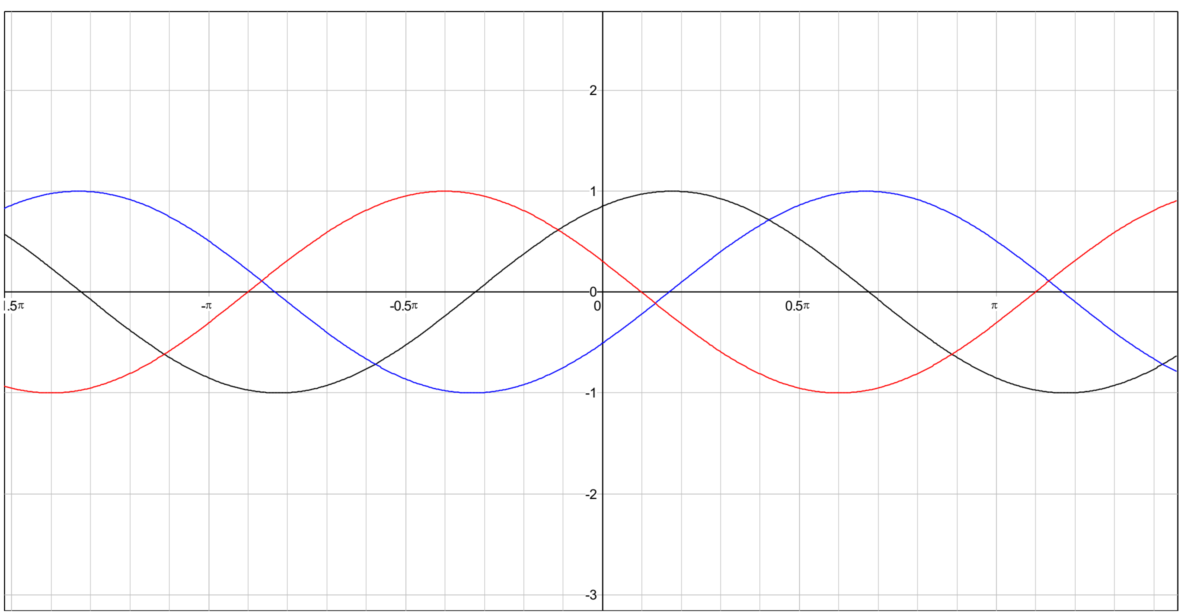
Part 3

Investigate the family of curves y = sin (x + C).

Part 3 is very similar to part 1 and part 2, only that now instead of varying the sine's coefficient, or the angle's coefficient, I am going to vary x by adding or subtracting more degrees to the angle. C is an angle and represents what has been added or subtracted to the angle.

The graph below shows three waves where C has been varied in order to investigate how this change affects the wave's position and shape.

Graph to show y = sin (x + 45º) ; y = sin (x + 100º) and y = sin (x – 60º)



In order to investigate what is the effect of C on the $y = sin\ x$ curve, the graph above has to be compared with the first graph of this assignment, the base graph of $y = sin\ x$. It can be noticed that C affects the position of the wave. When varying C, it can be noticed that the wave's position has been moved horizontally along the x axis. The angle that C represents is by how much the wave moves. If C is positive, then the wave moves to the left and vice versa. In other words, if it is negative the wave moves to the right. When varying *C* the amplitude and period remain the same as in the base graph of $y = \sin x$.

In terms of transformations, each wave acts upon a translation with vector $\begin{pmatrix} -C \\ 0 \end{pmatrix}$

For example, the transformation of the graph of $y = \sin x$ into the graph of $y = \sin(x + 45^\circ)$ that is shown in the graph on page 5, is a translation with vector $\begin{pmatrix} -45 \\ 0 \end{pmatrix}$

The same happens in the transformation of the graph of $y = \sin x$ in to the graph of $y = \sin(x - 60^\circ)$. This is a translation with vector $\begin{pmatrix} +60 \\ 0 \end{pmatrix}$

In terms of characteristics of the wave form, *C* makes the wave to change position always along the x axis, but remain the same in amplitude and period.

Part 4

Use your findings from parts 1, 2 and 3 to predict the shape and position of the graphs of y = 3 sin 2 (x + 90º); y = 1/2 sin 3 (x + 45º); y = - sin 1/2 (x - 45º).

Check your predictions.

If y = A sin B (x + C) explain how can you predict the shape and position of the graph for specific values of A, B and C.

Using my findings from parts 1, 2 and 3 I can predict what is going to happen with the shape and position of the waves of the following:

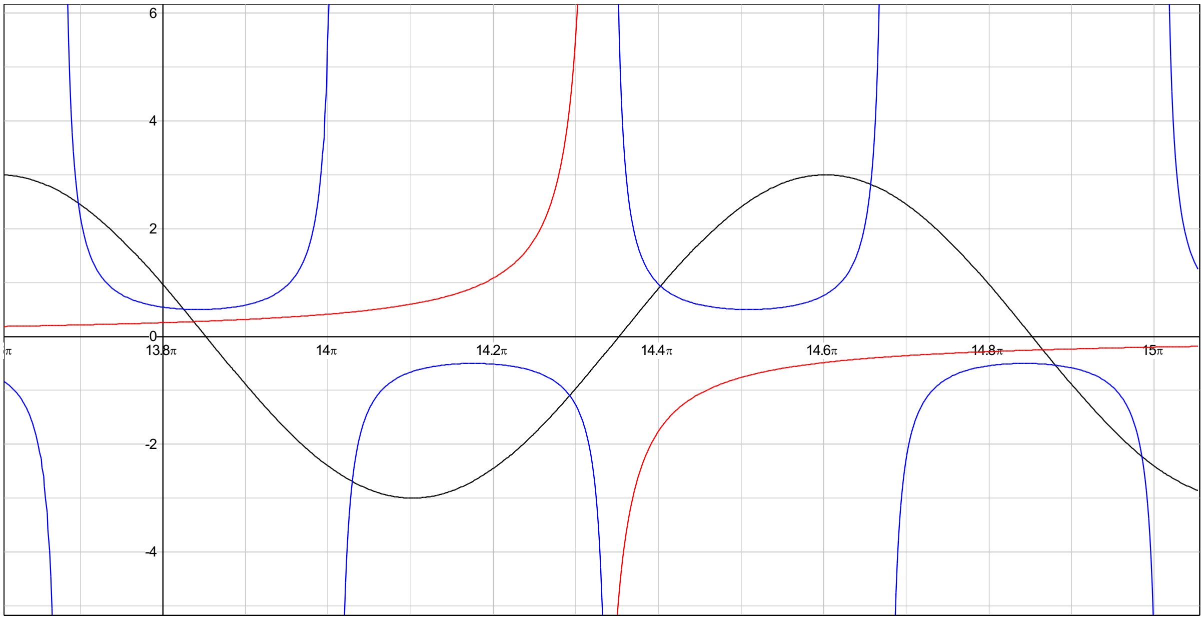
In the graph of $y = 3 \sin 2(x + 90^\circ)$, the following transformations will occur:
One way stretch, away from x axis, with a stretch factor of +3.
One way stretch, away from y axis, with a stretch factor of $+ \frac{1}{2}$
Translation with vector $\begin{pmatrix} -90 \\ 0 \end{pmatrix}$

In the graph of $y = 1/2 \sin 3(x + 45^\circ)$, the following transformations will occur:
One way stretch, away from x axis, with a stretch factor of + ½
One way stretch, away from y axis, with stretch factor of + 1/3
Translation with vector $\begin{pmatrix} -45 \\ 0 \end{pmatrix}$

In the graph of $y = -\sin 1/2(x - 45^\circ)$, the following transformations will occur:
One way stretch, away from x axis, with a stretch factor of -1.
One way stretch, away from y axis, with a stretch factor of + 2.
Translation with vector $\begin{pmatrix} +45 \\ 0 \end{pmatrix}$

The graph below proves that my predictions have been well made.

Graph to show y = 3 sin 2 (x + 90º) ; y = 1/2 sin 3 (x + 45º) and y = - sin 1/2 (x - 45º).



If *y = A sin B (x + C),* the shape and position of the graph will be different to the base graph and I know how the wave should be, by the findings I've made in this assignment. The amplitude of the wave will be *A*, the period of the wave will be *1/B* of 360º and the position of the wave will change by moving to the left as *C* is a positive number.

If *A* was a negative number, the wave's amplitude will be *A*, but the wave will be inverse as if it was a reflection in the x axis. If *B* was a negative number, the wave would be to the other side and it would be as a reflection in the y axis of the positive *B*. If *C* was a negative number, the curve instead of moving horizontally to the left, would move to the right.

If *A* was a fraction, the wave's amplitude would be *A* as well. We know that it would have a smaller amplitude than the base graph of y = sin x. If *B* was a fraction, and imagine the denominator is *d*, then the period will be *d* times as long as the one in the base graph of *y = sin x. C* being a fraction is only a very little angle, the movement will be to the same direction, but almost insignificant as the number is so little.

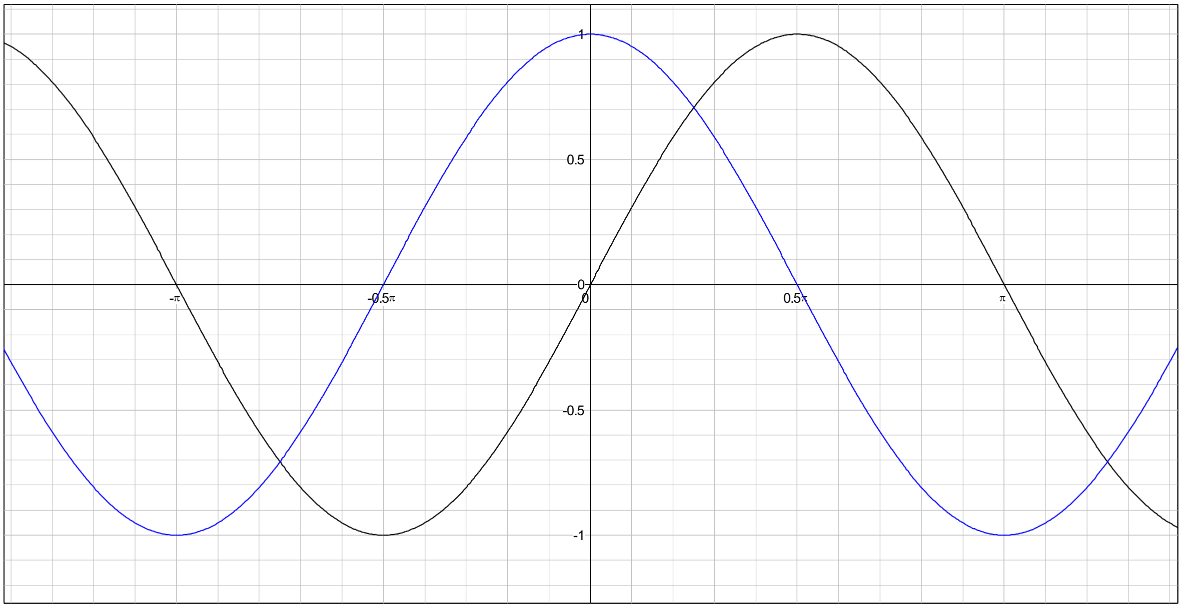
Part 5

How is the graph of y = cos x linked to the graph of y = sin x?

What is the relationship between these functions?

Part 5 is to compare and contrast the graph of sine and cosine.

Graph to show y = sin x and y = cos x

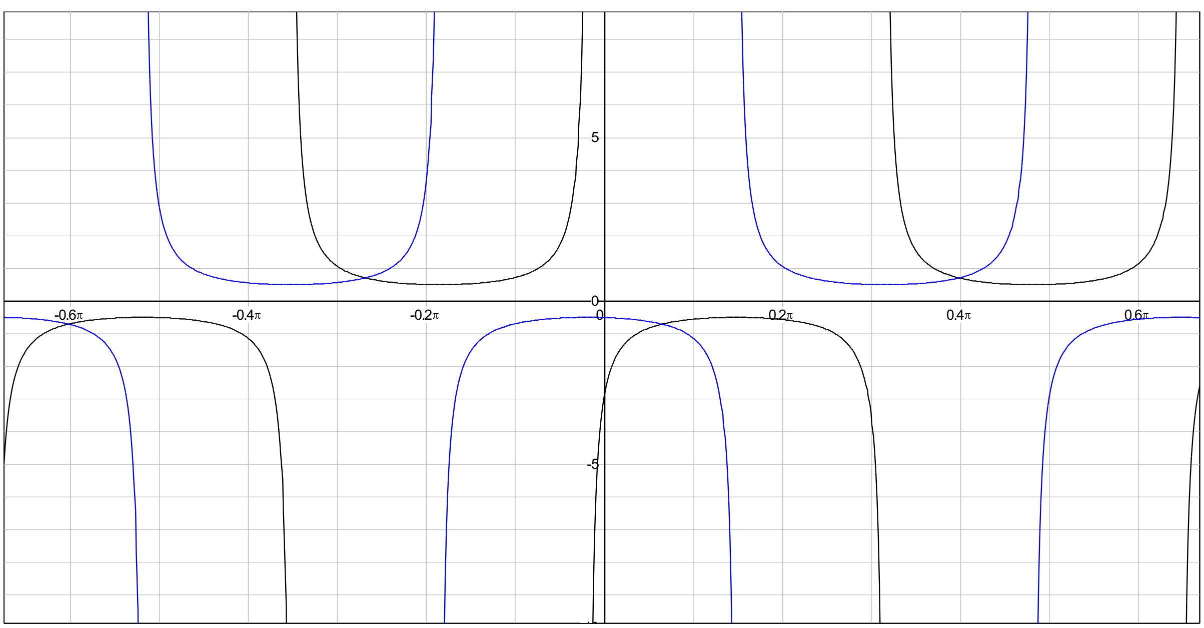


It can be noticed that both waves have the same amplitude and period, but a different position. It seems as if *y = sin x* has been added 180º to the *x*, so there has been a translation along the x axis of the vector $\begin{pmatrix}180\\0\end{pmatrix}$

It doesn't matter if *180°* is negative or positive because it will be the same as the period is 360° and 180° is its half.

To extend this investigation, and prove that it happens in every single graph of sine and cosine because all of them have the same characteristics, I am going to draw a graph showing this.

<u>Graph to show y = - ½ sin 3 (x – 90º) and y = - ½ cos 3 (x –90º)</u>



This graph proves what it was stated before as it can be clearly noticed that both waves have the same shape which includes amplitude and period, but the only that changes is position by *180º*, or in other words, by a translation of $\binom{180}{0}$